More General Axions

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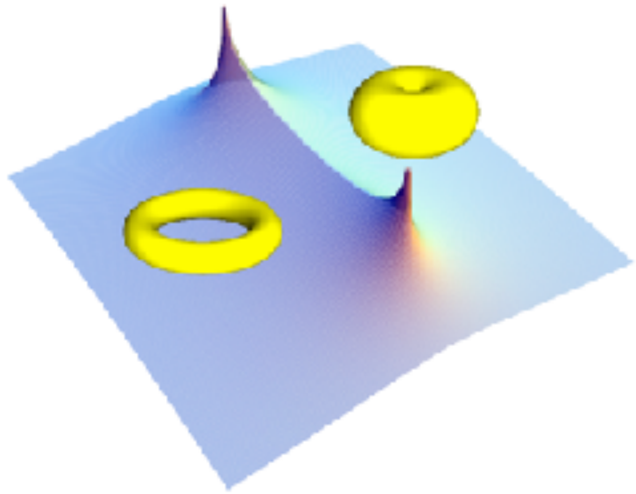
Ofri Telem, JT hep-ph/2411.15312

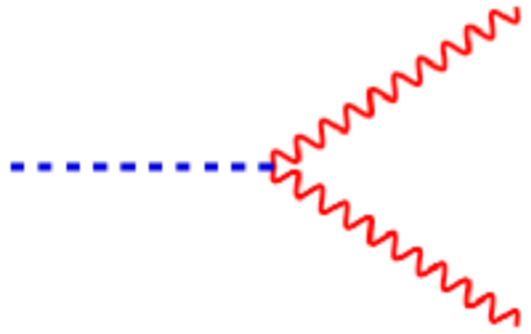
Outline

Claims, Counter Claims and Duality

Seiberg-Witten



Axion Coupling



Conclusions

Axion-gauge coupling quantization with a twist

Matthew Reece

ABSTRACT: The possible couplings of an axion to gauge fields depend on the global structure of the gauge group. If the Standard Model gauge group is minimal, or equivalently if fractionally charged color-singlet particles are forbidden, then the QCD axion's Chern-Simons couplings to photons and gluons obey correlated quantization conditions. Specifically, the photon coupling can have a fractional part which is a multiple of 1/3, but which is

Photophilic hadronic axion from heavy magnetic monopoles

Anton V. Sokolov and Andreas Ringwald

ABSTRACT: We propose a model for the QCD axion which is realized through a coupling of the Peccei-Quinn scalar field to magnetically charged fermions at high energies. We show that the axion of this model solves the strong CP problem and then integrate out heavy magnetic monopoles using the Schwinger proper time method. We find that the model discussed yields axion couplings to the Standard Model which are drastically different from the ones calculated within the KSVZ/DFSZ-type models, so that large part of the corre-

Shift Symmetry

$$\mathcal{L}_{\mathrm{free}} = -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} - \frac{\theta}{16\pi^2} F^{\mu\nu} \widetilde{F}_{\mu\nu}$$

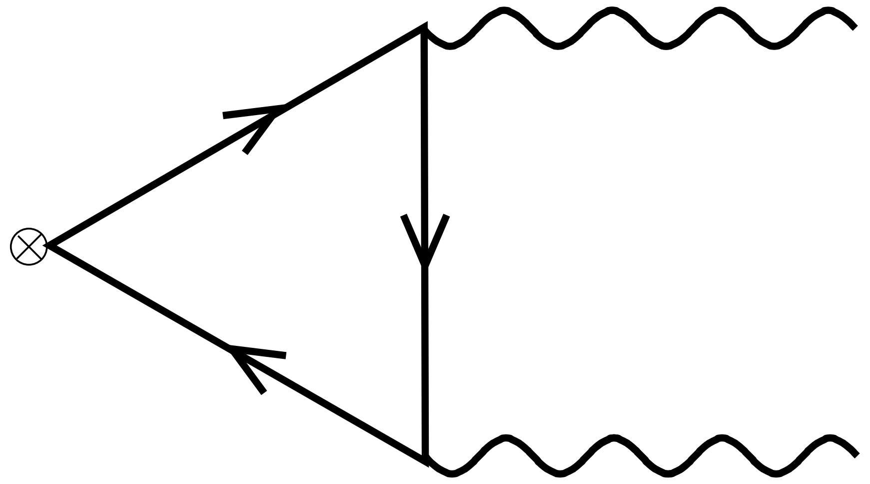
symmetry $\quad T: \ \theta \to \theta + 2\pi$

Atiyah-Singer

$$n_L - n_R = \frac{1}{2}\int d^4x\, \partial_\mu j^\mu_A(x) = \int d^4x \frac{q^2}{16\pi^2} F\tilde{F} = q^2 Ch_2$$

$$\theta \mapsto \theta + 2\pi : \quad e^{iS[A,\theta]} \to e^{iS[A,\theta]} \exp\left[-2\pi i \,\frac{1}{16\pi^2}\int F\widetilde{F}\right]$$

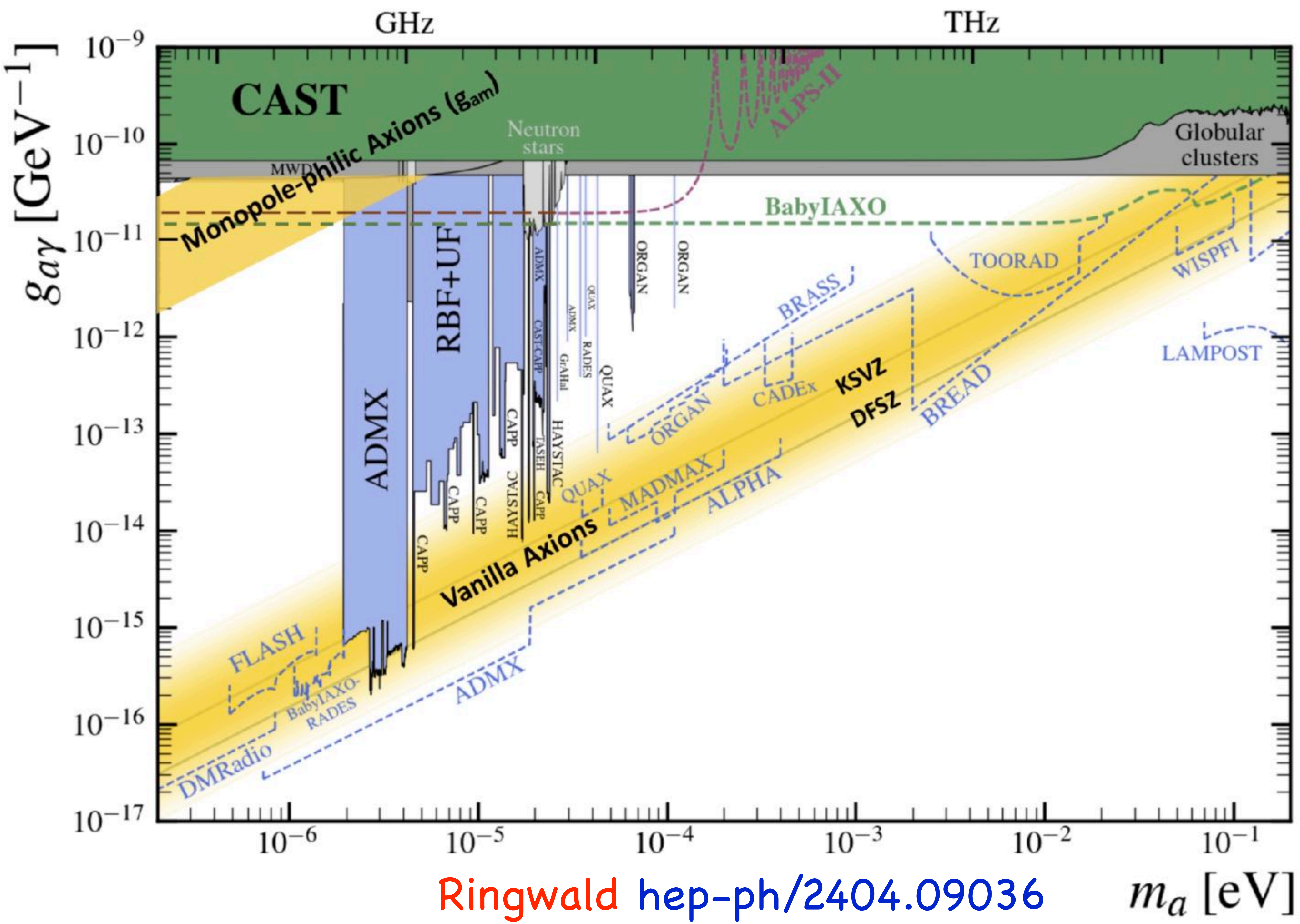
Axial Anomaly Revisited



$$\partial_\mu j^\mu_A = \sum_i \left(\frac{e^2 q_i^2}{16\pi^2} - \frac{g_i^2}{e^2} \right) F^{\mu\nu} \, \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$$

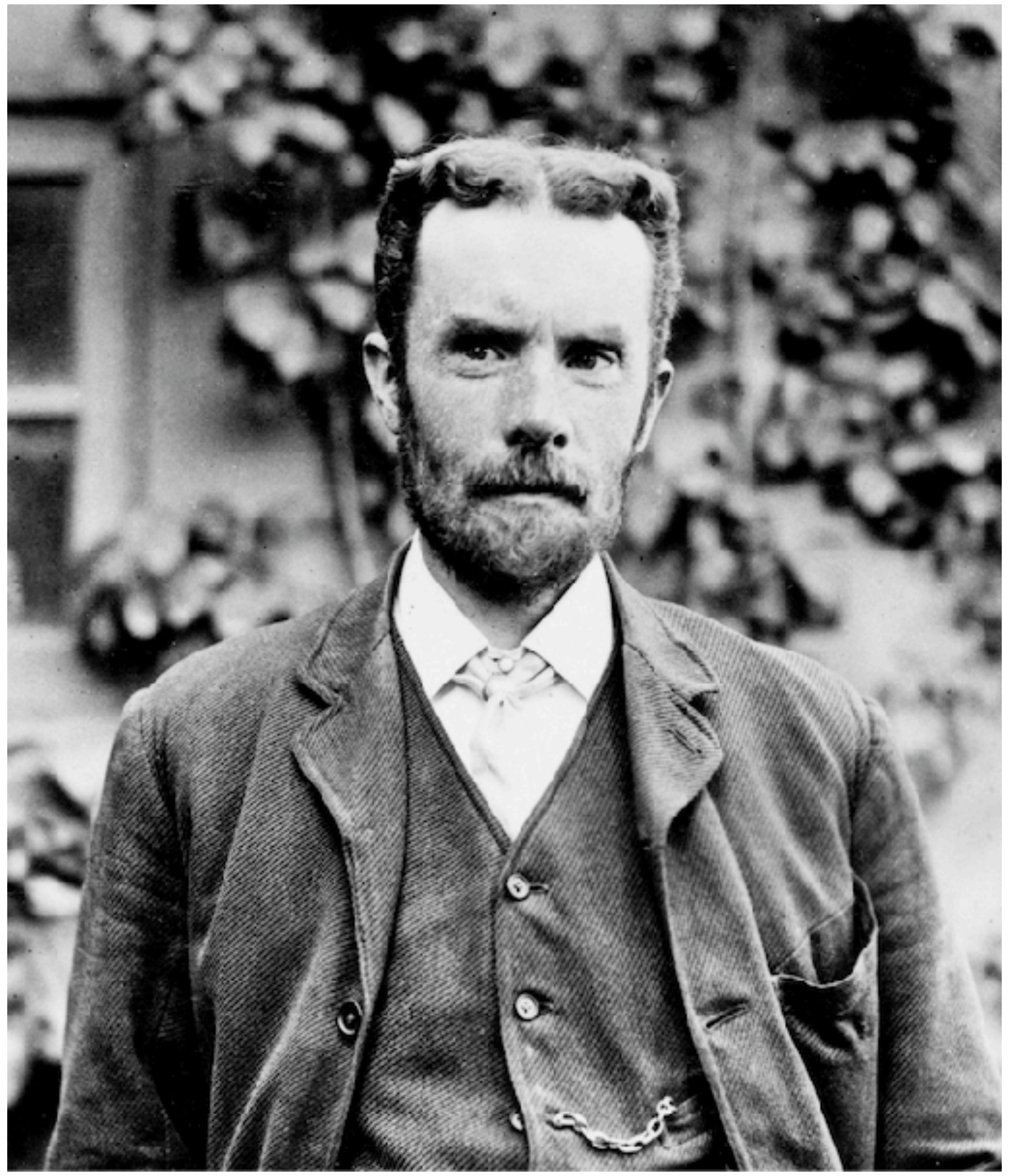
$$q_i \, g_j = \frac{n}{2} \qquad\qquad (e \, q_i) \left(\frac{4\pi}{e} g_j \right) = 2\pi n$$

Csaki, Shirman, JT hep-th/1003.0448



Ringwald hep-ph/2404.09036

E-M Duality



$$\vec{E} \rightarrow \vec{B}$$
$$\vec{B} \rightarrow -\vec{E}$$

$$\widetilde{F}^{\mu\nu} \rightarrow \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

$$F^{\mu\nu} \rightarrow \widetilde{F}^{\mu\nu}$$

Heaviside, Phil. Trans. Roy. Soc. A 183 (1893) 423

Shift + Duality

$$\mathcal{L}_{\text{free}} = -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} - \frac{\theta}{16\pi^2} F^{\mu\nu} \widetilde{F}_{\mu\nu}$$

symmetry $\quad T: \ \theta \to \theta + 2\pi$

$$\tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$$

EM duality $\quad S: \tau \to -\frac{1}{\tau}$

SL(2,Z) Duality

$$\tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{e^2} \qquad S : \tau \to -\frac{1}{\tau} \qquad T : \tau \to \tau + 1$$

$$\tau' = \frac{a\tau + b}{c\tau + d}$$

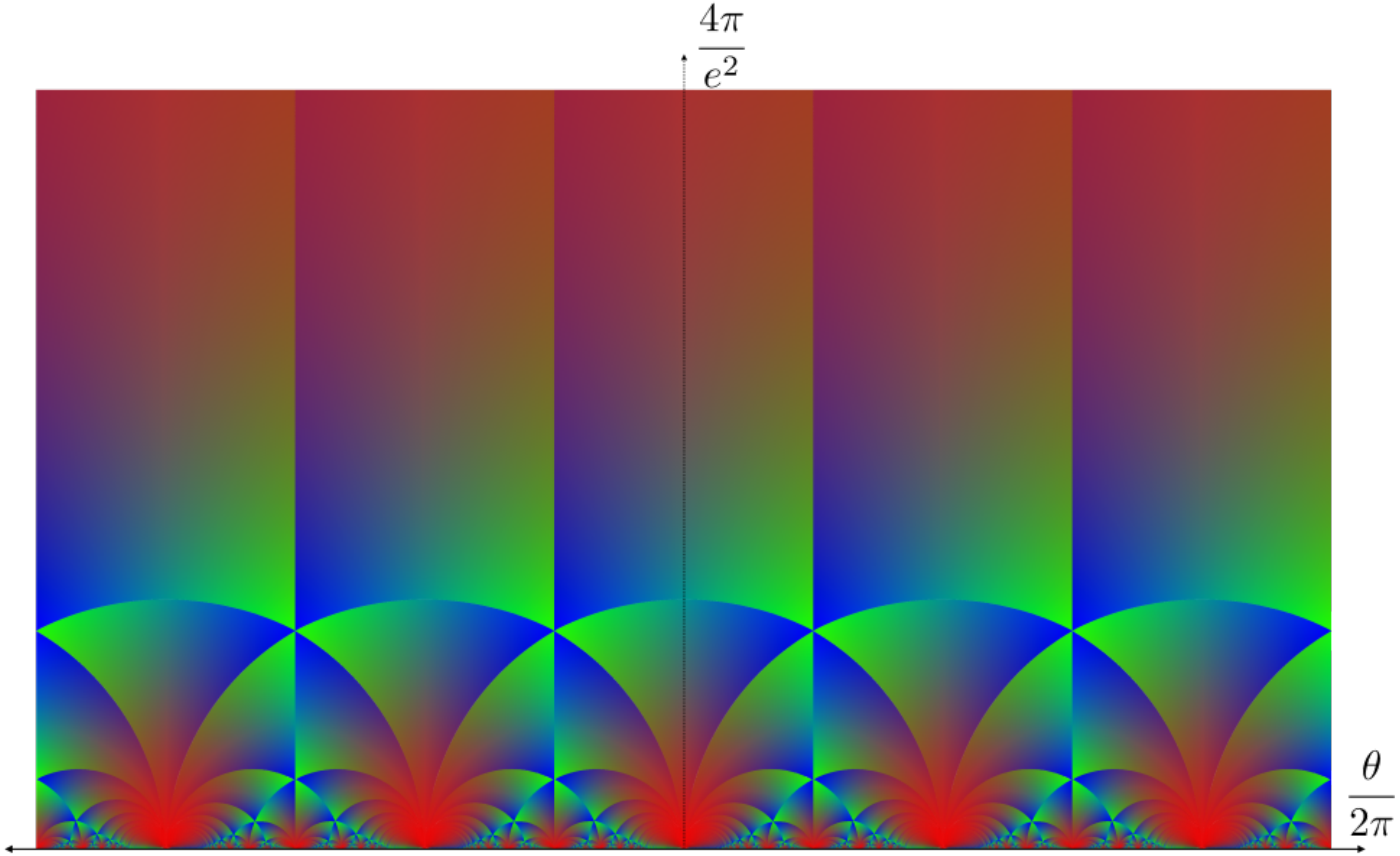
$$K^\mu \to aK'^\mu + cJ'^\mu \ , \quad J^\mu \to bK'^\mu + dJ'^\mu$$

$$ad - bc = 1$$

duality not a symmetry

Cardy and Rabinovici Nucl. Phys. B**205** (1982) 1

Tiling the coupling plane



Maxwell Eq.

$$\frac{\mathrm{Im}\,(\tau)}{4\pi}\,\partial_\mu\left(F^{\mu\nu}+i\,{}^*F^{\mu\nu}\right)=J^\nu-\bar{\tau}K^\nu$$

duality covariant

includes Witten effect

Maxwell Eq.

$$\frac{\mathrm{Im}\,(\tau)}{4\pi}\,\partial_\mu\left(F^{\mu\nu} + i\,{}^*F^{\mu\nu}\right) = J^\nu - \bar{\tau} K^\nu$$

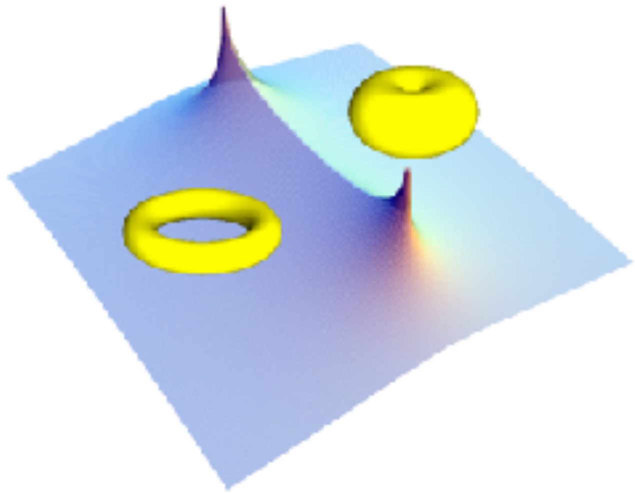
duality covariant

Sikivie-Maxwell Eq.

$$\partial_\nu F^{\mu\nu} = g\widetilde{F}^{\mu\nu}\partial_\nu a\ , \qquad \partial_\nu \widetilde{F}^{\mu\nu} = 0$$

naively duality violating!

Seiberg-Witten



$$\mathcal{N}=2, \quad SU(2)$$

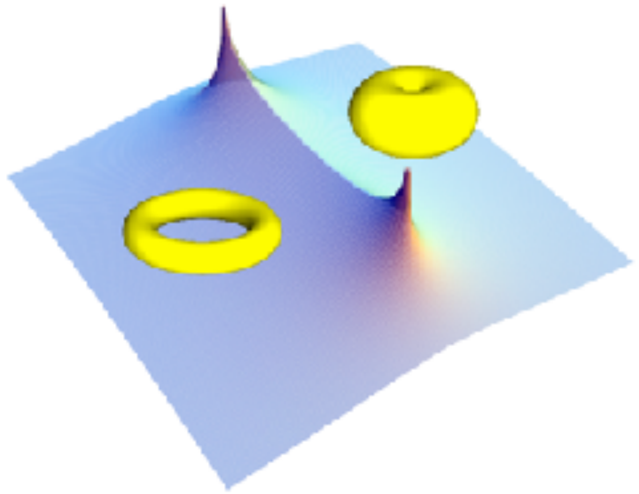
$$W_\alpha \quad A_\mu$$

$$\lambda \quad \psi$$

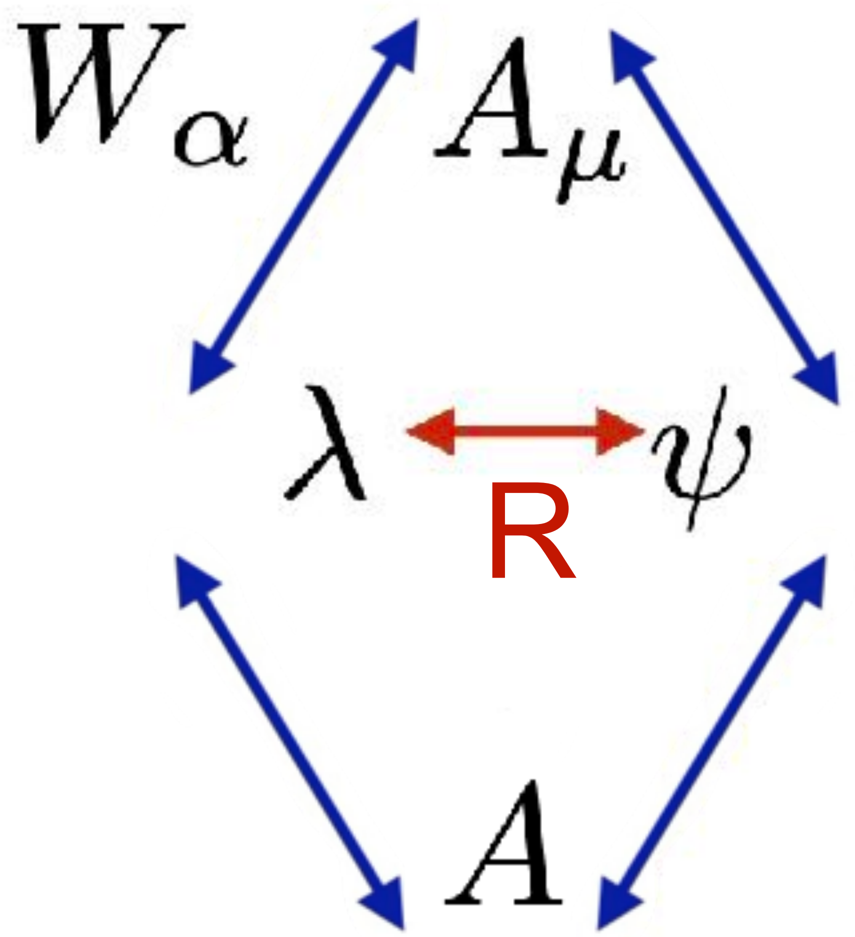
$$A$$

hep-th/9407087

Seiberg-Witten

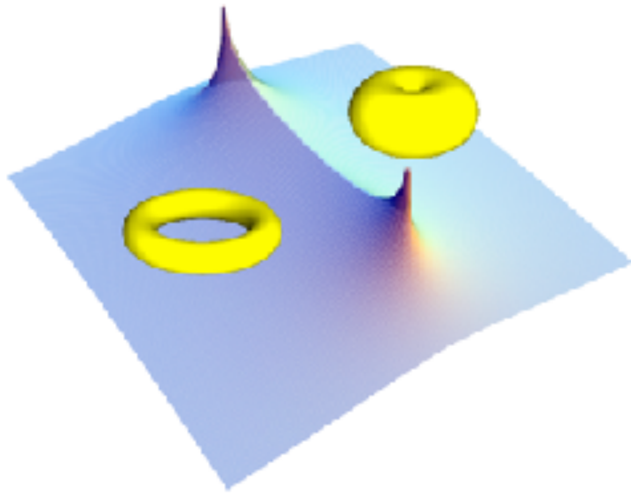


$$\mathcal{N} = 2, \quad SU(2)$$

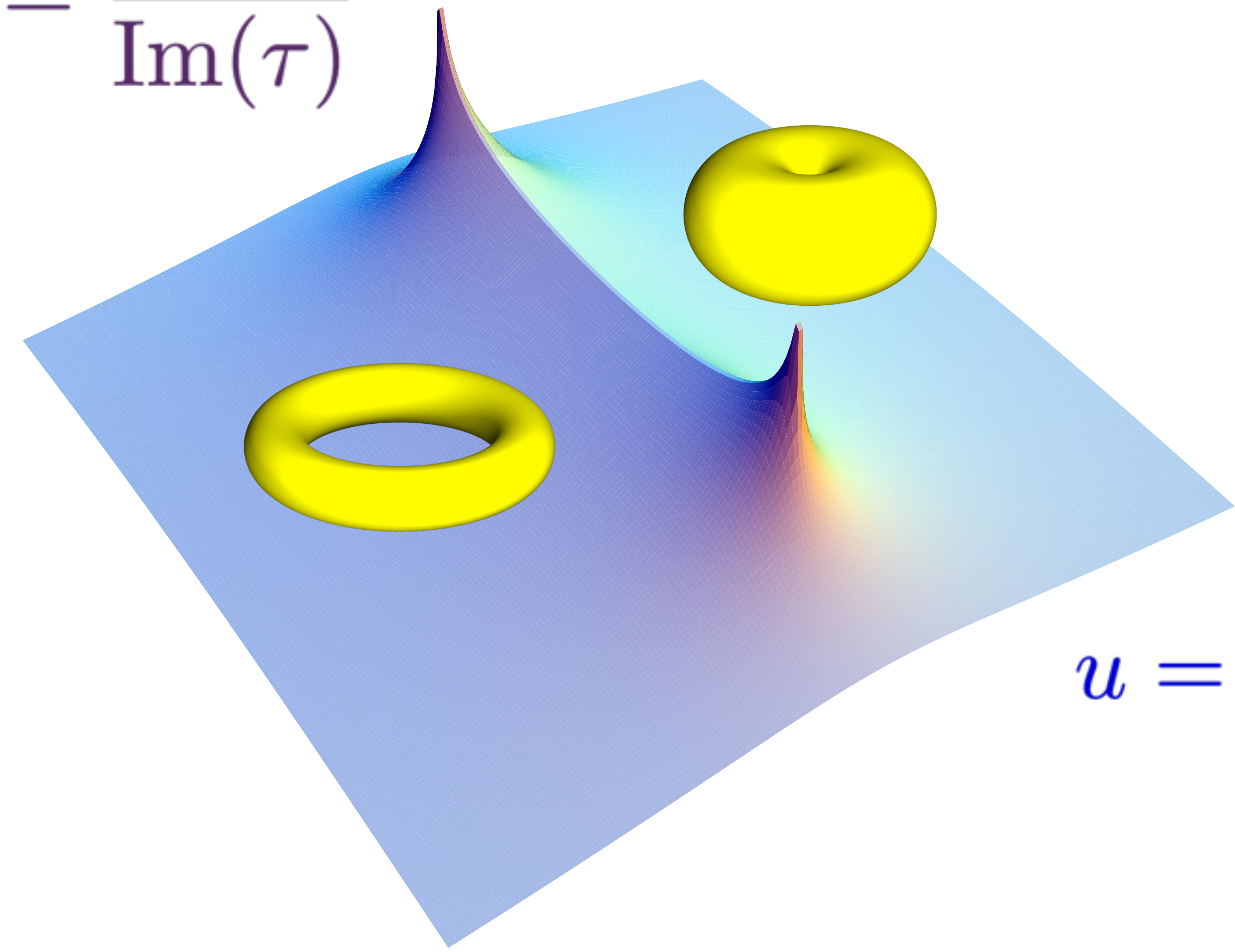


hep-th/9407087

Seiberg-Witten



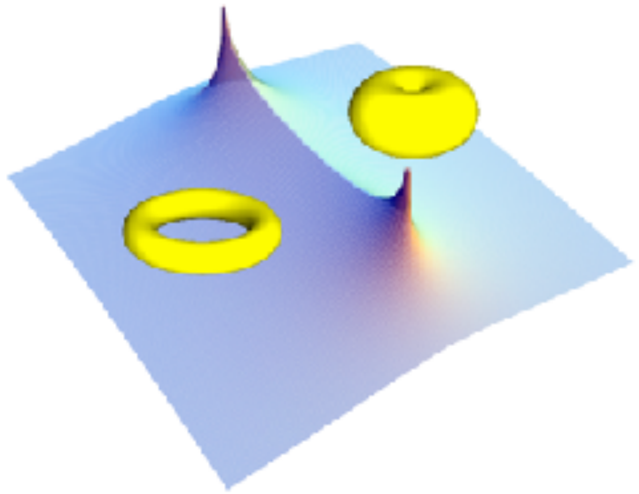
$$\frac{e^2}{4\pi} = \frac{1}{\mathrm{Im}(\tau)}$$



$$u = \frac{1}{2}\mathrm{Tr}A^2$$

hep-th/9407087

Seiberg-Witten



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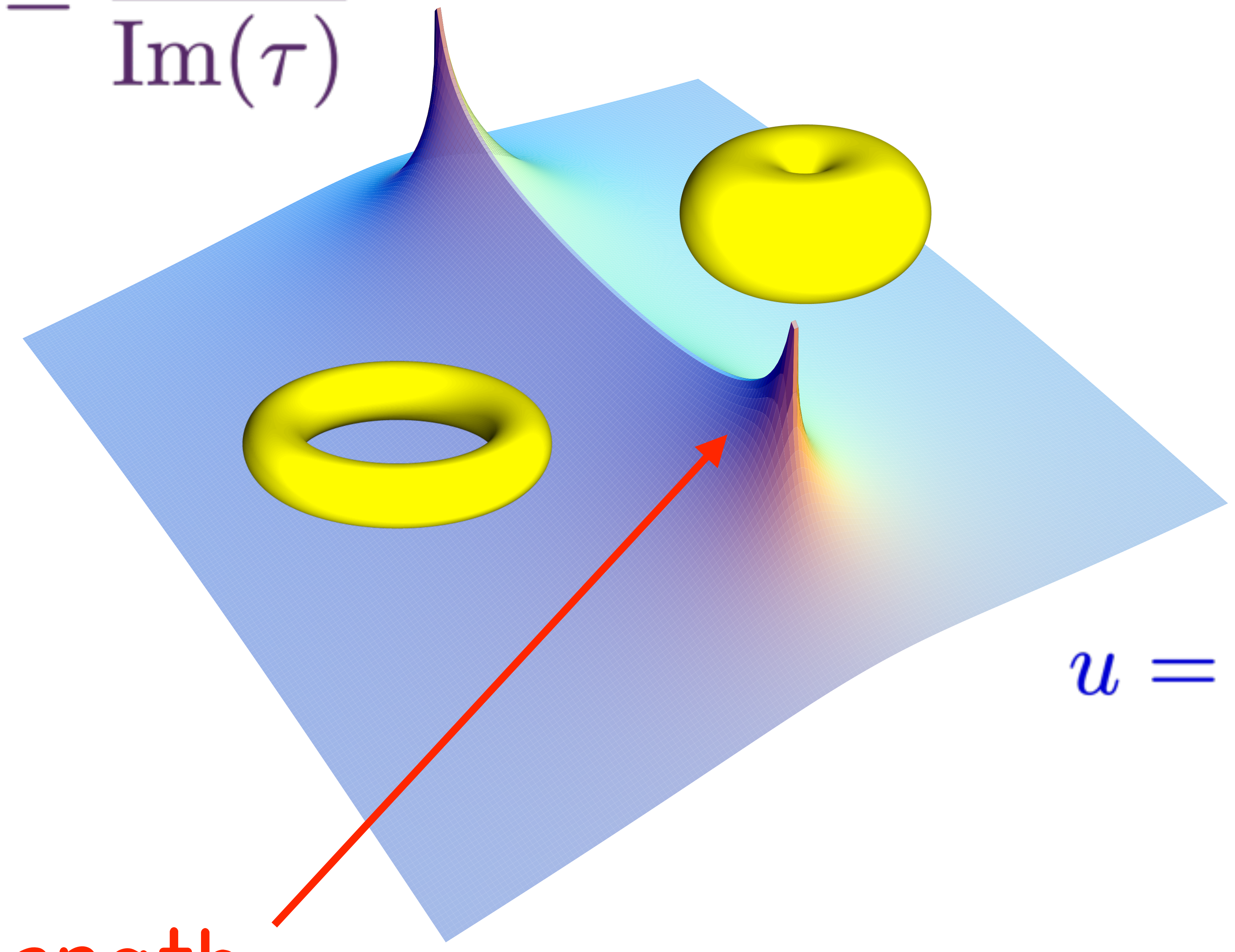


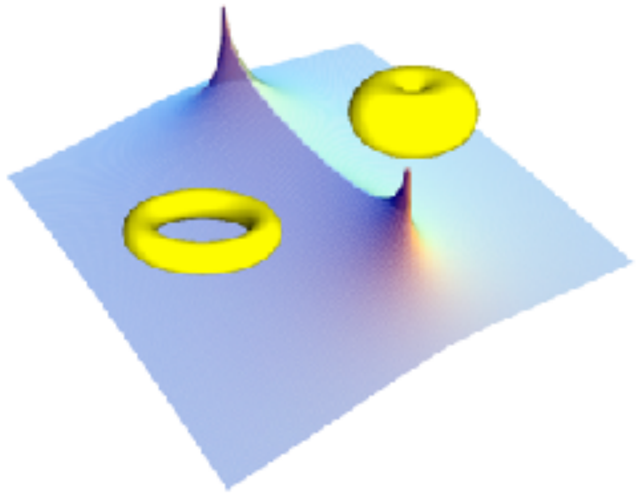
$$u = \frac{1}{2}\mathrm{Tr}A^2$$

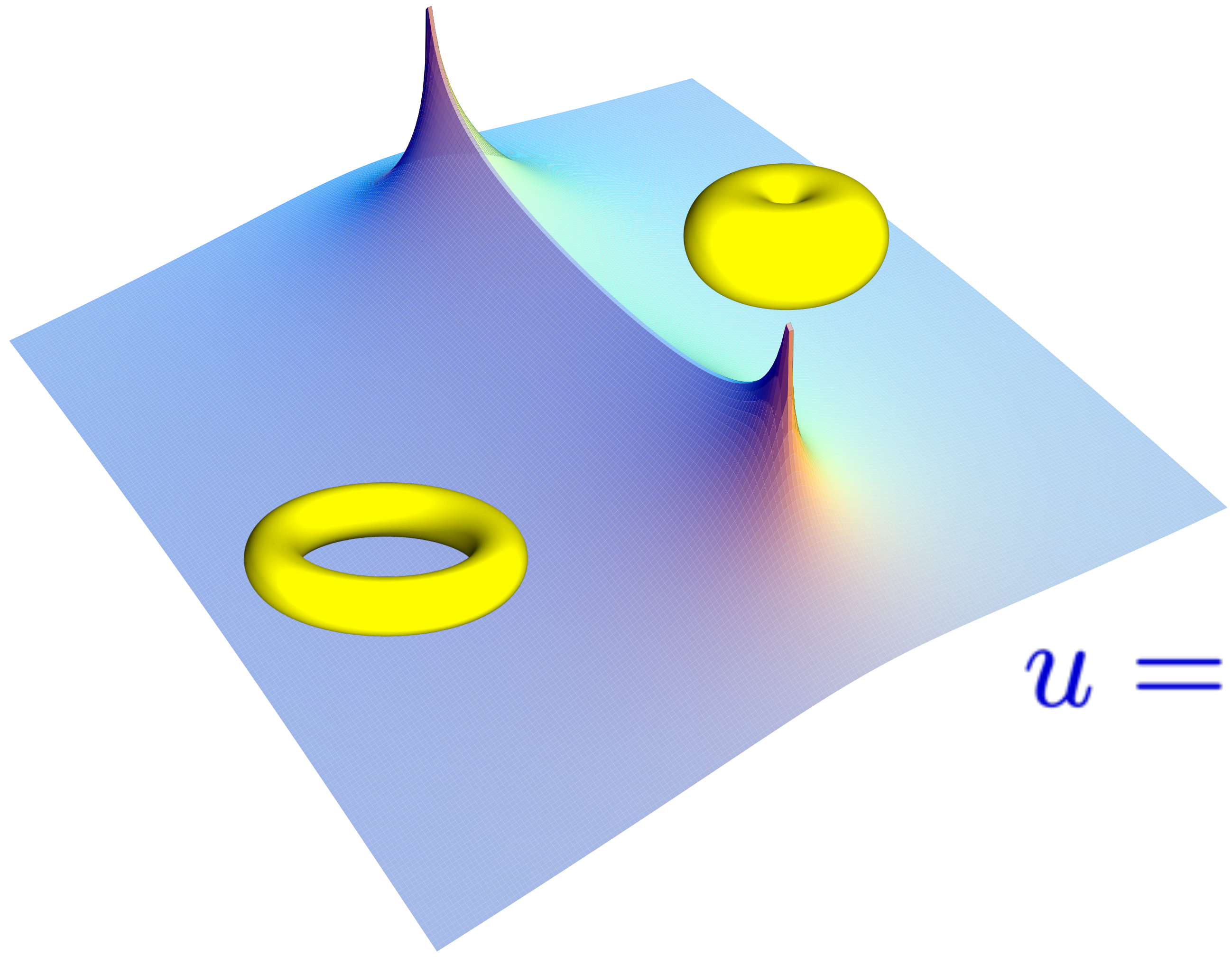
massless monopoles

size smaller than Compton wavelength

hep-th/9407087

Seiberg-Witten





Higgs $\quad SU(2) \to \mathrm{EM}$

possible vacua parameterized by u

$$u = \frac{1}{2}\mathrm{Tr}A^2$$

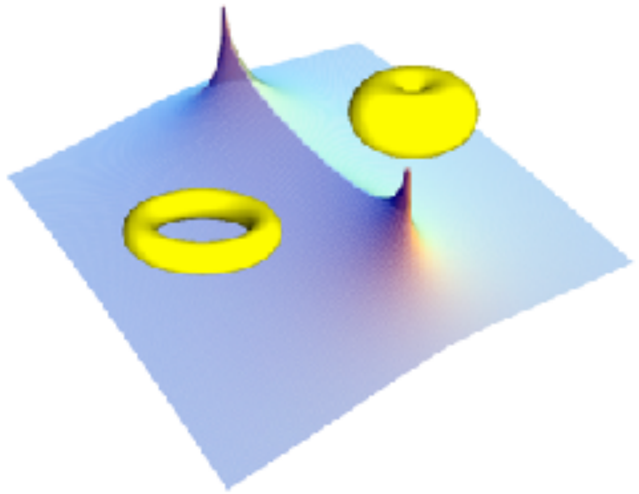
$$m(e,g;u) = \sqrt{2}|e\,A(u) + g\,A_D(u)|$$

$$A(u) \quad = \quad -\sqrt{2(\Lambda^2+u)}\,F\left(-\frac{1}{2},\frac{1}{2},1;\frac{2}{1+\frac{u}{\Lambda^2}}\right)$$

$$A_D(u) \quad = \quad -i\frac{1}{2}\left(\frac{u}{\Lambda}-\Lambda\right)F\left(\frac{1}{2},\frac{1}{2},2;\frac{1}{2}\left(1-\frac{u}{\Lambda^2}\right)\right)$$

hep-th/9407087

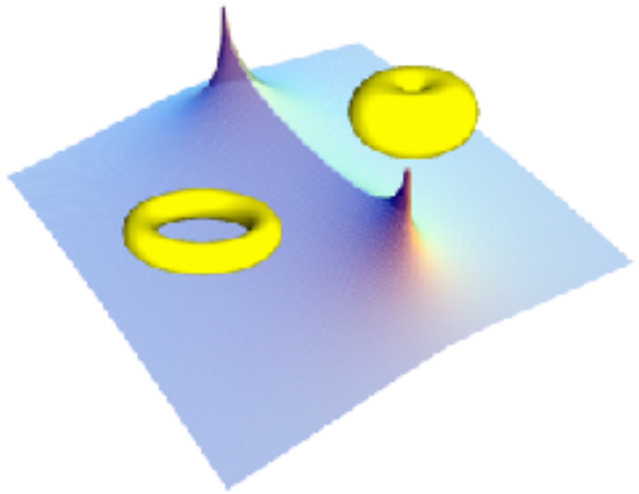
Low Energy EFT



$$\mathcal{F}(A) = \frac{1}{2\pi i}\left\{-4A^2\left(\ln\frac{2A}{\Lambda} - \frac{3}{2}\right) + A^2\sum_{k=1}^{\infty} d_k \left(\frac{\Lambda}{A}\right)^{4k}\right\}$$

$$\tau = \frac{\partial^2 \mathcal{F}}{\partial^2 A} \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$$

Low Energy EFT



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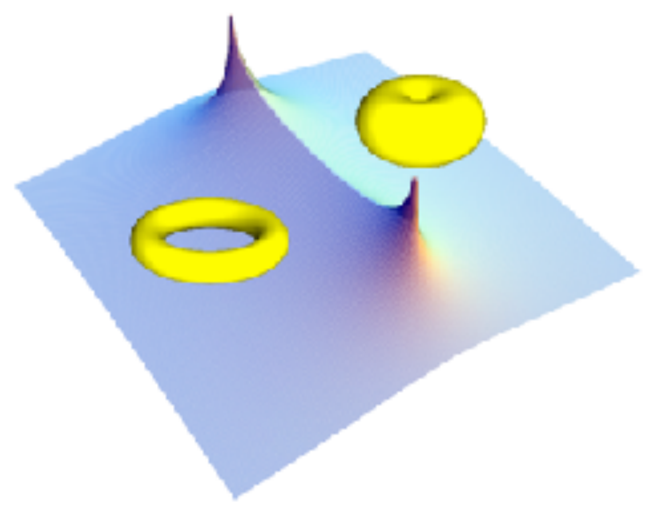
perturbative β function

$$\tau = \frac{\partial^2 \mathcal{F}}{\partial^2 A} \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$$

Low Energy EFT

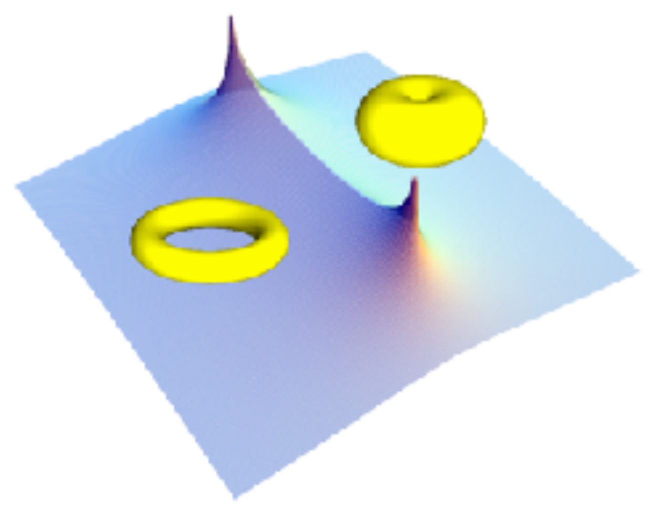


$$\mathcal{F}(A) = \frac{1}{2\pi i}\left\{-4A^2\left(\ln\frac{2A}{\Lambda}-\frac{3}{2}\right)+A^2\sum_{k=1}^{\infty} d_k\left(\frac{\Lambda}{A}\right)^{4k}\right\}$$

instantons

$$\tau = \frac{\partial^2 \mathcal{F}}{\partial^2 A} \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$$

Low Energy EFT

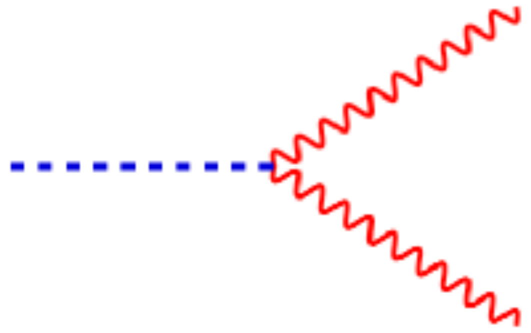


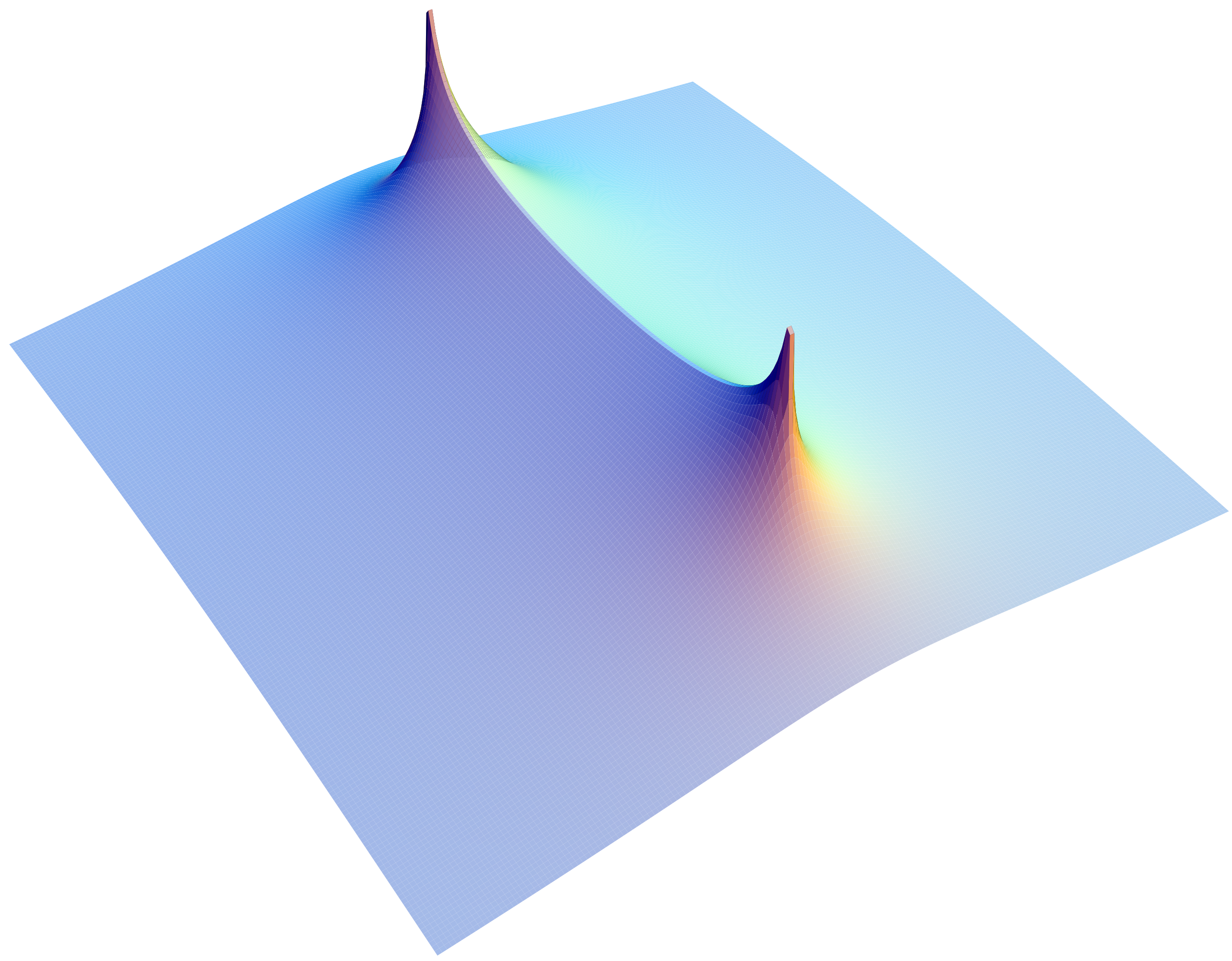
$$\mathcal{F}(A) = \frac{1}{2\pi i}\left\{-4A^2\left(\ln\frac{2A}{\Lambda} - \frac{3}{2}\right) + A^2\sum_{k=1}^{\infty} d_k \left(\frac{\Lambda}{A}\right)^{4k}\right\}$$

$$\mathcal{F}_D(A_D) = \frac{1}{4\pi i}\left\{A_D^2\left(\ln\frac{-iA_D}{32\Lambda} - \frac{3}{2}\right) - 16iA_D + A_D^2\sum_{k=1}^{\infty} d_k^D \left(\frac{A_D}{\Lambda}\right)^{k}\right\}$$

$$\tau = \frac{\partial^2\mathcal{F}}{\partial^2 A} \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$$

Seiberg-Witten Axion





large $\quad u = \frac{1}{2}\mathrm{Tr}A^2 \quad$ perturbative

small $\quad u = \frac{1}{2}\mathrm{Tr}A^2 \quad$ strongly coupled

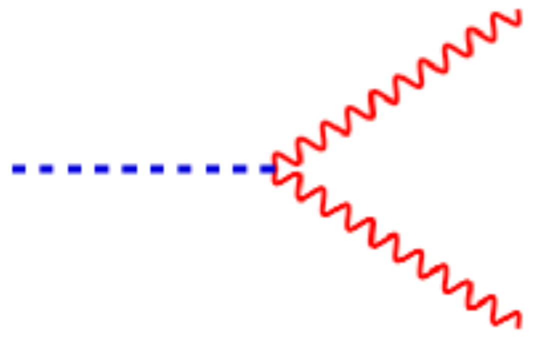
$SU(2) \to U(1)$ everywhere

A has R-charge=2

R-charge is anomalous and spontaneously broken

$\Longrightarrow$ axion

Low Energy EFT



expand around VEV:

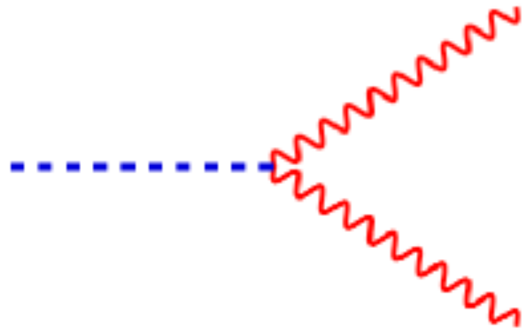
$$A(x) = A^v(u)\, e^{i\frac{a(x)}{f(u)}}$$

decay constant:

$$f(u) = \sqrt{2}|A^v(u)|/e(u)$$

$$-\frac{e^2}{16\pi^2 f} F_{\mu\nu}\widetilde{F}^{\mu\nu} \quad \left\{ N_a a - \sum_{k=1}^{\infty} \left[b_k \sin\left(\frac{4ka}{f}\right) + c_k \cos\left(\frac{4ka}{f}\right) \right] \right\}$$

Low Energy EFT



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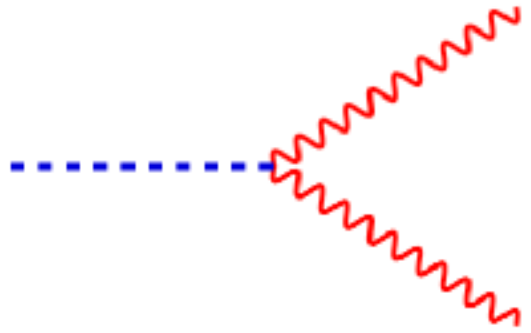
perturbative anomaly

Low Energy EFT



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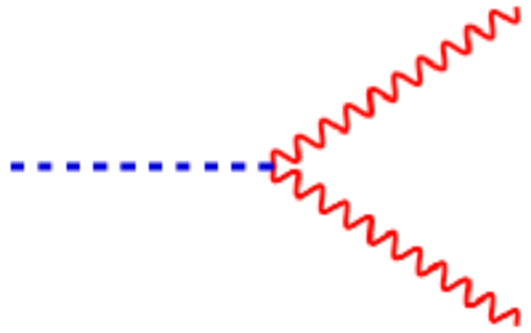
non-perturbative instanton/monopole effects

Low Energy EFT



expand to linear order in axion:

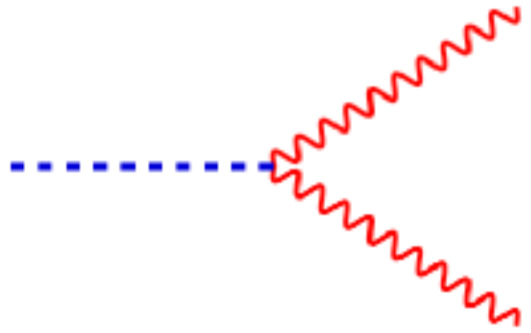
$$-\frac{a}{16\pi}\mathrm{Im}\left\{\left[i\frac{e^3}{\sqrt{2}}\frac{\partial\tau}{\partial A}\right]_{a=0}(F^{\mu\nu}+i\widetilde{F}^{\mu\nu})^2\right\}$$

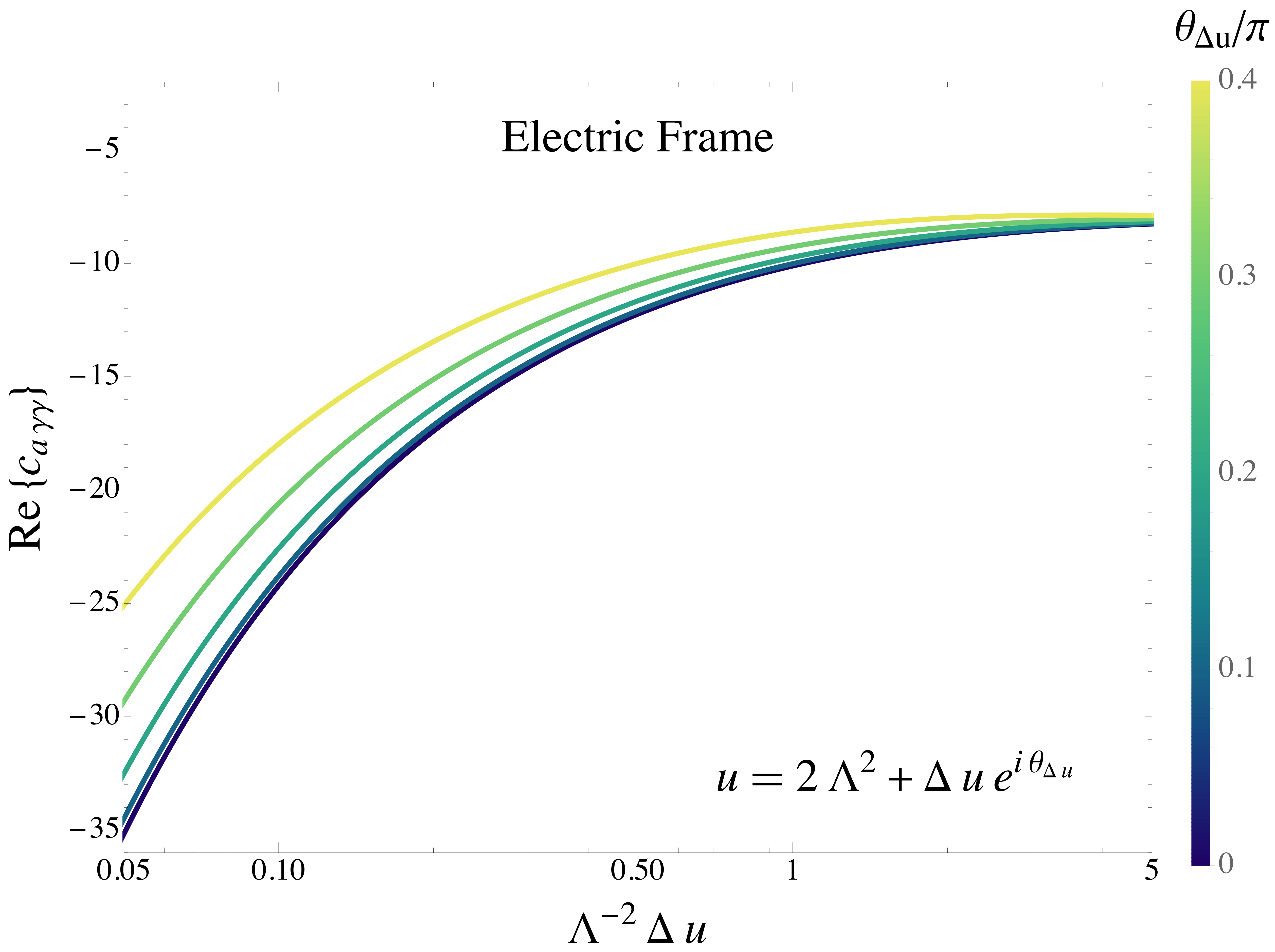
$$\frac{g_{a\gamma\gamma}}{4}=\frac{e^2}{16\pi^2 f}\left(-8-\sum_{k=1}^{\infty}4k(\tilde{b}_k-i\tilde{c}_k)\left(\frac{\Lambda}{A^v}\right)^{4k}\right)$$

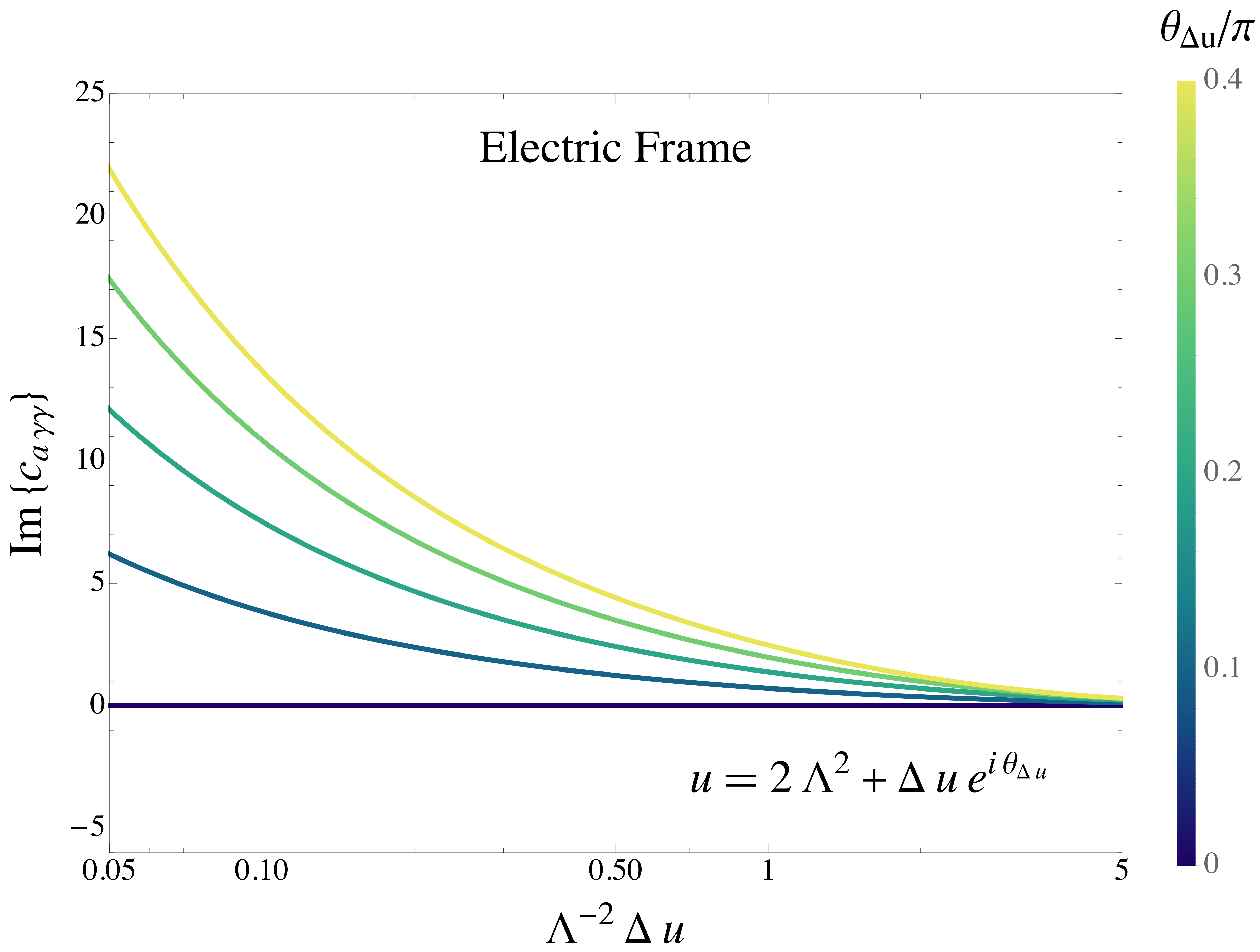
non-perturbative instanton effects

one-loop chargino contribution

Axion Coupling

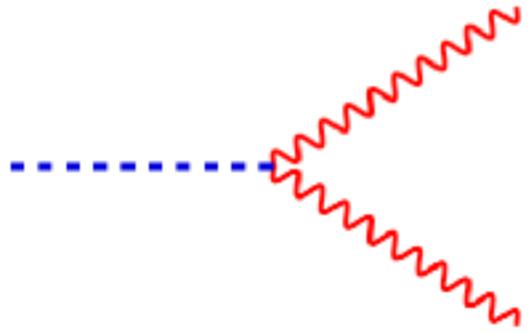






$$\frac{g_{a\gamma\gamma}}{4} = \frac{e^2}{16\pi^2 f} c_{a\gamma\gamma}$$

Dual Low Energy EFT



expand around VEV:

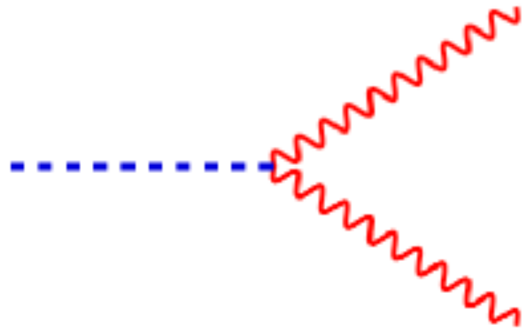
$$A_D(x) = A_D^v(u)\, e^{i\frac{a_D(x)}{f_D(u)}}$$

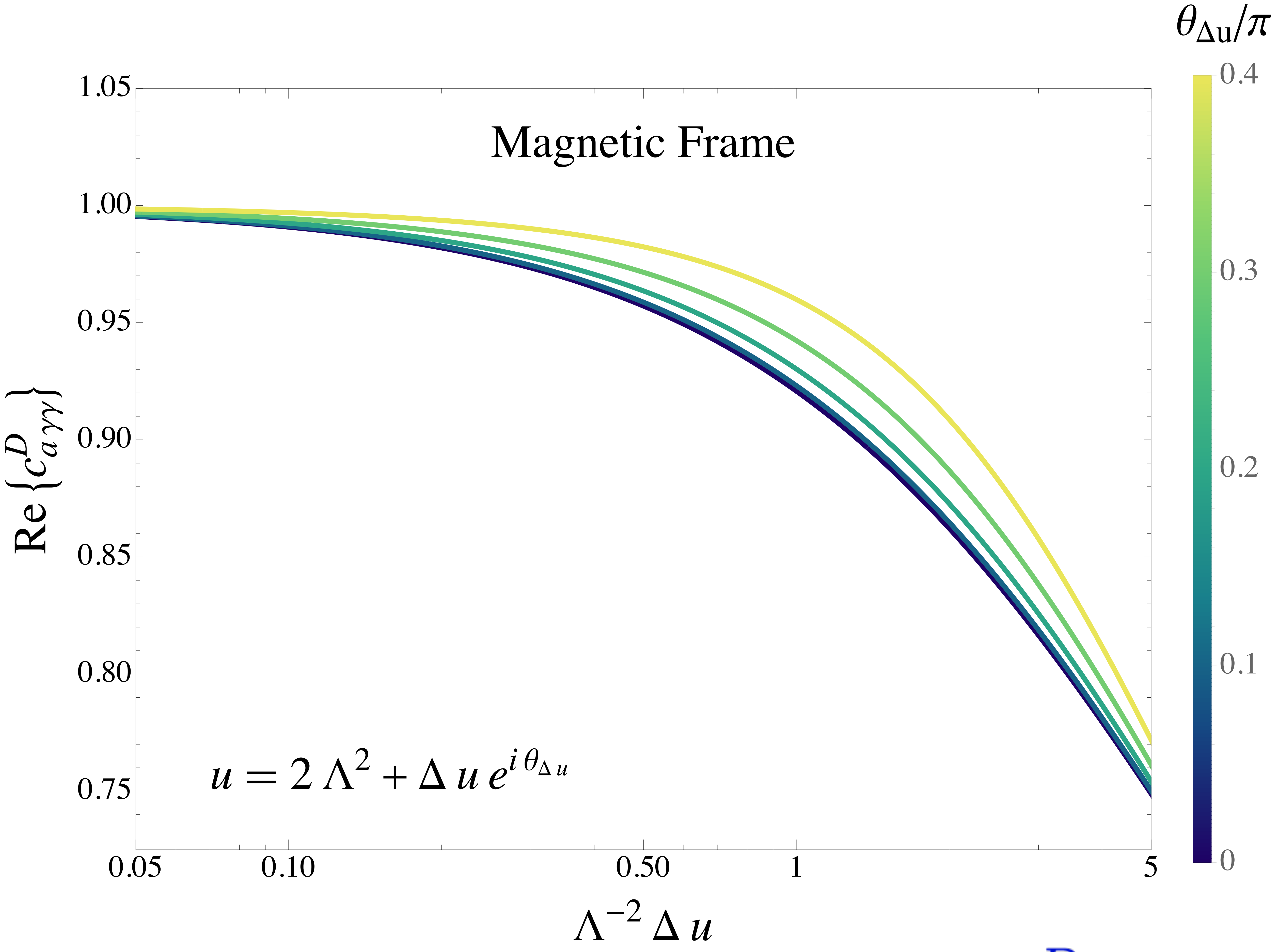
$$\frac{g^D_{a\gamma\gamma}}{4} = \frac{e_D^2}{16\pi^2 f_D}\left(1 + \sum_{k=1}^{\infty}\frac{k}{2}\left(\tilde{b}_k^D - i\tilde{c}_k^D\right)\left(\frac{A_D^v}{\Lambda}\right)^k\right)$$

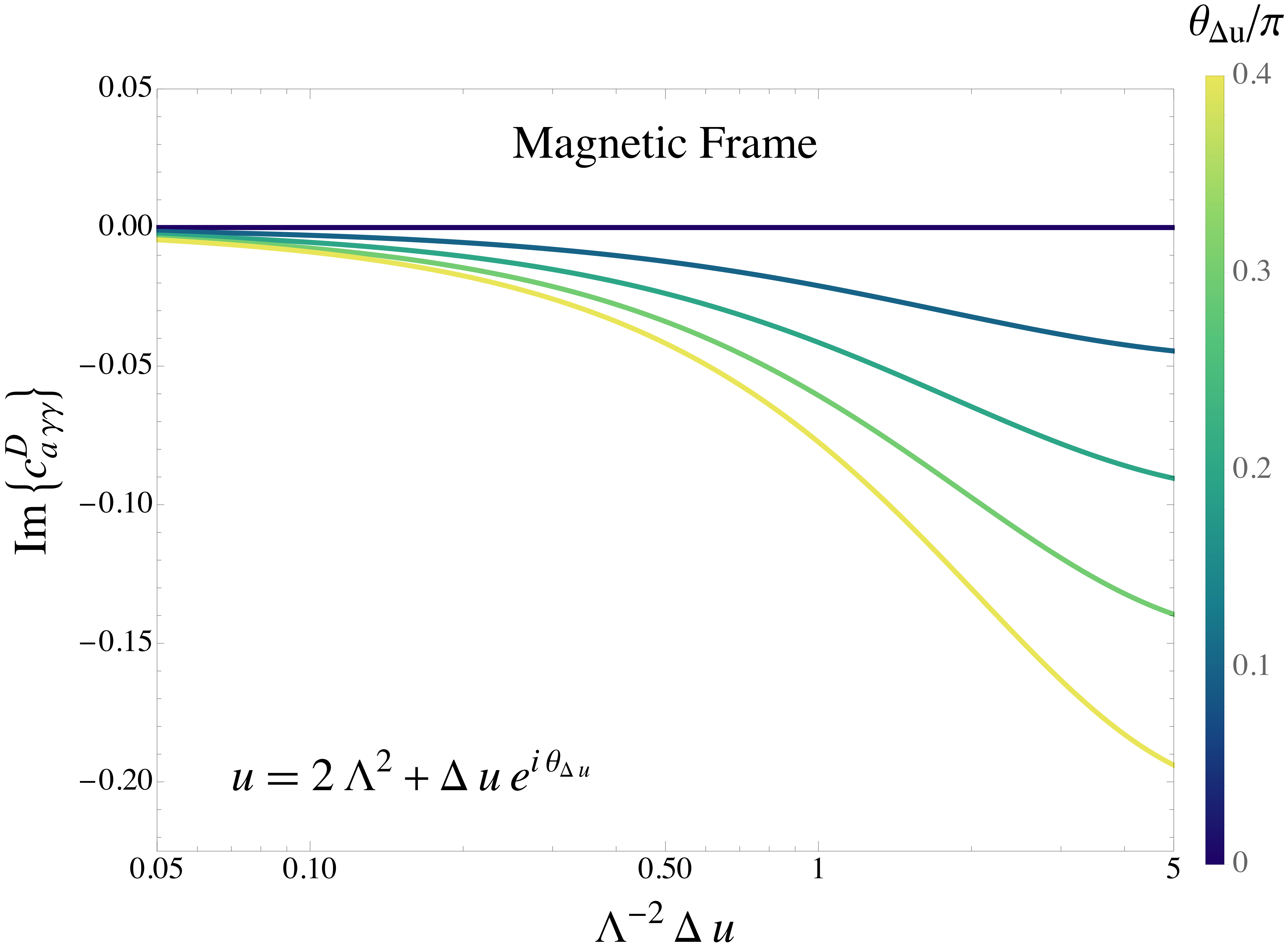
non-perturbative electric charge effects

one-loop monopole contribution

Dual Axion Coupling

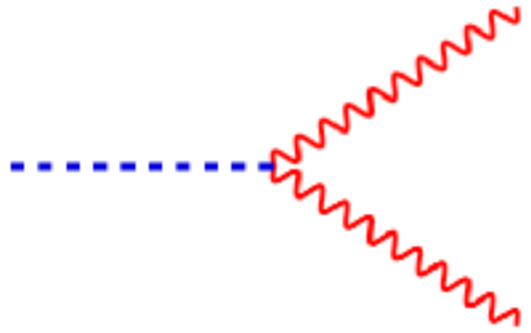


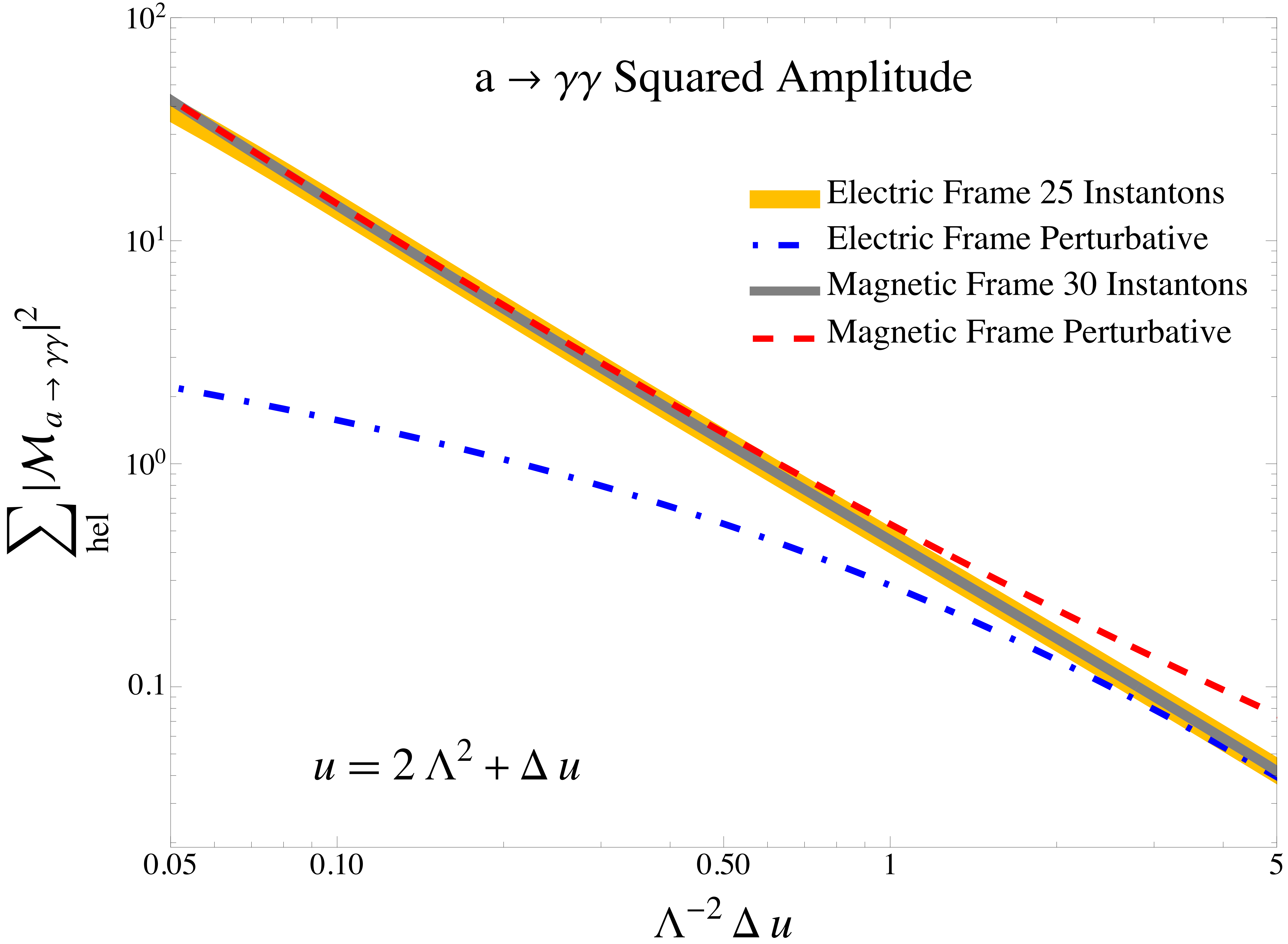




$$\frac{g^D_{a\gamma\gamma}}{4} = \frac{e_D^2}{16\pi^2 f_D} c^D_{a\gamma\gamma}$$

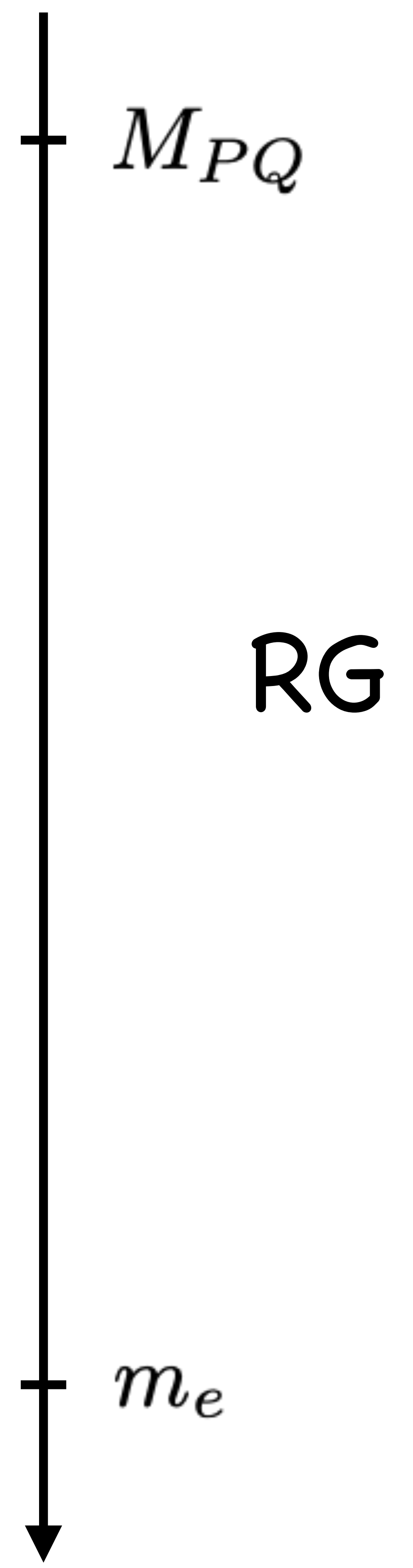
Duality




a → γγ Squared Amplitude
Electric Frame 25 Instantons
Electric Frame Perturbative
Magnetic Frame 30 Instantons
Magnetic Frame Perturbative
$u = 2\,\Lambda^2 + \Delta\,u$
$\sum_{\text{hel}} |\mathcal{M}_{a\to\gamma\gamma}|^2$
$\Lambda^{-2}\,\Delta\,u$
10^2
10^1
10^0
0.1
0.05
0.10
0.50
1
5

Without SUSY

PQ magnetic fermion
light electron



Without SUSY

PQ magnetic fermion
light electron

$\xrightarrow{\text{duality}}$

PQ electric fermion
light magnetic "electron"

M_{PQ}

RG

m_e

Without SUSY

PQ magnetic fermion
light electron

$\xrightarrow{\text{duality}}$

PQ electric fermion
light magnetic "electron"

Witten effect

$$Q = g\,\frac{\theta(a)}{2\pi}$$

$$\alpha_d\left(\theta(a)\right)$$

M_{PQ}

RG

m_e

Without SUSY

PQ magnetic fermion
light electron

$\xrightarrow{\text{duality}}$

PQ electric fermion
light magnetic "electron"

M_{PQ}

Witten effect

$$Q = g\frac{\theta(a)}{2\pi}$$

RG

$\alpha\left(\theta(a)\right)$ CP violating
axion couplings

$\xleftarrow{\text{duality}}$

$\alpha_d\left(\theta(a)\right)$

m_e

SL(2,Z) Covariance

$$\frac{\mathrm{Im}\,(\tau)}{4\pi}\,\partial_\mu\left(F^{\mu\nu} + i\,{}^*F^{\mu\nu}\right) = J^\nu - \overline{\tau}K^\nu$$

SL(2,Z) Covariance

$$\frac{\mathrm{Im}\,(\tau)}{4\pi}\,\partial_\mu\left(F^{\mu\nu}+i\,{}^*F^{\mu\nu}\right)=J^\nu-\bar{\tau}K^\nu$$

$$\frac{1}{e^2}\partial_\mu F^{\mu\nu}=J^\nu-\frac{\theta}{2\pi}K^\nu \qquad\qquad \frac{1}{4\pi}\partial_\mu\,{}^*F^{\mu\nu}=K^\nu$$

SL(2,Z) Covariance

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$$\frac{1}{e^2}\partial_\mu F^{\mu\nu}=J^\nu-\frac{\theta}{2\pi}K^\nu \qquad\qquad \frac{1}{4\pi}\partial_\mu\,{}^*F^{\mu\nu}=K^\nu$$

$$\frac{\partial_\mu}{4\pi}\begin{pmatrix} {}^*F^{\mu\nu} \\ \mathrm{Im}\,(\tau)\,F^{\mu\nu}+\mathrm{Re}\,(\tau)\,{}^*F^{\mu\nu}\end{pmatrix}=\begin{pmatrix} K^\nu \\ J^\nu \end{pmatrix}$$

SL(2,Z) doublets

SL(2,Z) Covariance

$$\frac{\mathrm{Im}\,(\tau)}{4\pi}\,\partial_\mu\left(F^{\mu\nu}+i\,{}^*F^{\mu\nu}\right)=J^\nu-\overline{\tau}K^\nu$$

$$\frac{1}{e^2}\partial_\mu F^{\mu\nu}=J^\nu-\frac{\theta}{2\pi}K^\nu \qquad\qquad \frac{1}{4\pi}\partial_\mu\,{}^*F^{\mu\nu}=K^\nu$$

$$\frac{\partial_\mu}{4\pi}\begin{pmatrix}{}^*F^{\mu\nu}\\ \mathrm{Im}\,(\tau)\,F^{\mu\nu}+\mathrm{Re}\,(\tau)\;{}^*F^{\mu\nu}\end{pmatrix}=\begin{pmatrix}K^\nu\\ J^\nu\end{pmatrix}$$

SL(2,Z) doublets

$$\frac{\partial_\mu}{4\pi}\mathrm{Im}\begin{pmatrix}F^{\mu\nu}_+\\ \tau F^{\mu\nu}_+\end{pmatrix}=\begin{pmatrix}K^\nu\\ J^\nu\end{pmatrix}$$

$$F^{\mu\nu}_+=F^{\mu\nu}+i\,{}^*F^{\mu\nu}$$

SL(2,Z) Covariance

$$\frac{\partial_\mu}{4\pi}\mathrm{Im}\begin{pmatrix} F_+^{\mu\nu} \\ \tau F_+^{\mu\nu} \end{pmatrix} = \begin{pmatrix} K^\nu \\ J^\nu \end{pmatrix}$$

SL(2,Z) Covariance

$$\frac{\partial_\mu}{4\pi}\mathrm{Im}\begin{pmatrix} F_+^{\mu\nu} \\ \tau F_+^{\mu\nu} \end{pmatrix} = \begin{pmatrix} K^\nu \\ J^\nu \end{pmatrix}$$

$$\tau \longrightarrow \tau(x)$$

SL(2,Z) Covariance

$$\frac{\partial_\mu}{4\pi}\mathrm{Im}\begin{pmatrix} F_+^{\mu\nu} \\ \tau F_+^{\mu\nu} \end{pmatrix} = \begin{pmatrix} K^\nu \\ J^\nu \end{pmatrix}$$

$$\tau \longrightarrow \tau(x)$$

SL(2,Z) doublets

$$\frac{\partial_\mu}{4\pi}\mathrm{Im}\begin{pmatrix} F_+^{\mu\nu} \\ \tau F_+^{\mu\nu} \end{pmatrix} + \mathrm{Im}\begin{pmatrix} F_+^{\mu\nu} \\ \tau F_+^{\mu\nu} \end{pmatrix} f(\tau,\bar\tau)\partial_\mu\tau = \begin{pmatrix} K^\nu \\ J^\nu \end{pmatrix}$$

SL(2,Z) invariant

Linearized EOM

$$\frac{1}{4\pi}\partial_\mu {}^*F^{\mu\nu} = K^\nu$$

$$\left(\frac{1}{e^2} + g'\theta a\right)\partial_\mu F^{\mu\nu} + g'\theta\partial_\mu a F^{\mu\nu} + g\partial_\mu a\, {}^*F^{\mu\nu} = J^\nu - \left(\frac{\theta}{2\pi} + 4\pi g a\right) K^\mu$$

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Challenge for Model Builders

this class of theories has two CP problems: θ_{QCD}, θ_{EM}

enhance g while keeping θ_{EM} small

Conclusions

axion couplings are not quantized

additional monopole/instanton terms can be large

generically leads to CP violation