A New Spin on Long-Range Interactions

Coupling Continuous Spin Fields to Matter

Natalia Toro

based on 2303.04816 (JHEP) with P. Schuster, K. Zhou
(see also 1302.1198, 1302.1577, 1404.0675 with Schuster)

Outline

- Everything you learned about massless particles' spin in QFT is a special case. Helicities mix under Lorentz – controlled by ***spin scale*** ρ [Wigner 1939]
- Superspace-like formalism for gauge theories of any massless particle [1404.0675]
- **Coupling to matter particles** is a predictive and well-behaved deformation of familiar theories
- Open questions and future directions

Massless Spin, Covariantly

Spin state — Internal charges

Physical states take the form $|p^\mu, \sigma, n\rangle$

Spin σ characterizes state's transformation under **little group**.

Little group generators correspond with 3 components of $W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}J_{\nu\rho}p_\sigma$

(For massive particle at rest, $W^\mu = (0, m\mathbf{J})$, $W^2 = -m^2\mathbf{J}^2 = -m^2 s(s+1)$)

$W^0, \mathbf{W}\cdot\hat{\mathbf{p}}$ are proportional to familiar helicity generator $R = \mathbf{J}\cdot\hat{\mathbf{p}}$.

Transverse spatial components (W_x, W_y for $\mathbf{p} \propto \hat{\mathbf{z}}$) are less familiar:

$W_x \propto J_x + K_y$ and $W_y \propto -J_y + K_x$ generate transverse boost **and** rotation

Because these "translation" generators commute, massless little group is ISO(2)

The natural relativistic invariant is $W^2 = -(W_x^2 + W_y^2)$ – independent of helicity R!

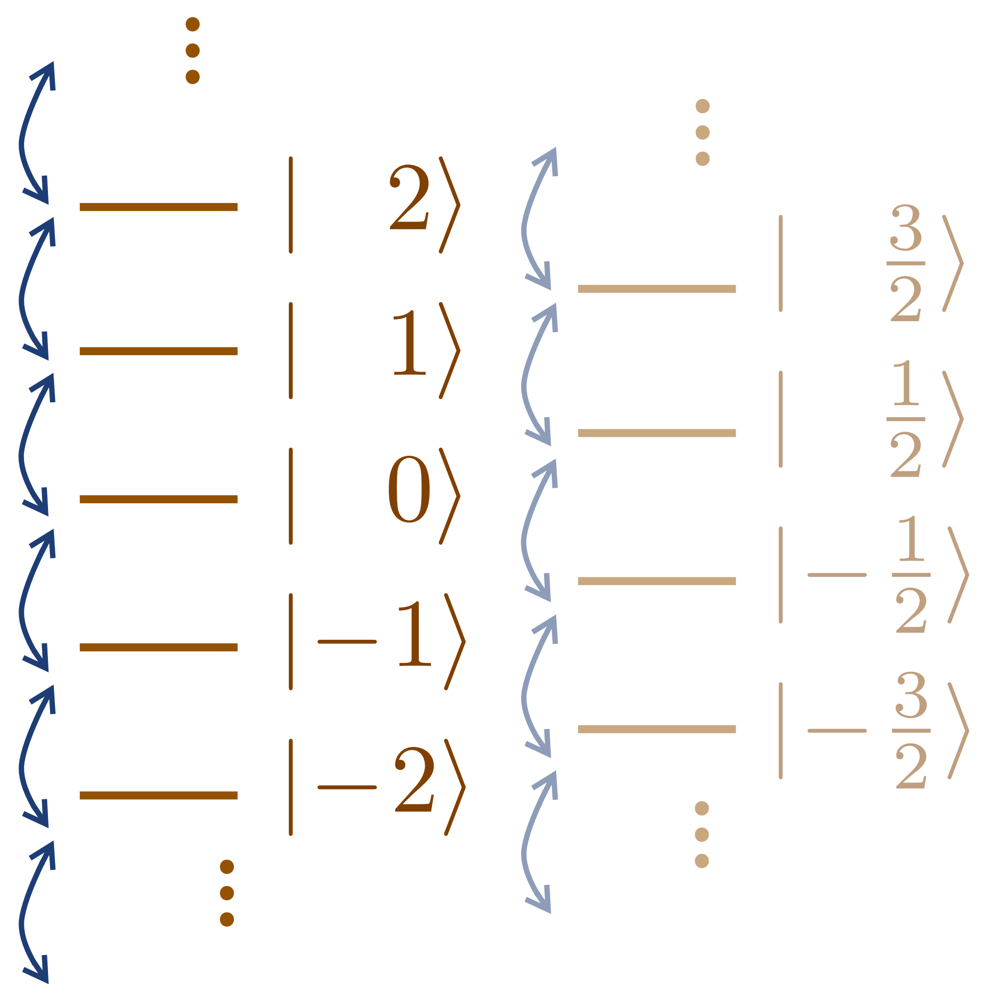
Massless Spin, Covariantly

It's convenient to work in a helicity eigenstate basis: $\mathbf{J}\cdot\hat{\mathbf{p}}\,|p,\sigma\rangle = \sigma\,|p,\sigma\rangle$,

Eigenvalues σ must be (half-)integer so that 4π rotation returns state to itself, since Lorentz group is doubly connected.

Can group "translations" into raising/lowering operators $W_\pm = W_x \pm iW_y$, which act as $W_\pm\,|p,\sigma\rangle = \rho\,|p,\sigma\pm 1\rangle$ where the invariant $W^2 = -\rho^2$ sets the **spin-scale** ρ.

σ-independent coefficient
$\Rightarrow$ ***infinite*** ladder of states



Exception: if $\rho = 0$ the states decouple. Each $|\sigma\rangle$ is a singlet representation, related only to $|-\sigma\rangle$ by CPT.

"Continuous spin" (or "CSP") refers to the general case $\rho \neq 0$, where integer helicities mix under Lorentz – just as they do for massive particles.

Why has this possibility been ignored?

Continuous spin includes high helicity states. Massless high spin is sick. Aren't these?

Robust constraints on high helicities (e.g. Weinberg-Witten, Weinberg soft theorems) all rely deeply on boost-invariance of helicity, so don't apply when $\rho \neq 0$.

Massive high spin is better analogy, and can be consistent – *e.g.* nuclei and string theory

Incompatible with field theory?

Early analyses didn't allow for gauge redundancy – problem is solved

Are infinitely many states at fixed energy a problem? (Cross-sections, thermodynamics)

Very interesting resolution follows from Lorentz symmetry (at least for best-controlled calculations)

At frequencies $\gg \rho$, **all but one helicity have parametrically suppressed interactions.**

Why should we care?

- Theorist: “Because it’s there”

  Falls out simply from postulates of relativity and quantum mechanics ⇒ worth understanding!

- Phenomenologist: “Because it might **really** be there”

  Can build experiments to measure/constrain the spin-scale of photons and gravitons

  All SM fields are either fundamentally massless (before EWSB) or unnaturally light.

  Thinking about models with non-zero spin scales may illuminate new approaches to many SM problems.

Outline

- Everything you learned about massless particles' spin in QFT is a special case. Helicities mix under Lorentz – controlled by ***spin scale*** ρ [Wigner 1939]
- Superspace-like formalism for gauge theories of any massless particle [1404.0675]
- **Coupling to matter particles** is a predictive and well-behaved deformation of familiar theories
- Open questions and future directions

A Field Theory for All Helicities

- Helicity $\pm h$ modes typically described by gauge theory of rank-h tensor fields
  - Notably subtle – many components: 2 physical, rest are pure gauge
- Continuous spin field should, in $\rho \to 0$ limit, decompose into similar modes

$$\Psi(\eta, x) \equiv \phi^{(0)}(x) + \eta^\mu \phi^{(1)}_\mu(x) + \eta^\mu \eta^\nu \phi^{(2)}_{\mu\nu}(x) + \dots$$

Lorentz acts as $x \to \Lambda x, \eta \to \Lambda \eta$

- Action:

$$\mathscr{L} = \frac{1}{2}\int_\eta \delta'(\eta^2+1)(\partial_x \Psi)^2 + \frac{1}{2}\delta(\eta^2+1)(\Delta\Psi)^2 \quad \text{with } \Delta\Psi \equiv \partial_\eta \cdot \partial_x + \rho$$

- Naively divergent integral – implicit division by "volume" $\int_\eta \delta(\eta^2+1)$ but fully fixed by symmetry & divergence can be regulated by analytically continuing η^0.

introduced in 1404.0675 – complementary pedagogical discussion in 2303.04816

What is Vector Superspace?

- A space for encoding little group action on-shell, and Lorentz off-shell

  Basis states' orthonormality, tree unitarity of CSP exchange, and little group covariance of matrix elements all follow from one identity:

  Whenever $\delta(\eta^2+1)k\cdot\partial_\eta F(\eta)=0,$ η-integration reduces to a 1D integral over a circle!

  $$\int_\eta \delta'(\eta^2+1)F(\eta)=\int_C F(\eta)$$

  C is unit circle of $\vec{\eta}$'s orthogonal to $\vec{k}$ (in any frame) ~ unit circle in "little group E_2 plane"

- A space enabling an enlarged spacetime symmetry

  Free action is invariant under a "bosonic supertranslation" $\delta x^\mu=\omega^{\mu\nu}\eta_\nu$ [Rivelles '14].

- Like Grassmann superspace, can use symmetry to show equivalence between $\int_\eta \delta'(\eta^2+1)$ and differential operator $J_0(\partial_\eta^2)$:

  Toy example: $\displaystyle\int_\eta \delta(\eta^2+1)\eta^\mu\eta^\nu=\frac{1}{4}g^{\mu\nu}\int_\eta \delta(\eta^2+1)\eta^2=-\frac{1}{4}g^{\mu\nu}$

A Field Theory for All Helicities

Using $\int_\eta \delta(\eta^2+1)\eta^\mu\eta^\nu = \frac{1}{4}g^{\mu\nu} \int_\eta \delta(\eta^2+1)\eta^2 = -\frac{1}{4}g^{\mu\nu},$

action with $\rho = 0$ reduces to sum of familiar massless actions, e.g.

$$\mathscr{L}[\Psi \to \phi(x)] = \frac{1}{2}\int_\eta \underbrace{\delta'(\eta^2+1)}_{\text{gives } 1}\underbrace{(\partial_x\Psi)^2}_{\partial_x\phi} + \frac{1}{2}\delta(\eta^2+1)(\partial_x \cdot \underbrace{\partial_\eta \Psi)^2}_{=0} \ = \frac{1}{2}(\partial_x\phi)^2$$

$$\mathscr{L}[\Psi \to \sqrt{2}\eta^\mu A_\mu] = \frac{1}{2}\int_\eta \delta'(\eta^2+1)\underbrace{(\partial_x\Psi)^2}_{2\,(\eta_\mu\partial_x A^\mu)^2} + \frac{1}{2}\delta(\eta^2+1)\underbrace{(\partial_x \cdot \partial_\eta \Psi)^2}_{2\,(\partial_\mu A^\mu)^2} \ = -\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}(\partial_\mu A^\mu)^2$$

But working in η-space directly is simple and powerful.

A Field Theory for All Helicities

Analogy with Maxwell Action

Action	$\int_x -\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}(\partial\cdot A)^2$	$\frac{1}{2}\int_{x,\eta} \delta'(\eta^2+1)(\partial_x\Psi)^2 + \frac{1}{2}\,\delta(\eta^2+1)(\Delta\Psi)^2$
Equation of Motion	$\Box A_\mu - \partial_\mu\partial\cdot A = 0$	$\delta'(\eta^2+1)\Box\,\Psi(\eta,x) - \frac{1}{2}\Delta(\delta(\eta^2+1)\Delta\Psi) = 0$
Gauge Invariance	$A_\mu \simeq A_\mu + \partial_\mu\epsilon(x)$	$\Psi(\eta,x) \simeq \Psi(\eta,x) + \overbrace{\left(\eta\cdot\partial_x - \frac{1}{2}(\eta^2+1)\Delta\right)}^{D}\epsilon(\eta,x) + (\eta^2+1)^2\chi(\eta,x)$

For $\rho = 0$, a nice re-packaging of Fronsdal actions for all rank-h tensor fields.

For $\rho \neq 0$, Δ mixes tensor components of $\Psi \Rightarrow$ **much** simpler in η-space

Each result has 1- or 2-line proof, using standard IBP and two δ-fn identities:

$\delta'(\eta^2+1)Df = \frac{1}{2}\Delta\left(\delta(\eta^2+1)f\right),\ \delta(\eta^2+1)\Delta D\epsilon = \Box\,\epsilon$

A Field Theory for All Helicities

Gauge fixing and physical states

Covariant Gauge Fixing	$\partial \cdot A = 0$	$\Delta\Psi(\eta, x) = 0$
Gauge-Fixed EOM	$\Box A_\mu = 0$	$\Box \Psi = 0$
Residual Gauge Freedom	$\Box \epsilon = 0$	$\Box \epsilon = \Delta\epsilon = 0$
Physical states (helicity basis)	$\psi_{\pm,k}(x) = e^{-ik\cdot x}\, \epsilon_\pm^\mu$	$\psi_{h,k}(\eta, x) = e^{-ik\cdot x}\, (\pm i\eta \cdot \epsilon_\pm)^{\lvert h \rvert}\ e^{-i\rho\eta\cdot q}$
	$\epsilon_- = \epsilon_+^*, \epsilon_\pm \cdot k = 0, \epsilon_+ \cdot \epsilon_- = -2$	$q\,.\,k = 1, q\,.\,\epsilon_\pm = 0$

Helicity basis states with $\rho \neq 0$ are simple functions of η, but **not** tensors!
Another reason to work in vector superspace

A Field Theory for All Helicities

Coupling to currents

Current Term in Action	$\delta S = -\int_x A^\mu(x) J_\mu(x)$	$\delta S = \int_{x,\eta} \delta'(\eta^2+1)\Psi(\eta,x)J(\eta,x)$
Continuity condition from gauge-invariance	$\partial_\mu J^\mu = 0$	$\delta(\eta^2+1)\Delta J(\eta,x) = 0$
EOM in suitable gauge	$\Box A^\mu = J^\mu$	$\Box \Psi(\eta,x) = J(\eta,x)$

Once we have found a suitable current, can use familiar machinery to compute radiation and CSP-exchange forces.

Solving the Continuity Condition

Ansatz for worldine-local current coupled to continuous spin field, in momentum space:

$$J(\eta, x) = \int d\tau \; d^4k \; e^{ik\cdot(z(\tau)-x)} f(k, \dot{z}, \eta), \text{ continuity condition } \; (-ik \cdot \partial_\eta + \rho) f = 0$$

(up to total derivative terms)

Simple example solution: $f = g e^{-i\rho \frac{\eta \cdot \dot{z}}{k \cdot \dot{z}}}$.

Note $\rho \to 0$ limit: $f = g \to J(\eta, x) = \int d\tau \delta(x - z(\tau))$ – standard worldline coupling to scalar field.

Radiation from a Moving Particle

$$\frac{dP_h}{d\omega d\hat{\mathbf{r}}} = \frac{\omega^2}{8\pi^2}\int_\eta \delta'(\eta^2+1)\,|\,\psi^*_{h,\omega\hat{\mathbf{r}}}(\eta)J(\eta,\omega\hat{\mathbf{r}})\,|^2$$

For our ansatz current, and simple harmonic motion:

Power radiated in minimally coupled scalar

$$P = \frac{g^2\omega^2}{24\pi}\times\begin{cases} v_0^2\left(1-\frac{(\rho v/\omega)^2}{20}+\dots\right) & h=0 \\ (\rho v/\omega)^2/2+\dots & h=\pm 1 \\ \mathcal{O}(\rho v/\omega)^{2|h|} & |h|\geq 2 \end{cases}$$

Subleading in $\rho v/\omega$

$$\bar{P} = \begin{cases} g^2 a_0^2/24\pi & a_0 \gg \rho v_0^2 \\ g^2\rho a_0/8\pi^2 & a_0 \ll \rho v_0^2 \end{cases}$$

Power falls off at low acceleration, but ***slower*** than for ordinary scalar

General Solutions

Most general solution to continuity condition (up to total derivative terms) can be written as

$$f(k, \dot{z}, \eta) = e^{-i\rho\frac{\eta\cdot\dot{z}}{k\cdot\dot{z}}}\hat{g}(k\cdot\dot{z}) + \underline{\mathcal{O}X(k, \dot{z}, \eta)}$$

"Shape" terms:
Similar to charge radius and higher-order operators
But *can* "move" singularity in first term

where free eom is $\delta'(\eta^2 + 1)\mathcal{O}\Psi = 0$

$\Rightarrow$ Shape terms do not couple to continuous spin radiation modes (as in E&M)

$\Rightarrow$ Worldline interactions with radiation **fully** determined by $\hat{g}$.

$$\hat{g} = \begin{cases} g & \text{scalar-like current} \\ \frac{e}{\rho} k\cdot\dot{z} & \text{vector-like current} \\ (k\cdot\dot{z})^n/\Lambda^n & \text{non-minimal currents*} \end{cases}$$

Classical results in these cases (scalar-like and vector-like) are main focus of 2303.04816

*Expect a special case is tensor-like, i.e. coupling dominantly to worldline $T^{\mu\nu}$ plus terms that cancel on worldline equations of motion

Radiation from a Moving Particle

Simple Harmonic Motion with Vector-Like Current

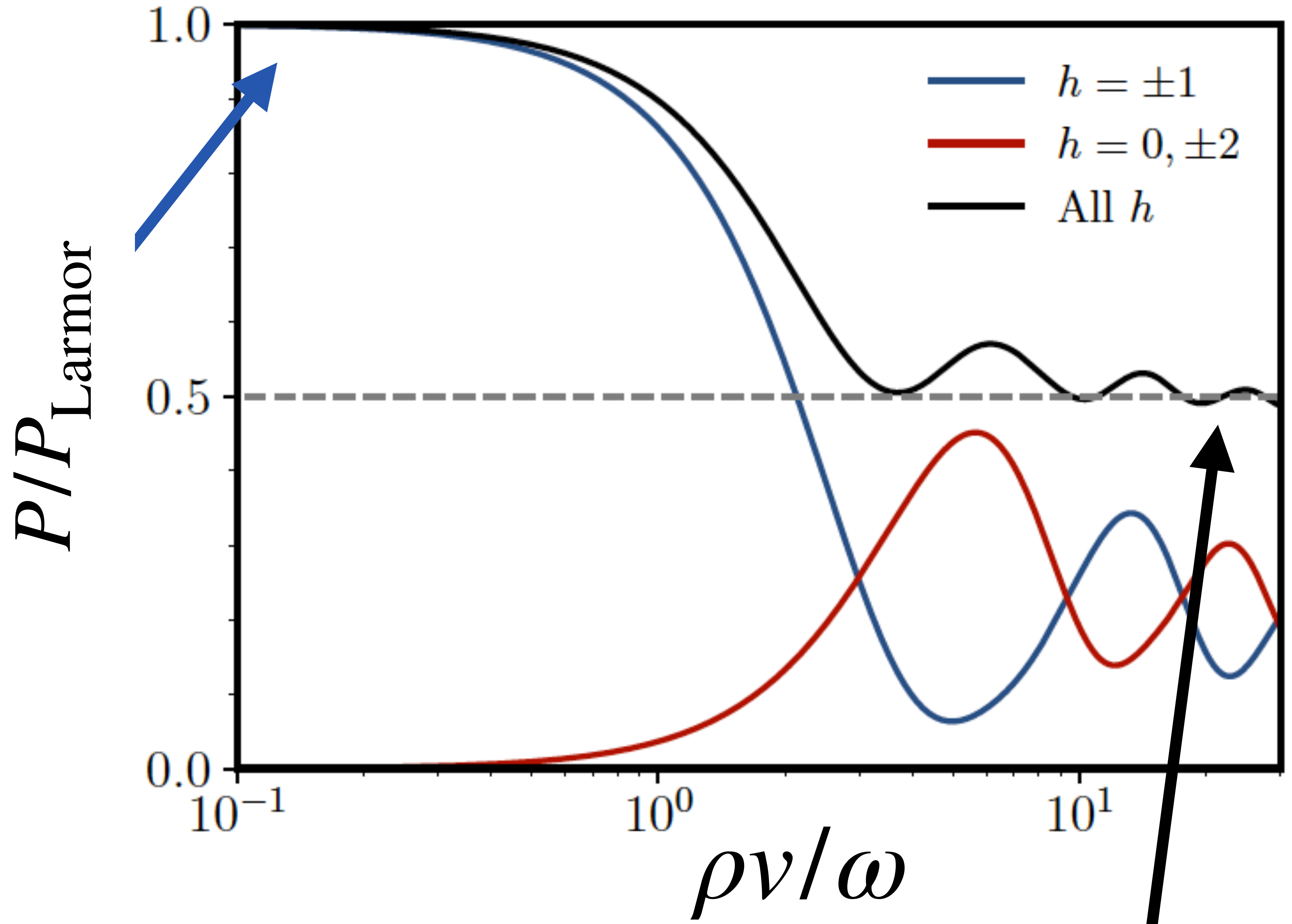
Indeed, for vector-like currents,

$$P = \frac{e^2\omega^2 v_0^2}{12\pi}\left(1 - \frac{9}{80}\frac{\rho^2 v_0^2}{\omega^2} + \dots\right)$$

Standard Larmor power

For small $\rho v/\omega$, power **matches Larmor** and **dominated by h=±1 modes**

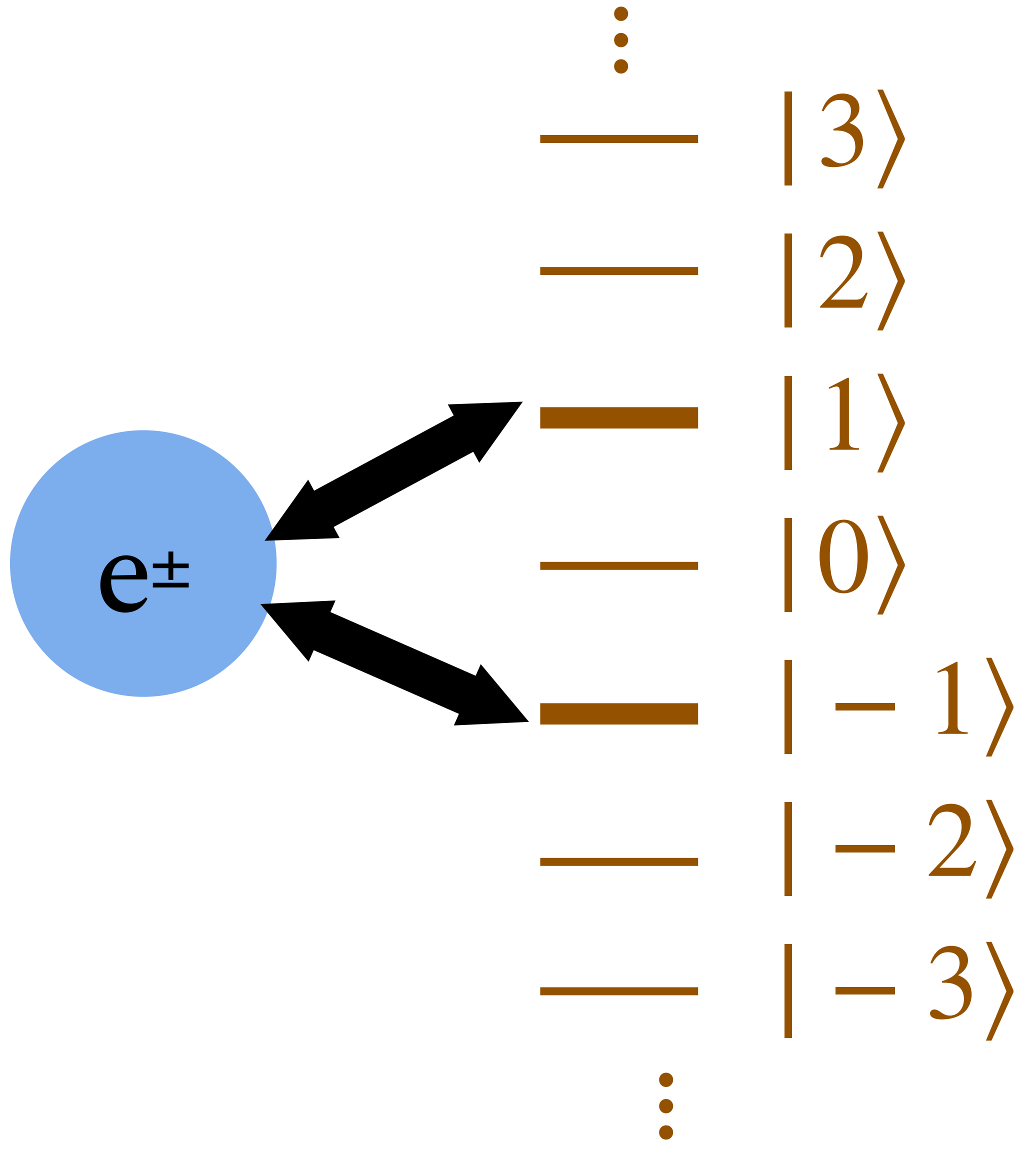
(Physical manifestation of formal correspondence noted earlier)



At large $\rho v/\omega$, power spread among many modes, harmonics **but total power emitted has finite limit.**

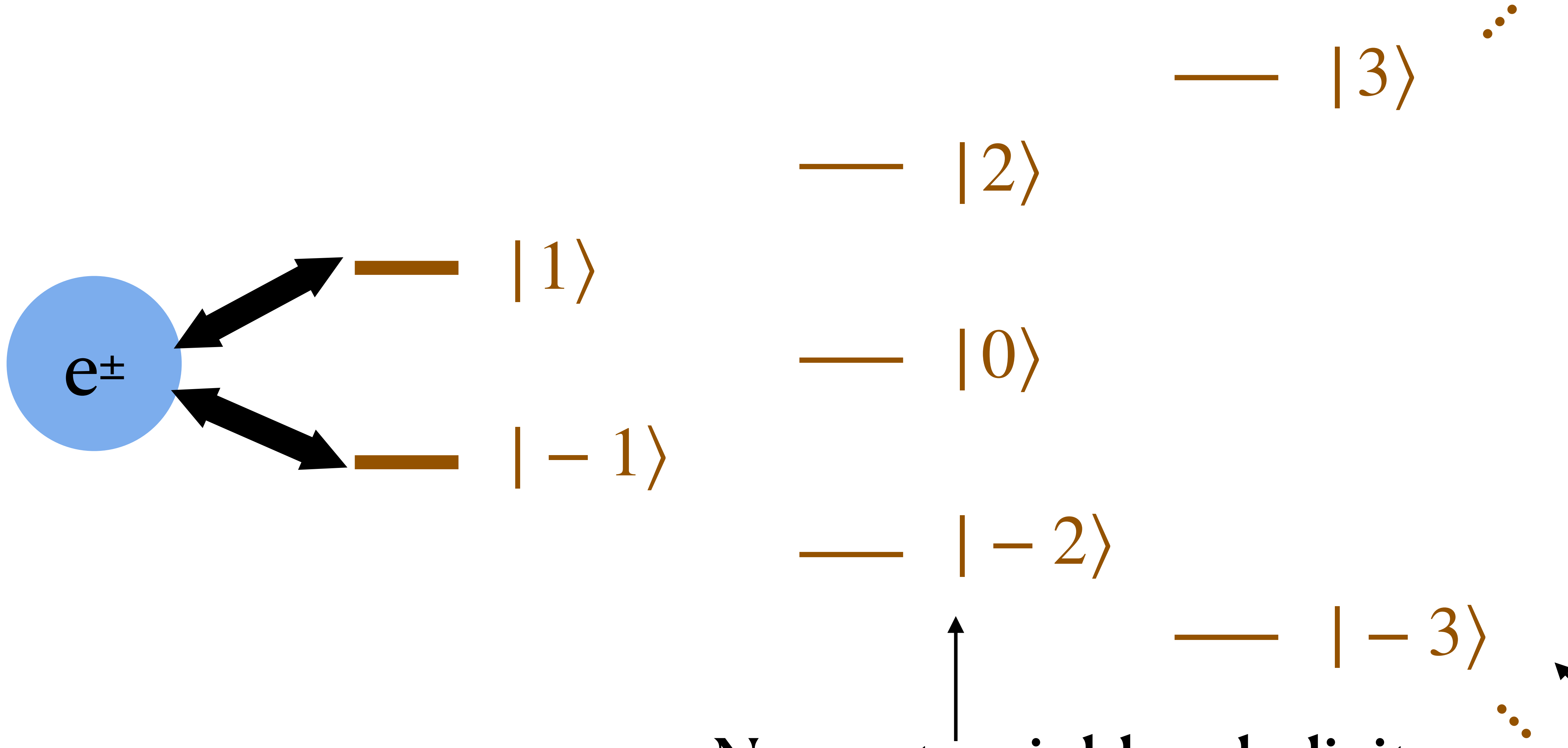
Continuous Spin Particles are like familiar massless particles with an associated dark sector

Covariant interactions single out **one** helicity with unsuppressed coupling (e.g. |h|=1)



Continuous Spin Particles are like familiar massless particles with an associated dark sector

Covariant interactions single out **one** helicity with unsuppressed coupling (e.g. |h|=1)



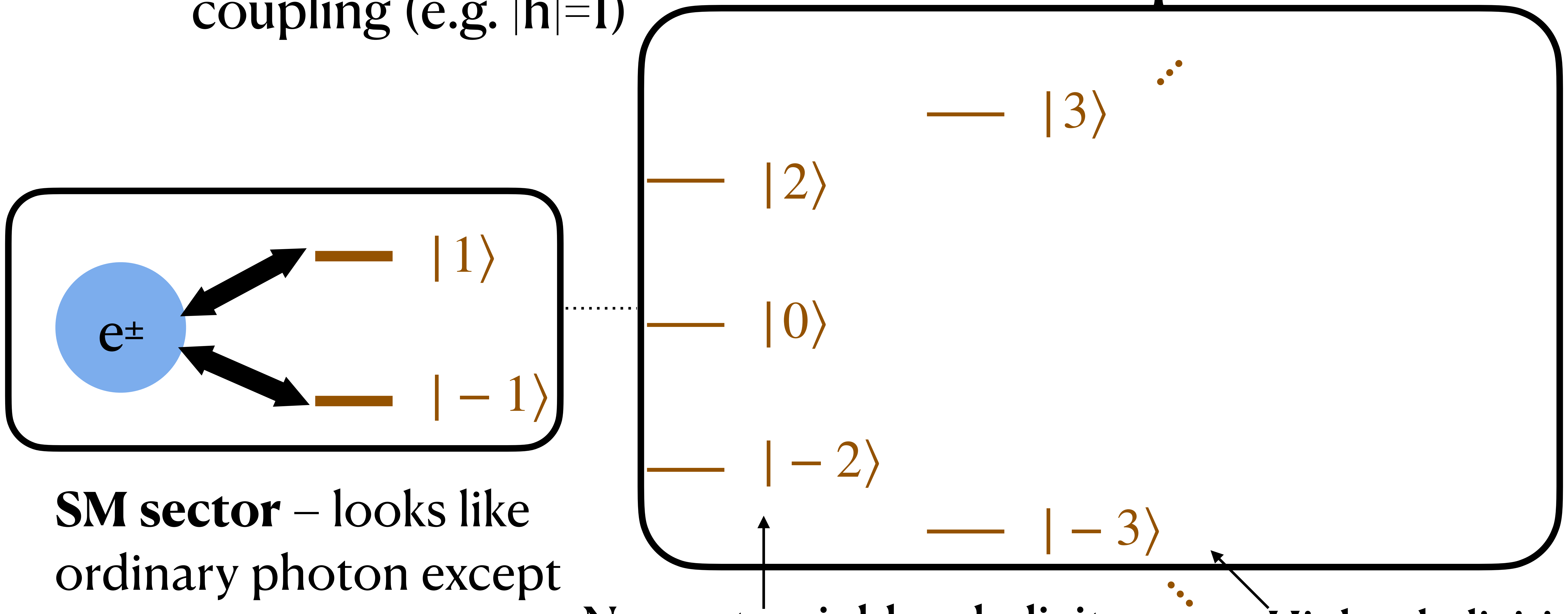
Nearest-neighbor helicity states' interactions suppressed by ρ/ω

Higher helicities' interactions suppressed by more powers of ρ/ω

Continuous Spin Particles are like familiar massless particles with an associated dark sector

Covariant interactions single out **one** helicity with unsuppressed coupling (e.g. |h|=1)

Continuous spin “dark” sector



SM sector – looks like ordinary photon except in deep IR $\omega \lesssim \rho$

Nearest-neighbor helicity states' interactions suppressed by ρ/ω

Higher helicities' interactions suppressed by more powers of ρ/ω

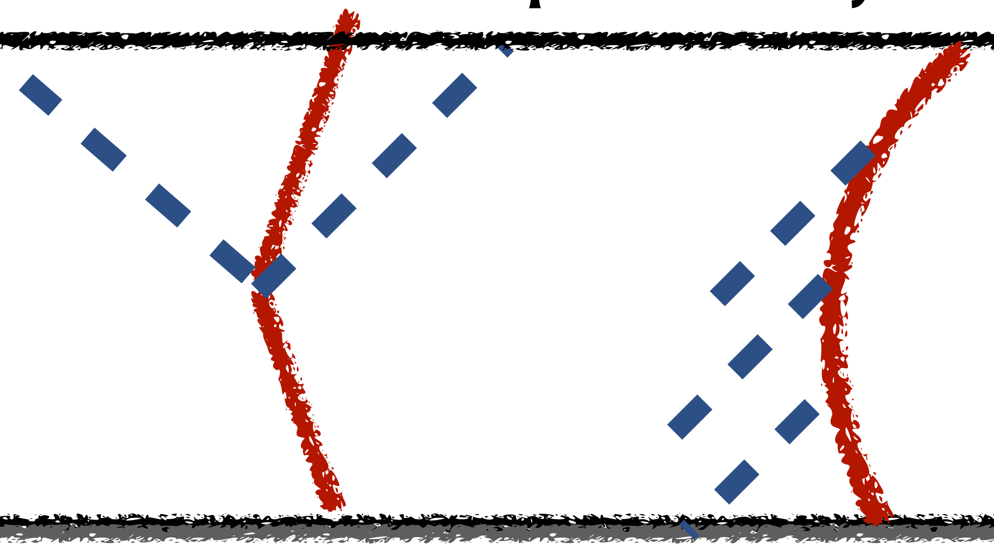
This feature renders radiation cross-sections and associated realistic (finite-time) thermodynamics safe.

Also implies that known massless particles could have non-zero spin scale – our goal: make this an empirical question.

What else can one compute?

Universal:
depend only on $\hat{g}$

Scope of our paper



- Radiation from worldline undergoing a single instantaneous kick –reproduces soft factor results found in 2013.
- Force on a particle induced by plane-wave radiation background
- Continuous spin field $\Psi(\eta, x)$ sourced by a particle at rest or in motion
- Inter-particle force laws (static force-law is just 1/r for our simple ansatz – phases cancel out – but other ansatz currents can give $O(\rho r)$ corrections)
- Response of macroscopic detectors to continuous spin radiation
- QM in path integral (defined, but need new computational tools)
  - Scattering involving only external continuous spin radiation
  - Wavefunction renormalization of continuous spin field
  - Continuous spin fields in loops

Depend on "shape terms"– we have only looked at simplest example currents

Currents in Space-Time

Although shape terms don't couple to continuous spin radiation, they **can** change the spacetime localization of the current – e.g. family of currents

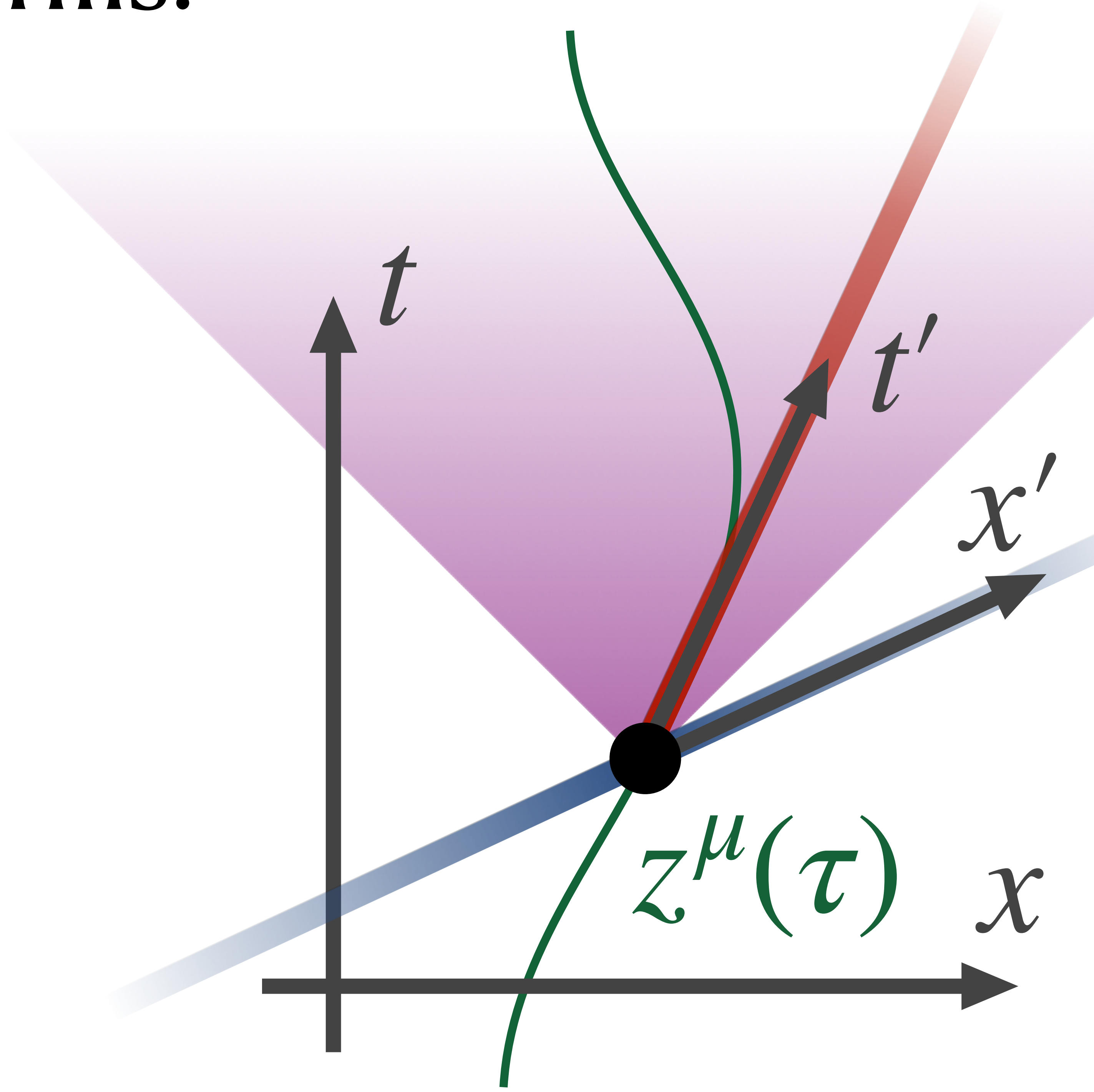
$$f(k, \dot{z}, \eta) \propto e^{-i\rho \frac{\eta \cdot V(\dot{z}, k)}{k \cdot V(\dot{z}, k)}}$$

Satisfy continuity condition for any V – differ only by shape terms.
But they have different localization properties:

$V = \dot{z}$ "temporal"

$V = k + \beta\rho\dot{z}$ "inhomogeneous"

$V = k - (k \cdot \dot{z})\, \dot{z}$ "spatial"

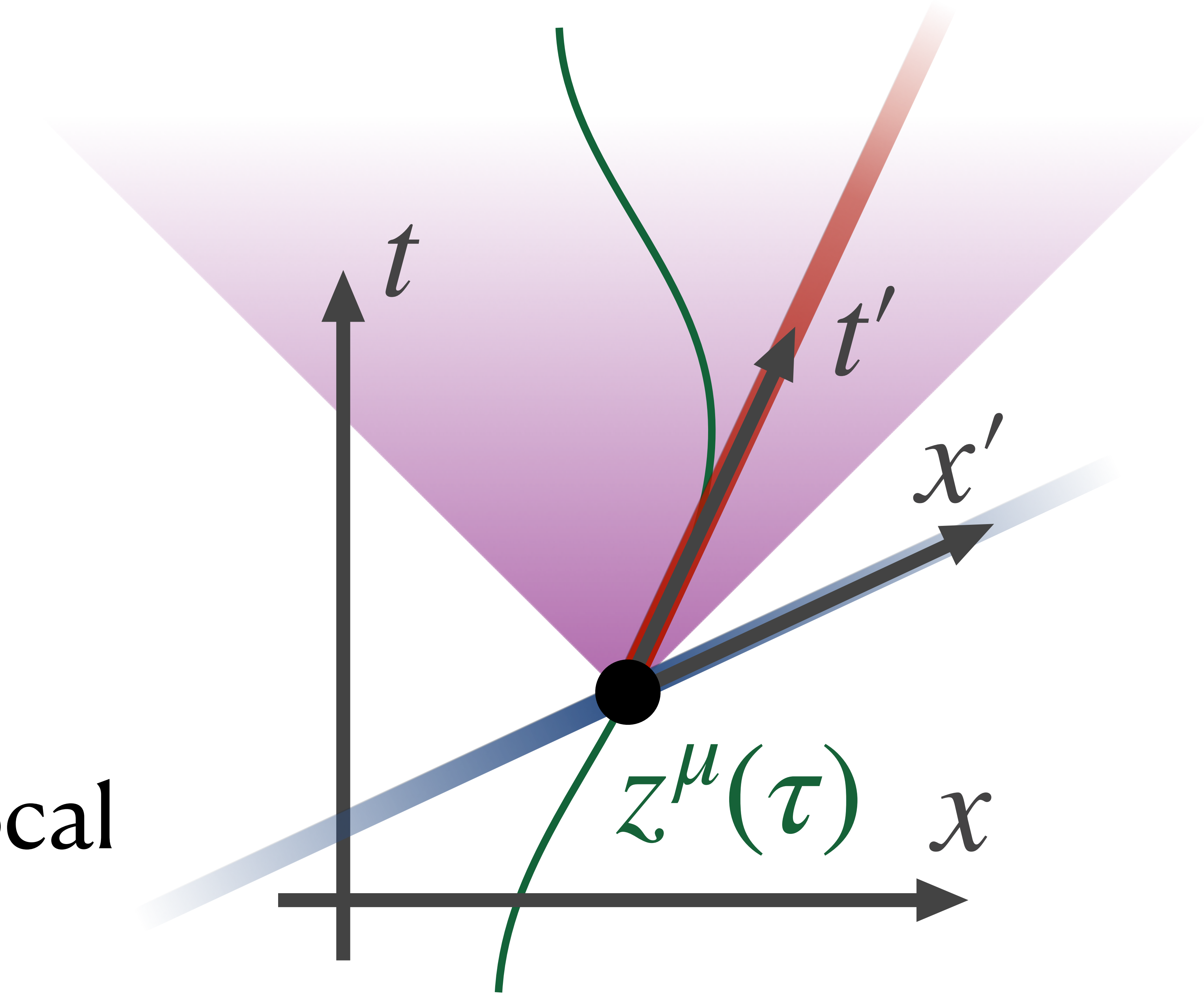


Currents in Space-Time: Causality

Some ansatz currents **admit retarded/advanced forms supported in source's forward/ backward lightcone → manifestly causal equations of motion**

$$\partial_x^2 \Psi = \sum_i \int d\tau\, j_R(\eta, x - z_i(\tau_i), \dot{z}_i(\tau_i)),$$

$$m_i \ddot{z}_i^\mu = -\int d^4x\, [d^4\eta]\, \delta'(\eta^2+1)\, \Psi(\eta, x) \int d\tau \left(\partial_x^\mu + \frac{d}{d\tau_i} \frac{\partial}{\partial \dot{z}_i^\mu} \right) j_A(\eta, x - z_i(\tau_i), \dot{z}_i(\tau_i)).$$

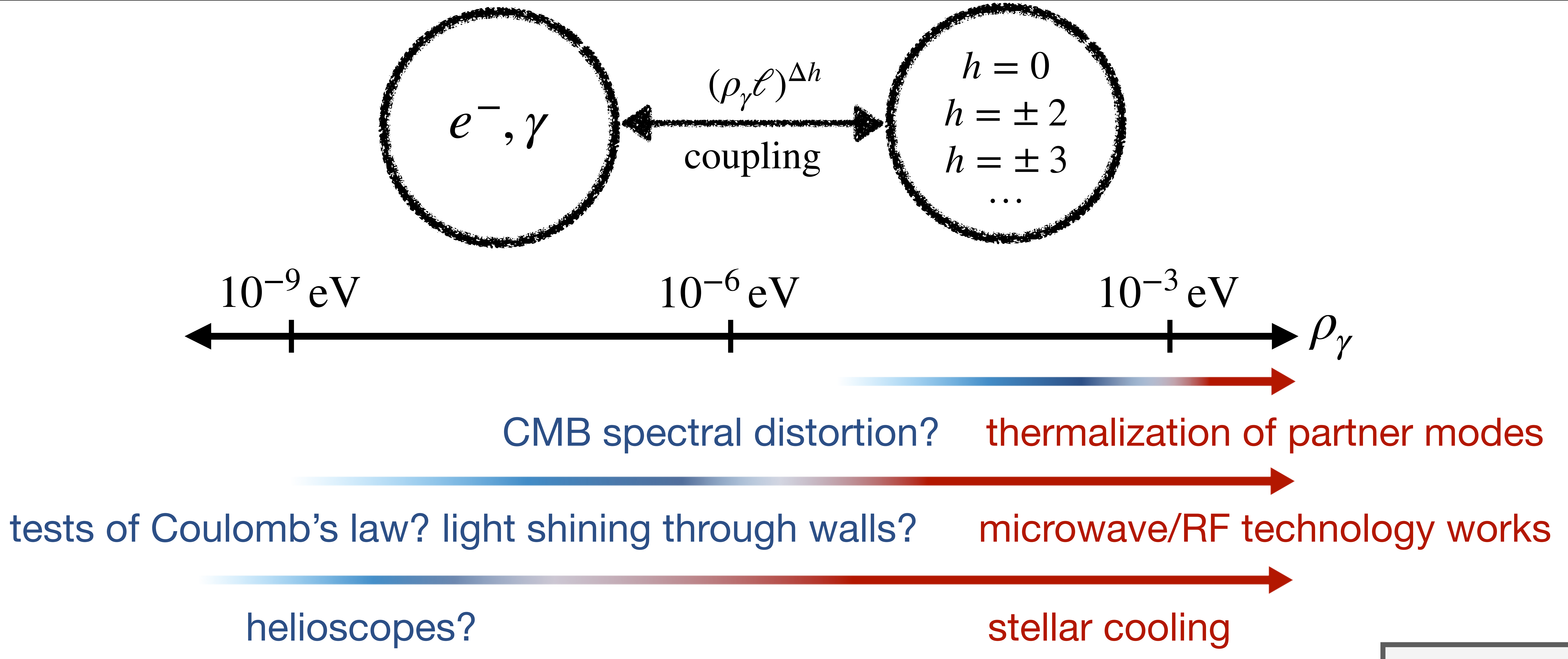


This form, and specific non-local structure, suggestive of integrating out intermediate fields.
We suspect this can be done at Lagrangian level to obtain local & manifestly causal action, but no concrete realization yet.
(Could Rivelles' supertranslation-like symmetry be a hint?)

Even equal-time interactions (c.f. Coulomb-gauge QED) can yield causal dynamics – could this happen for less causal-looking continuous spin currents?

Experimental Opportunities

Continuous spin field with vector-like coupling looks like photon + a dark sector.
Could our photon have non-zero ρ?



Very rough, preliminary estimates!

Can also study graviton's spin scale !

Model-Building Opportunities

One tantalizing (but very speculative and premature!) potential application:

Hierarchy Problem ≃ "scalars that generate long-range 1/r potentials are unnatural"

Scalar with non-derivative couplings to heavy matter receives radiative mass corrections

Goldstones have shift symmetry – protects mass terms, but also forbids non-derivative couplings

?

Continuous spin field admits non-derivative scalar-like interactions, **and** Lorentz requires it to stay massless to avoid d.o.f. discontinuity (aka gauge "symmetry" protects mass)

If these models are renormalizable, they admit something close to a radiatively stable massless scalar.

Perhaps related mechanism can stabilize massive but light Higgs?

Conclusions

- Lorntz invariance → massless particles have a spin-scale. **Is it zero or non-zero?**
- The non-zero option makes more sense than previously thought, and has testable consequences
- If inconsistent, deserves a proper burial
- If viable, we should think of the Standard Model as an effective theory with both UV and IR completions.