Hard & Forward Scattering: New Tools from EFT

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Bay Area Theory Seminar
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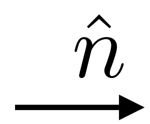
Outline

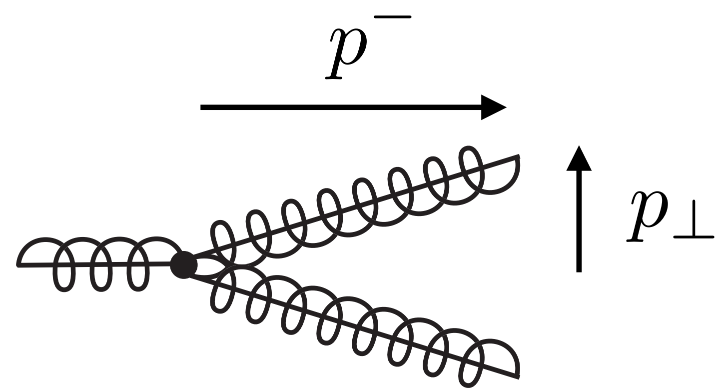
- Intro: Collider Dynamics, EFT, and Factorization
- New Tools For Hard Scattering (Jet Substructure, Multiple Vars)
  - Application to Top Mass Measurements at the LHC
- New Tools for Forward Scattering (Glauber Operators)
  - Application to understand BFKL evolution as operator ren.
  - Lagrangian description of Factorization Violation
- Conclude

Introduction

Relevant Momentum Regions:

- Collinear Splittings



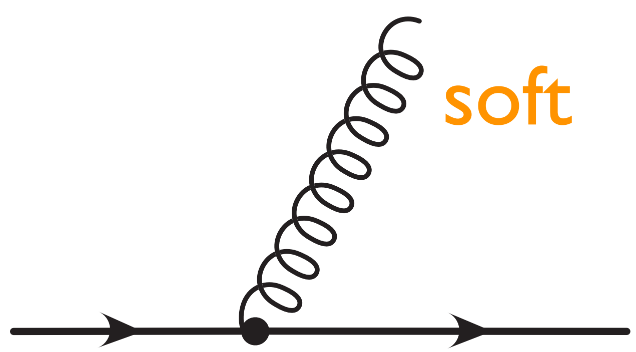


"n-collinear"

$$p^- \gg p_\perp \gg p^+$$

$$\text{onshell: } p^+ p^- = \vec{p}_\perp^{\,2}$$

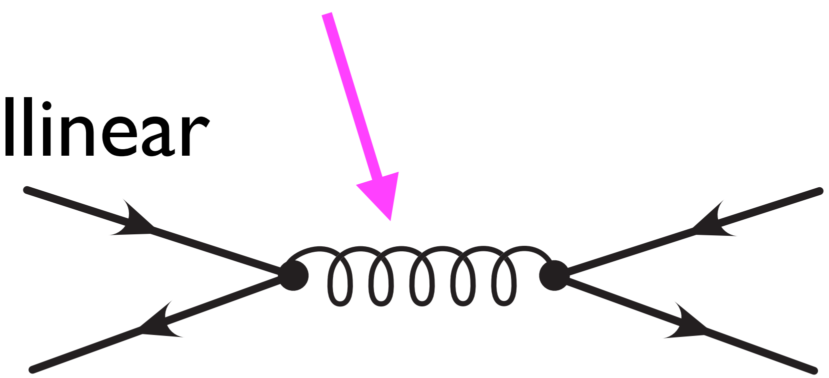
- Soft Emission



n-collinear n-collinear

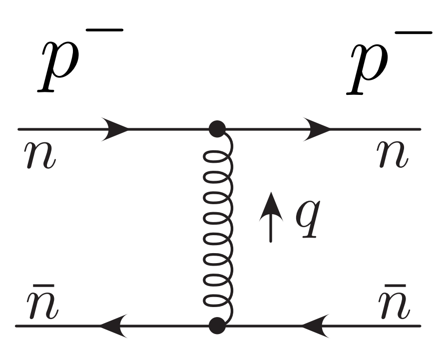
- Hard Propagators (short dist.)

n̄-collinear



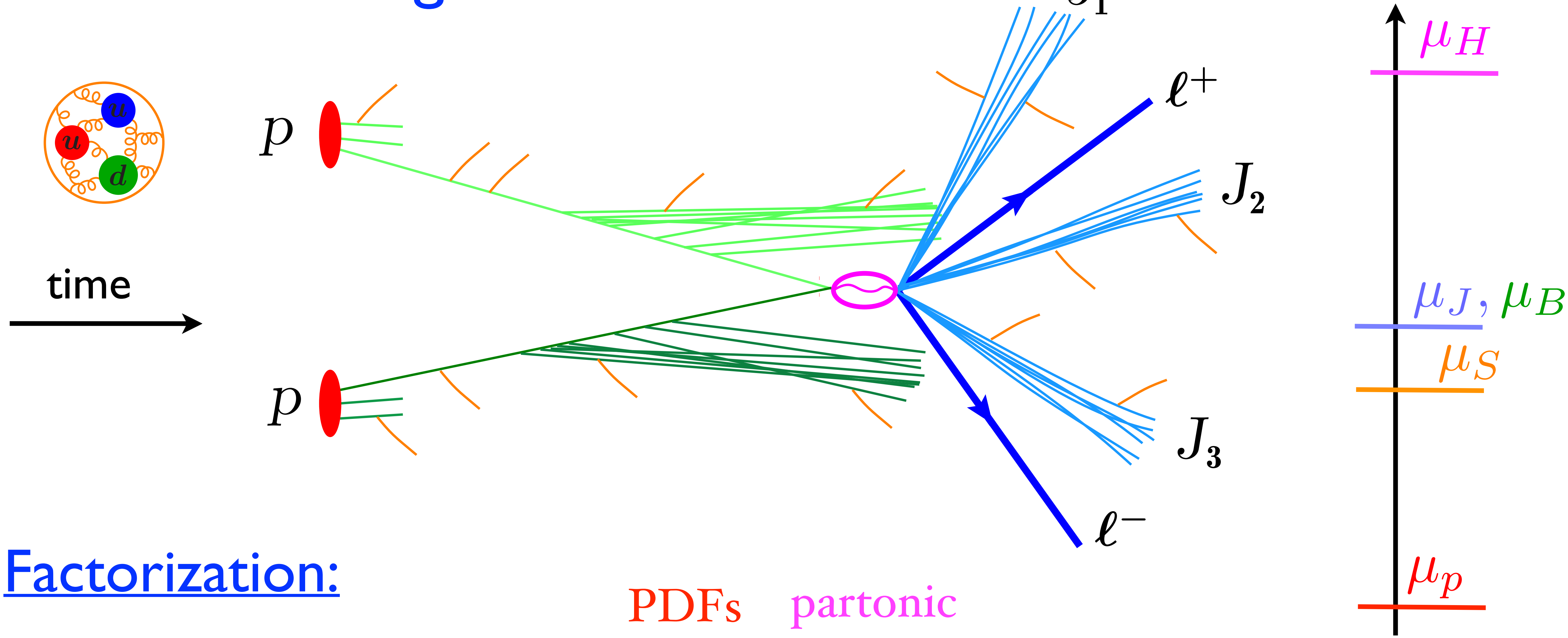
n-collinear

- Glauber Exchange



forward scattering

Hard Scattering Collisions:



Factorization:

PDFs partonic

Nonperturbative: $d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$ ← hadronization

$\mu_p \simeq \Lambda_{\mathrm{QCD}}$

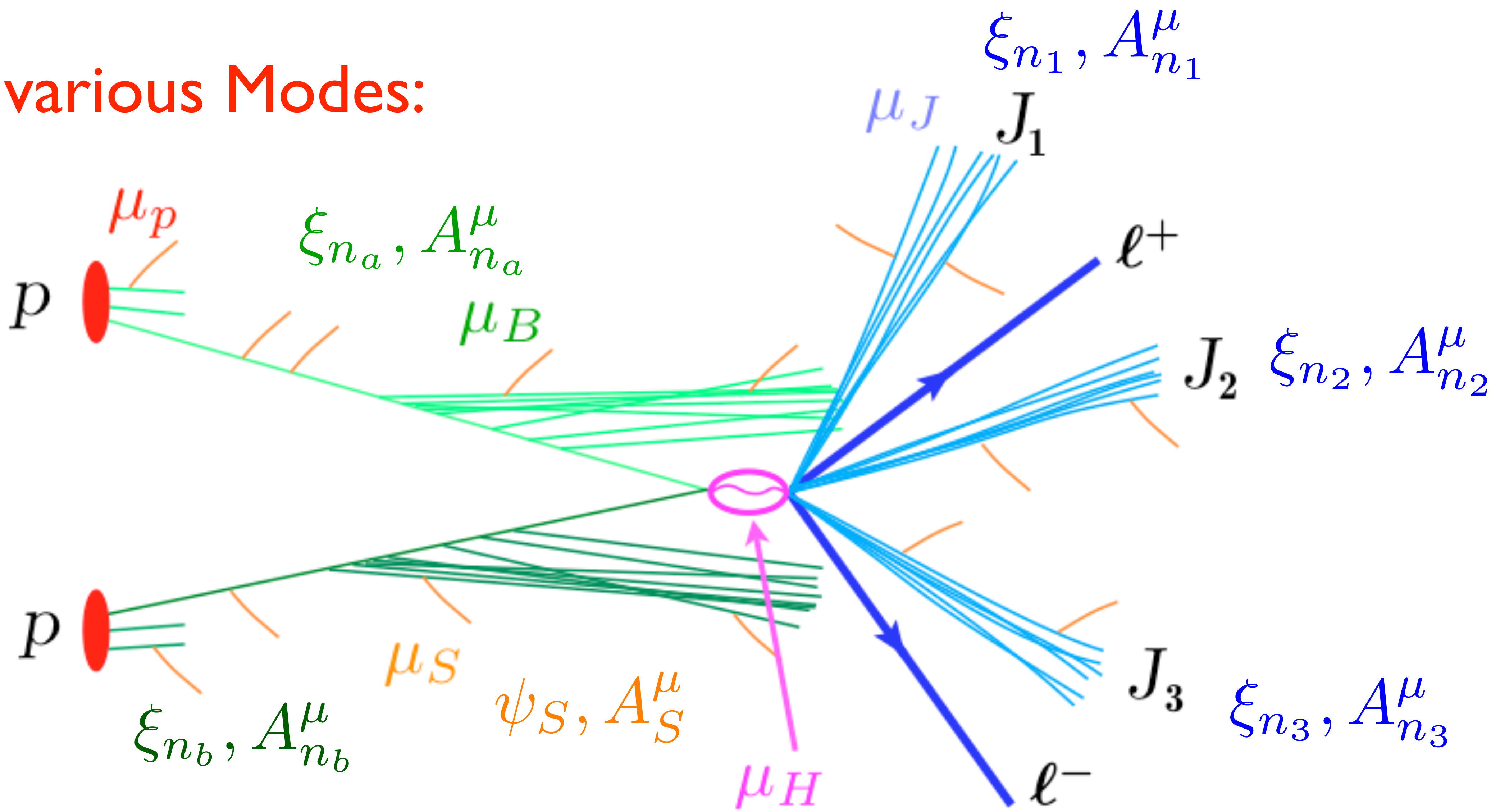
(In some cases by Operators, or is power suppressed)

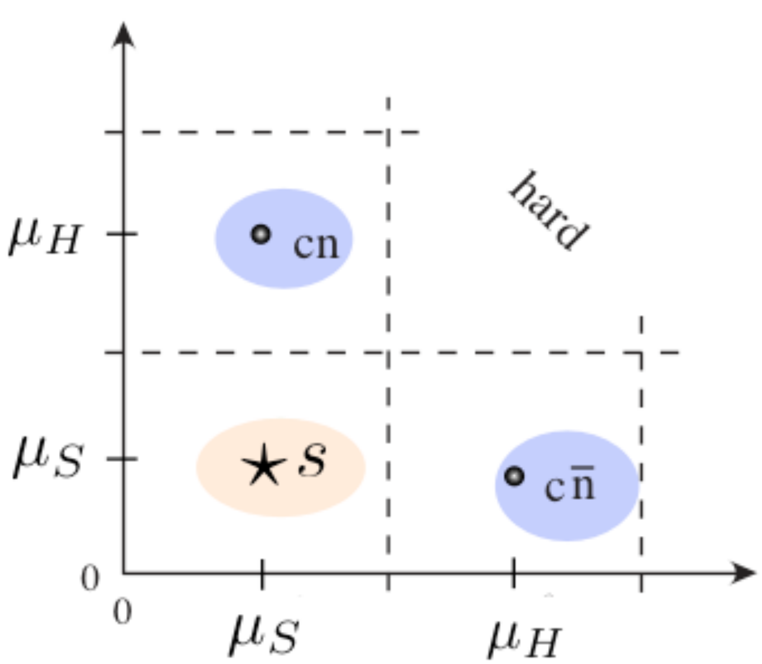
Perturbative: $\hat{\sigma}_{\mathrm{fact}} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$ Used to Sum Logs

or $\hat{\sigma}_{\mathrm{fact}} = \text{parton shower}$

EFT for collider physics = Soft Collinear Effective Theory

Fields for various Modes:





- dominant contributions from isolated regions of momentum space
- use subtractions rather than sharp boundaries to preserve symmetry

EFT Principles used for SCET

- Matching — QCD & SCET must agree at long distances
  short distance encoded by coefficients, C
- Power Counting — for fields, states, amplitudes with loops
  Rigorously track expansions
  Power counting theorems
- Symmetry — Gauge symmetry within sectors
  Lorentz & Reparameterization symmetries

Relevant Modes

$\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
n_a-collinear	$\xi_{n_a},\ A^\mu_{n_a}$	$(n_a \cdot p, \bar{n}_a \cdot p, p_{\perp a}) \sim Q(\lambda^2, 1, \lambda)$	collinear initial state jet a	onshell
n_b-collinear	$\xi_{n_b},\ A^\mu_{n_b}$	$(n_b \cdot p, \bar{n}_b \cdot p, p_{\perp b}) \sim Q(\lambda^2, 1, \lambda)$	collinear initial state jet b	onshell
n_j-collinear	$\xi_{n_j},\ A^\mu_{n_j}$	$(n_j \cdot p, \bar{n}_j \cdot p, p_{\perp j}) \sim Q(\lambda^2, 1, \lambda)$	collinear final state jet in $\hat{n}_j$	onshell
soft	$\psi_S,\ A^\mu_S$	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{us},\ A^\mu_{us}$	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda)$, $a + b > 2$	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

Integrate out these modes

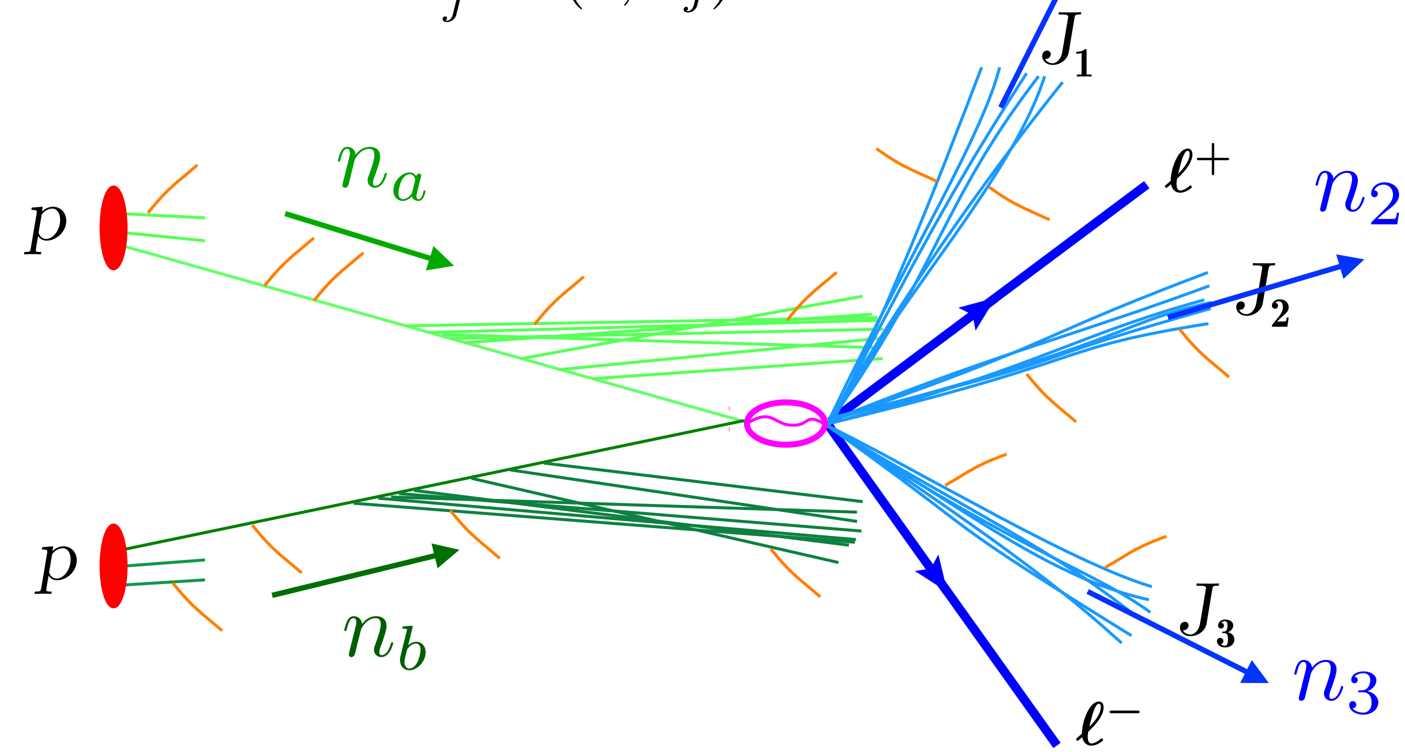
$$n_j^\mu = (1, \hat{n}_j)$$

$$p^\mu = \bar{n}_i \cdot p \, \frac{n_i^\mu}{2} + n_i \cdot p \, \frac{\bar{n}_i^\mu}{2} + p_\perp^\mu$$

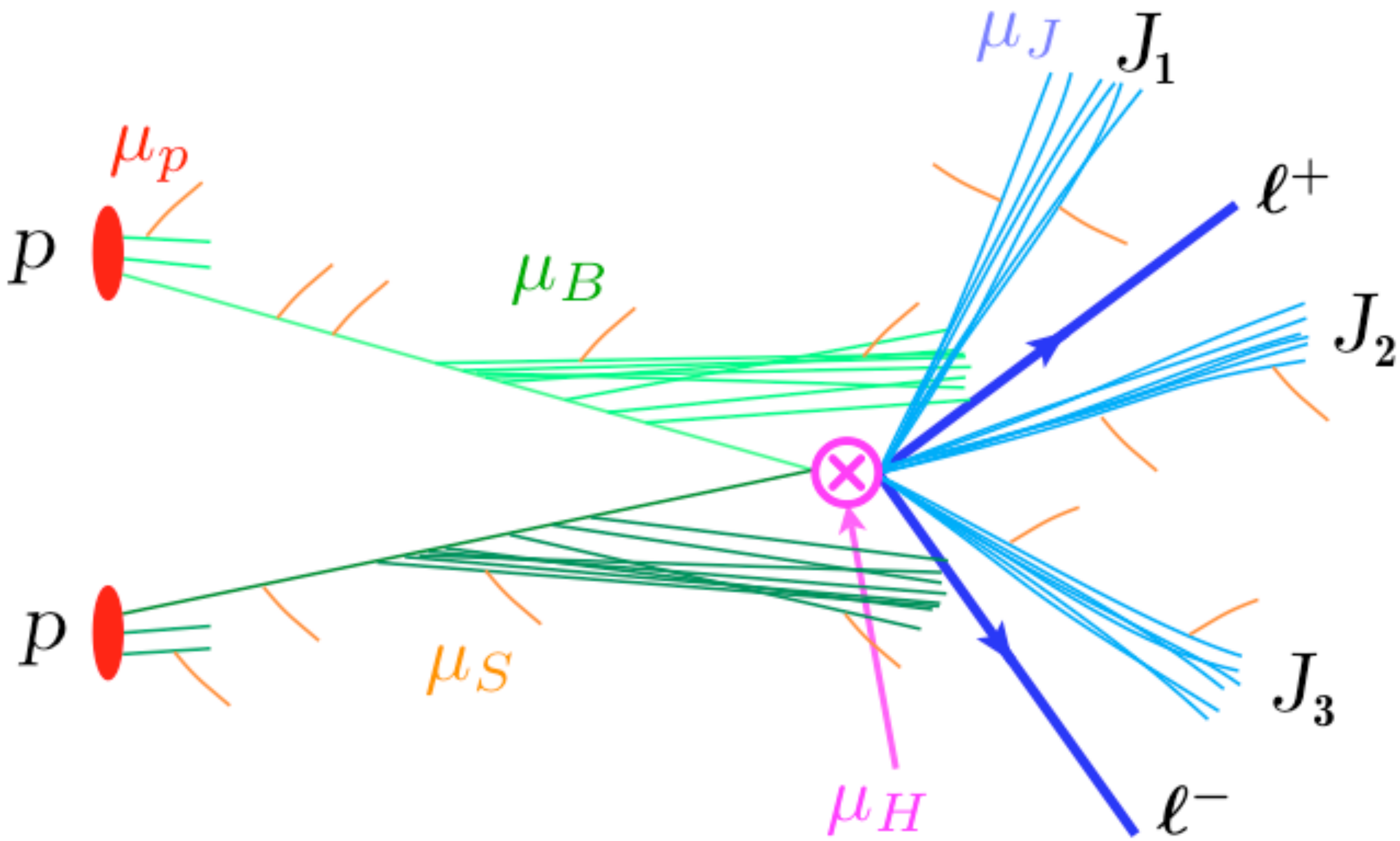
$$n_i^2 = 0$$

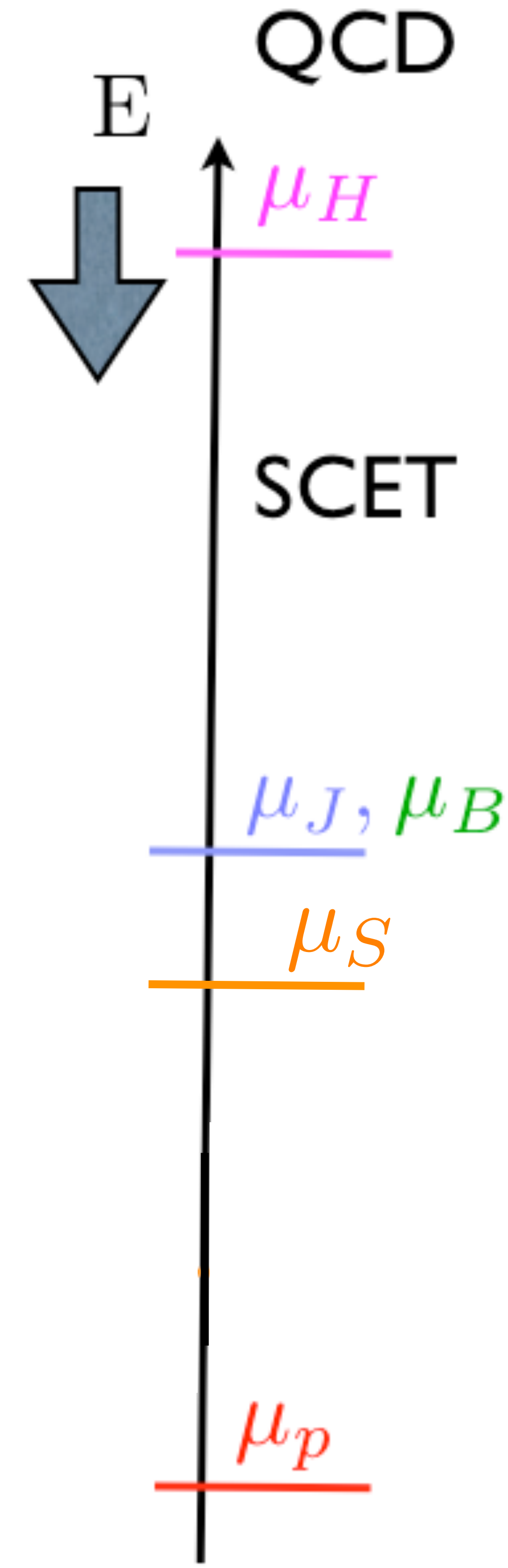
$$\bar{n}_i^2 = 0$$

$$n_i \cdot \bar{n}_i = 2$$



Hard-collinear factorization

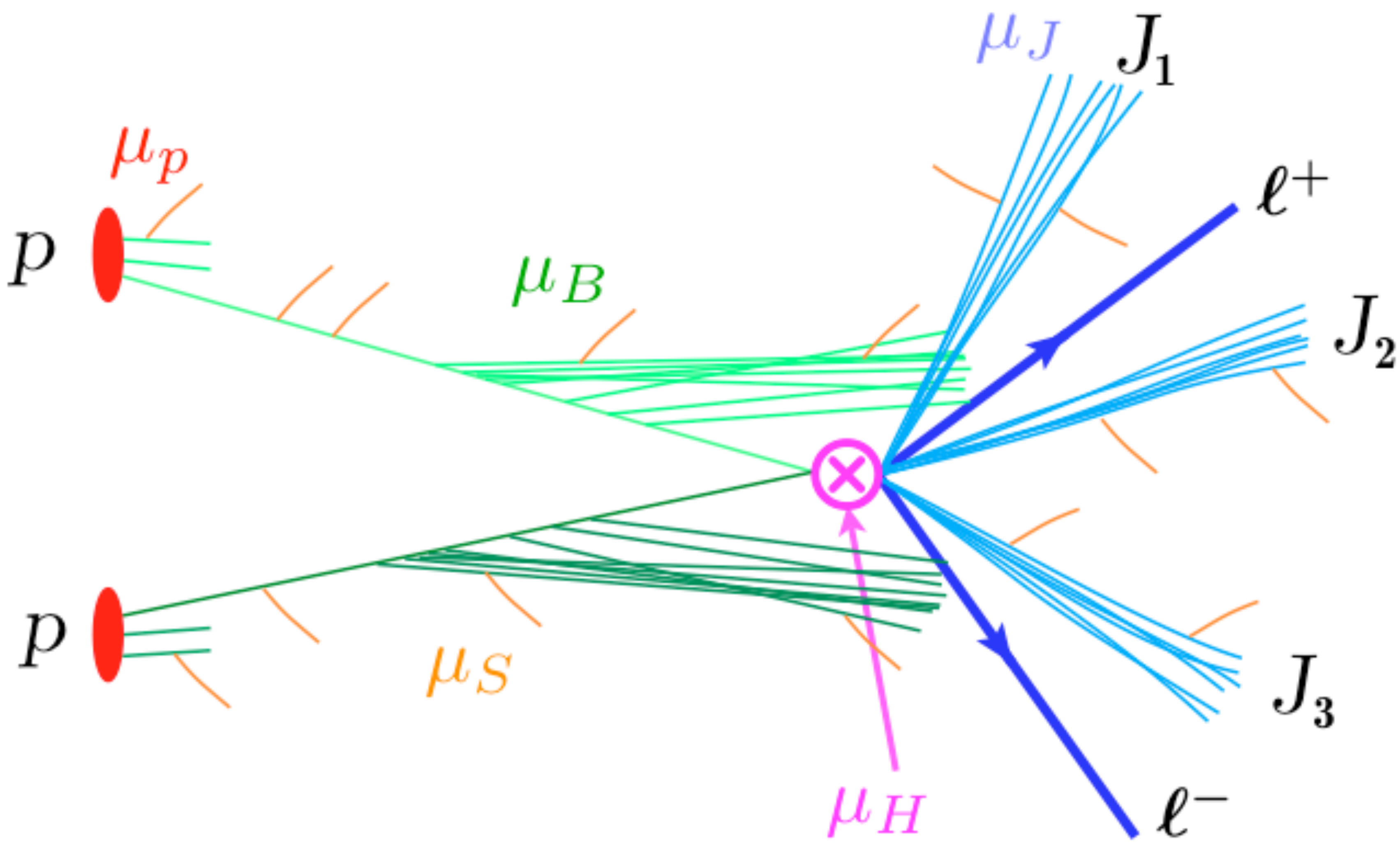


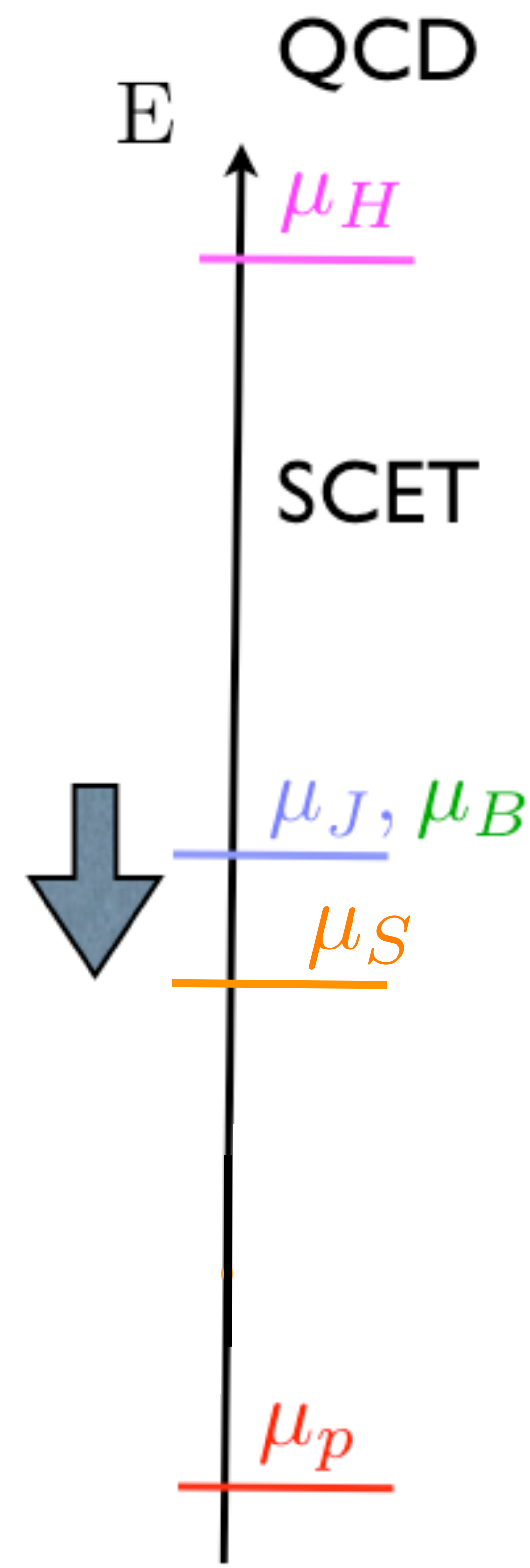


μ_H: Wilson coefficients for SCET Hard Scattering Operators

$$C \otimes \mathcal{O}$$

Hard-collinear factorization





Operators are built of building block fields:

$$\mathcal{O} = (\mathcal{B}_{n_a\perp})(\mathcal{B}_{n_b\perp})(\mathcal{B}_{n_1\perp})(\bar{\chi}_{n_2})(\chi_{n_3})$$

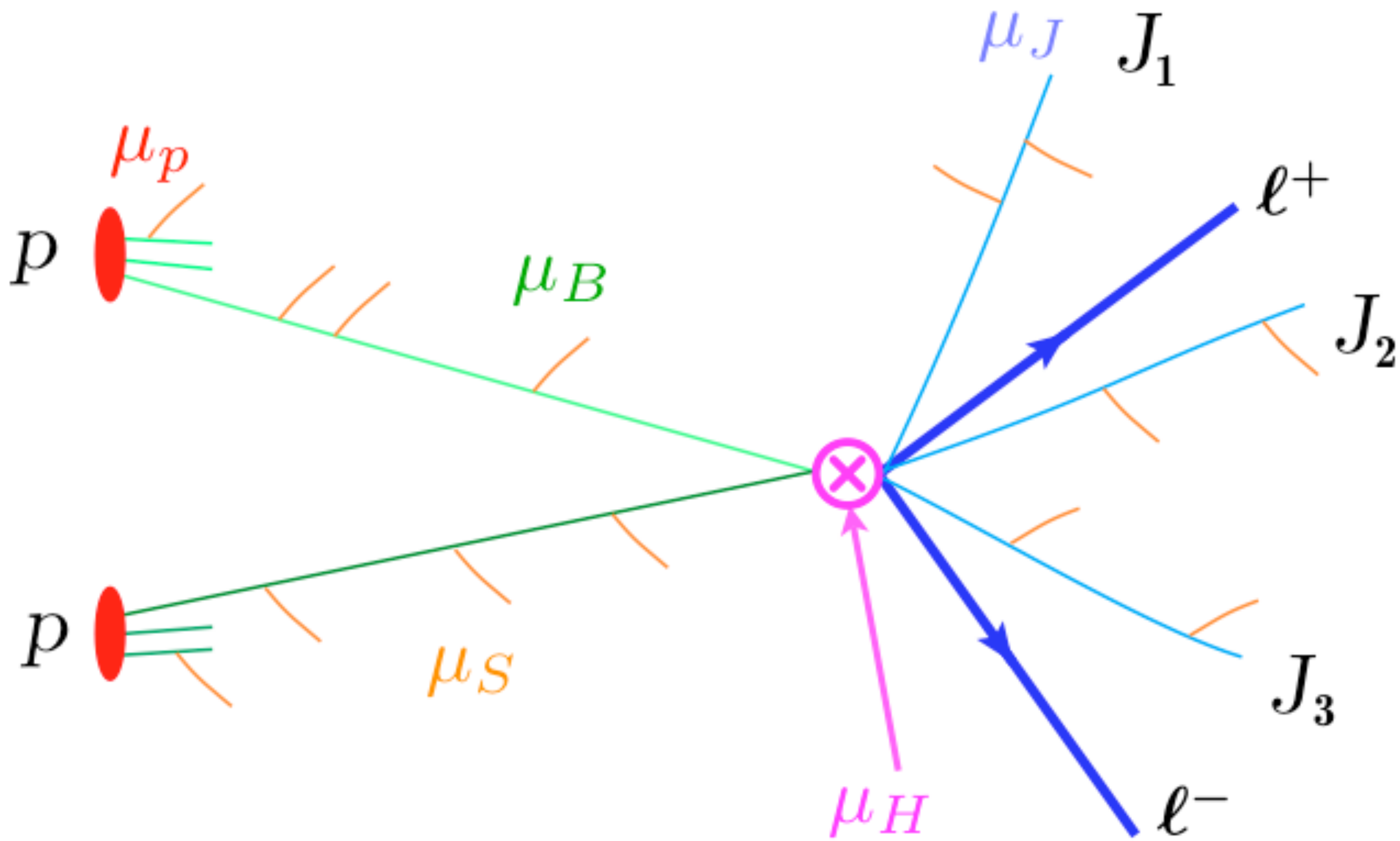
$\chi_n = (W_n^\dagger \xi_n)$ "quark jet"

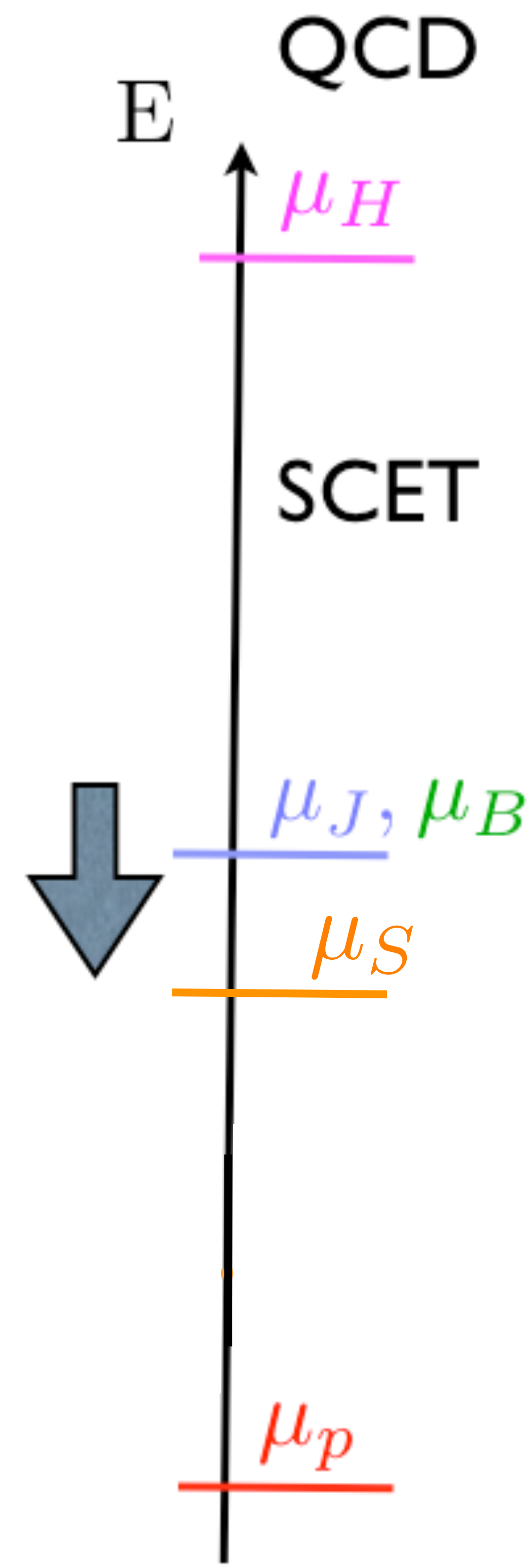
$\mathcal{B}_{n\perp}^\mu = [W_n^\dagger i D_\perp^\mu W_n]$ "gluon jet"

Wilson lines

$$W_n = P \exp\left(ig \int_{-\infty}^0 ds\, \bar{n} \cdot A_n(x + \bar{n}s)\right)$$

Soft-collinear factorization



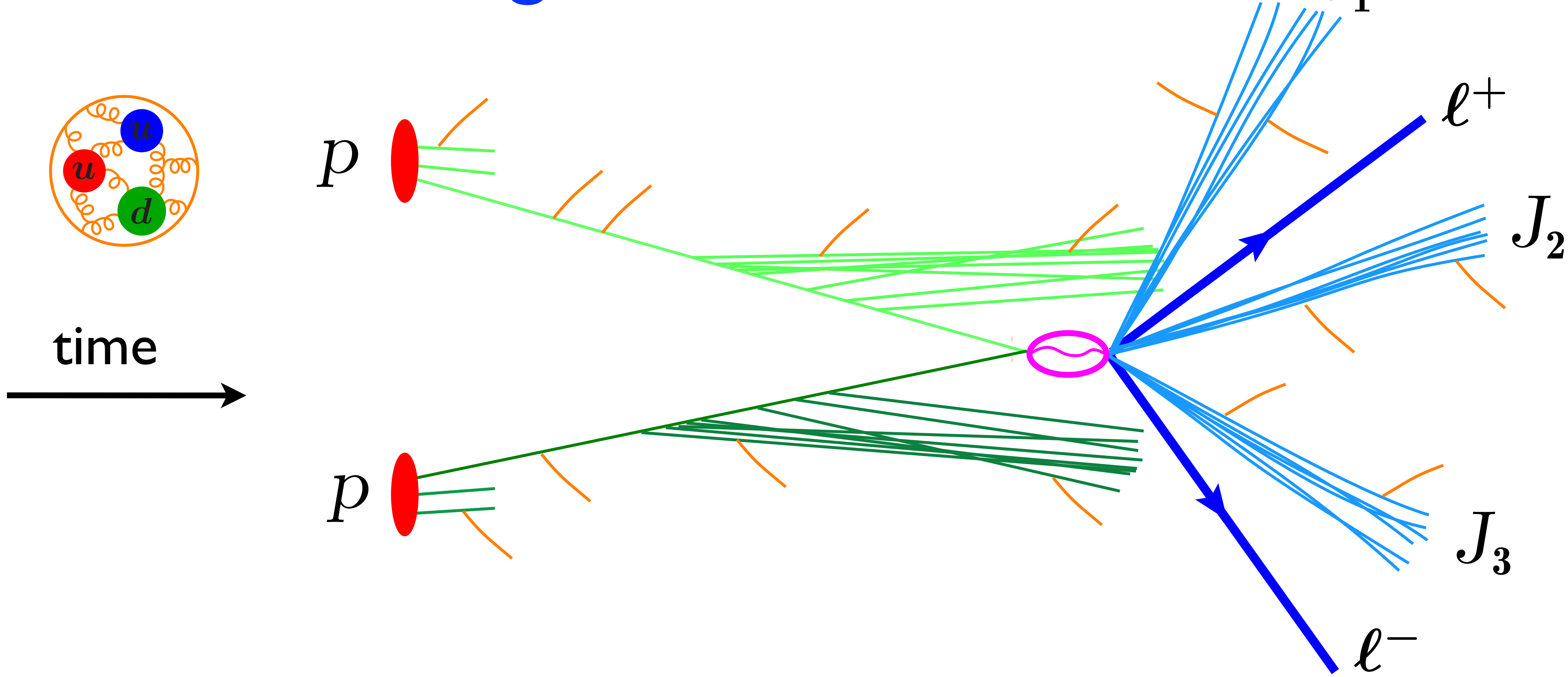


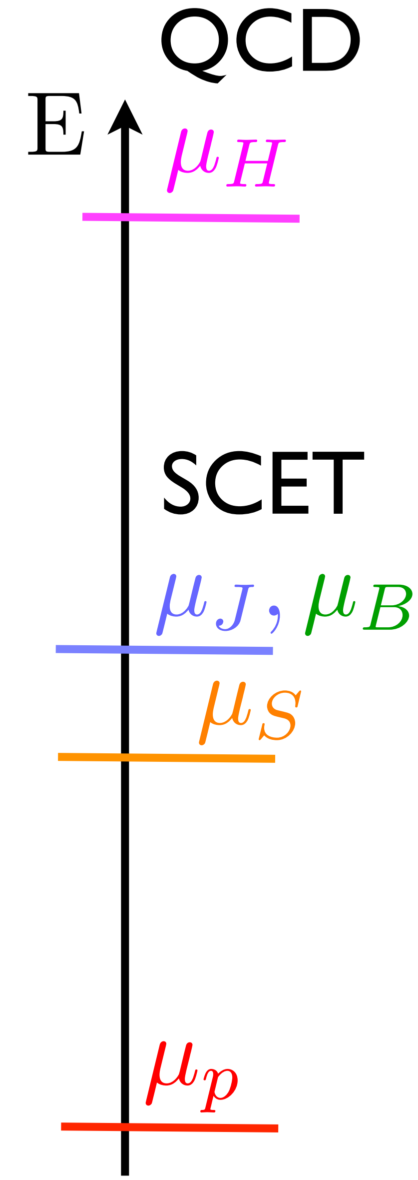
Soft radiation knows only about bulk properties of radiation in the jets

$$(S_{n_a} S_{n_b} S_{n_1} S_{n_2} S_{n_3})$$

Soft Wilson Lines

Hard Scattering Factorization:



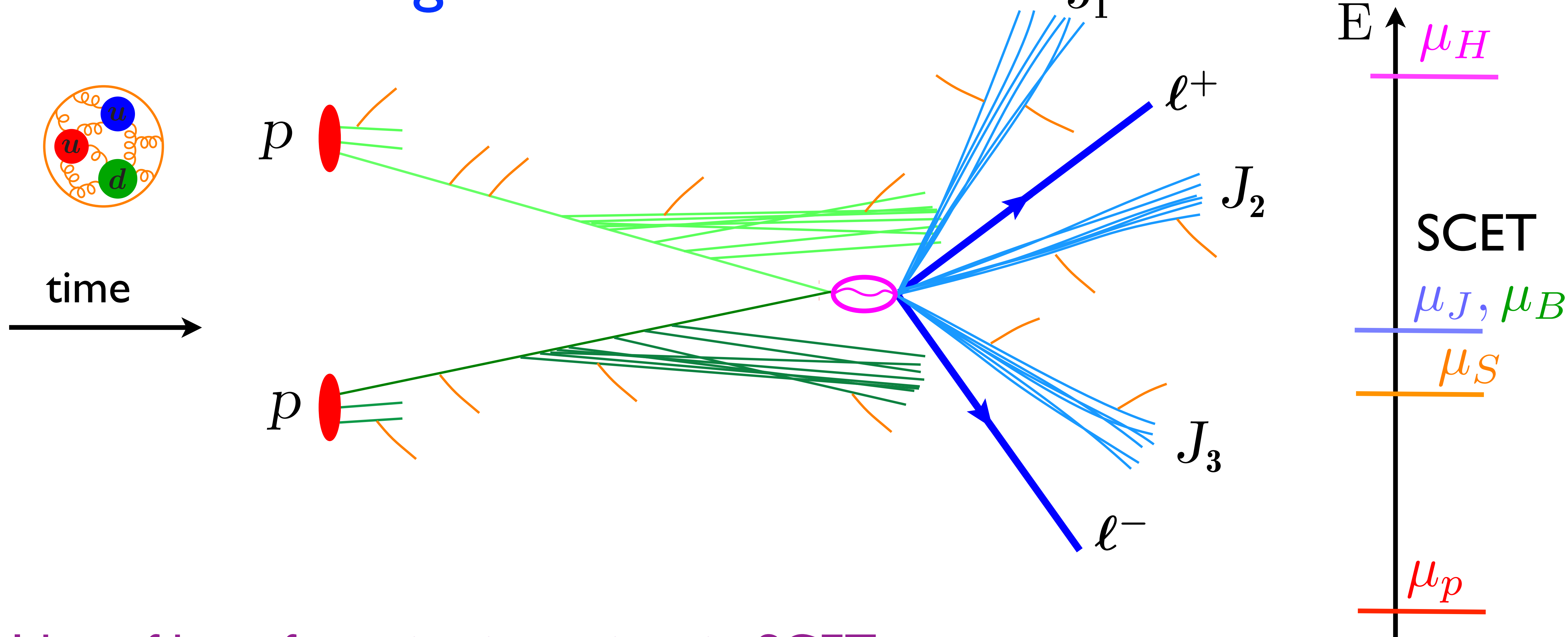


Idea of how factorization arises in SCET:

factorized Lagrangian: $\mathcal{L}^{(0)}_{\mathrm{SCET_{II},S,\{n_i\}}} = \mathcal{L}^{(0)}_S(\psi_S, A_S) + \sum_{n_i} \mathcal{L}^{(0)}_{n_i}(\xi_{n_i}, A_{n_i}) \quad \cancel{+\mathcal{L}^{(0)}_G}$

factorized Hard Ops: $C \otimes (\mathcal{B}_{n_a\perp})(\mathcal{B}_{n_b\perp})(\mathcal{B}_{n_1\perp})(\bar{\chi}_{n_2})(\chi_{n_3})(S_{n_a} S_{n_b} S_{n_1} S_{n_2} S_{n_3})$

Hard Scattering Factorization:



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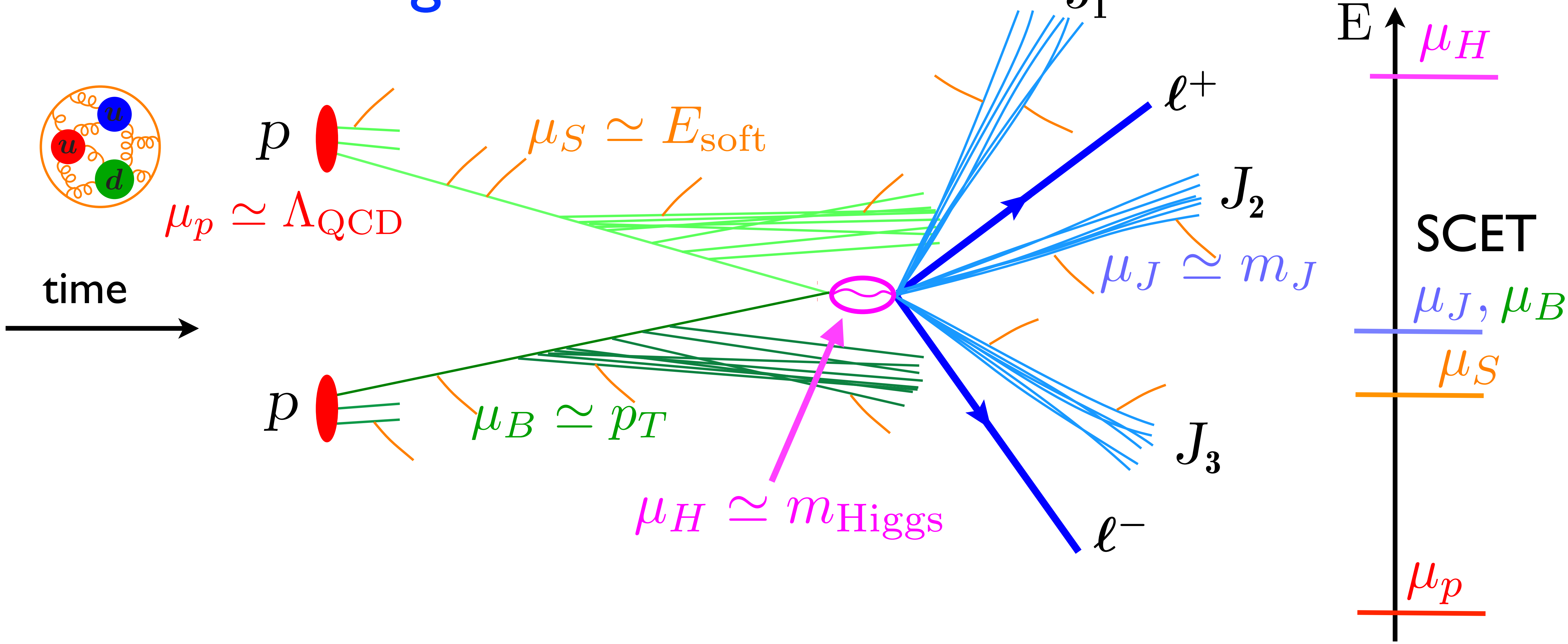
→ factorized squared matrix elements defining jet, soft, … functions

Hard Scattering Factorization:



Nonperturbative: $d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$ ← hadronization

$\mu_p \simeq \Lambda_{\rm QCD}$

(In some cases by Operators, or is power suppressed)

eg. Perturbative: $\hat{\sigma}_{\rm fact} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$

Used to Sum Logs

	μ_B	μ_H	μ_J	μ_S
Universal Functions:	beam	hard	jet	pert. soft

Examples of Factorization:

- Inclusive Higgs production $\qquad pp \to \text{Higgs} + \text{anything}$

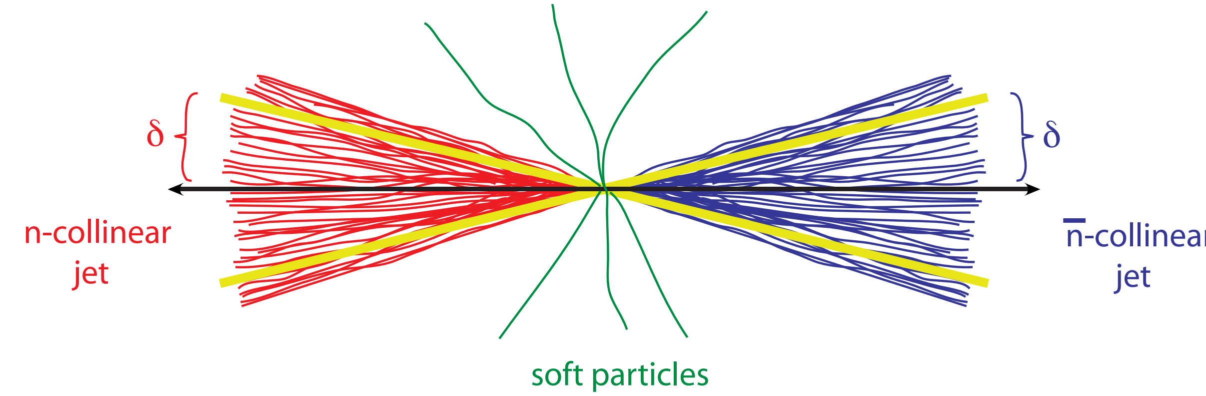
$$d\sigma \;=\; \int dY \sum_{i,j} \int \frac{d\xi_a}{\xi_a}\frac{d\xi_b}{\xi_b} f_i(\xi_a,\mu) f_j(\xi_b,\mu)\; H_{ij}^{\rm incl}\Big(\frac{m_H e^{Y}}{E_{\rm cm}\xi_a}, \frac{m_H e^{-Y}}{E_{\rm cm}\xi_b}, m_H, \mu\Big)$$

(Collins, Soper, Sterman)

(PDFs contribute, No Glaubers, No Softs)

- Dijet production $e^+e^- \to 2$ jets

  thrust $\tau \ll 1$

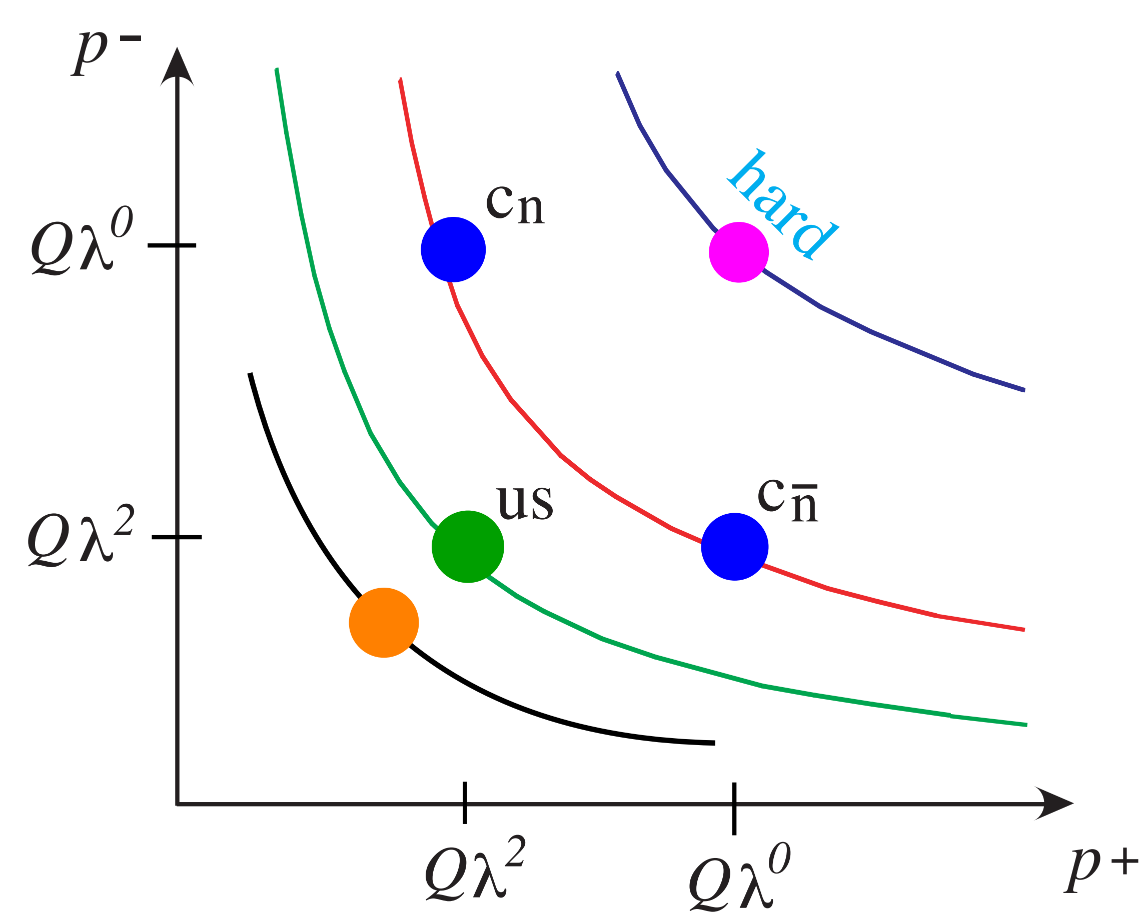


$$\frac{d\sigma}{d\tau} = \sigma_0 H(Q,\mu)\, Q \int d\ell\, d\ell'\; J_T(Q^2\tau - Q\ell,\mu)\, S_T(\ell-\ell',\mu)\, F(\ell')$$

hard function | jet functions | perturbative soft function | non-perturbative soft function

(No PDFs, No Glaubers, Softs contribute)

Modes:



- **Dijet production** $e^+e^- \to 2 \text{ jets}$

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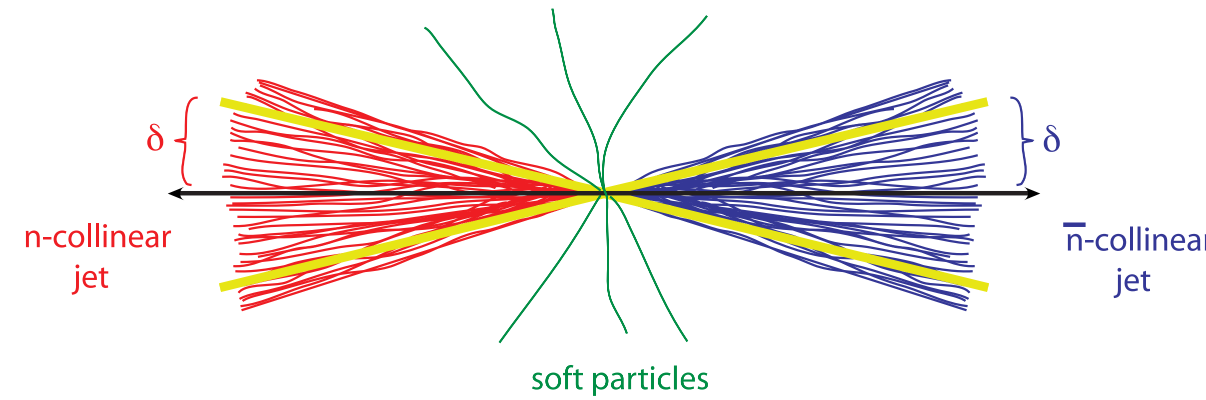

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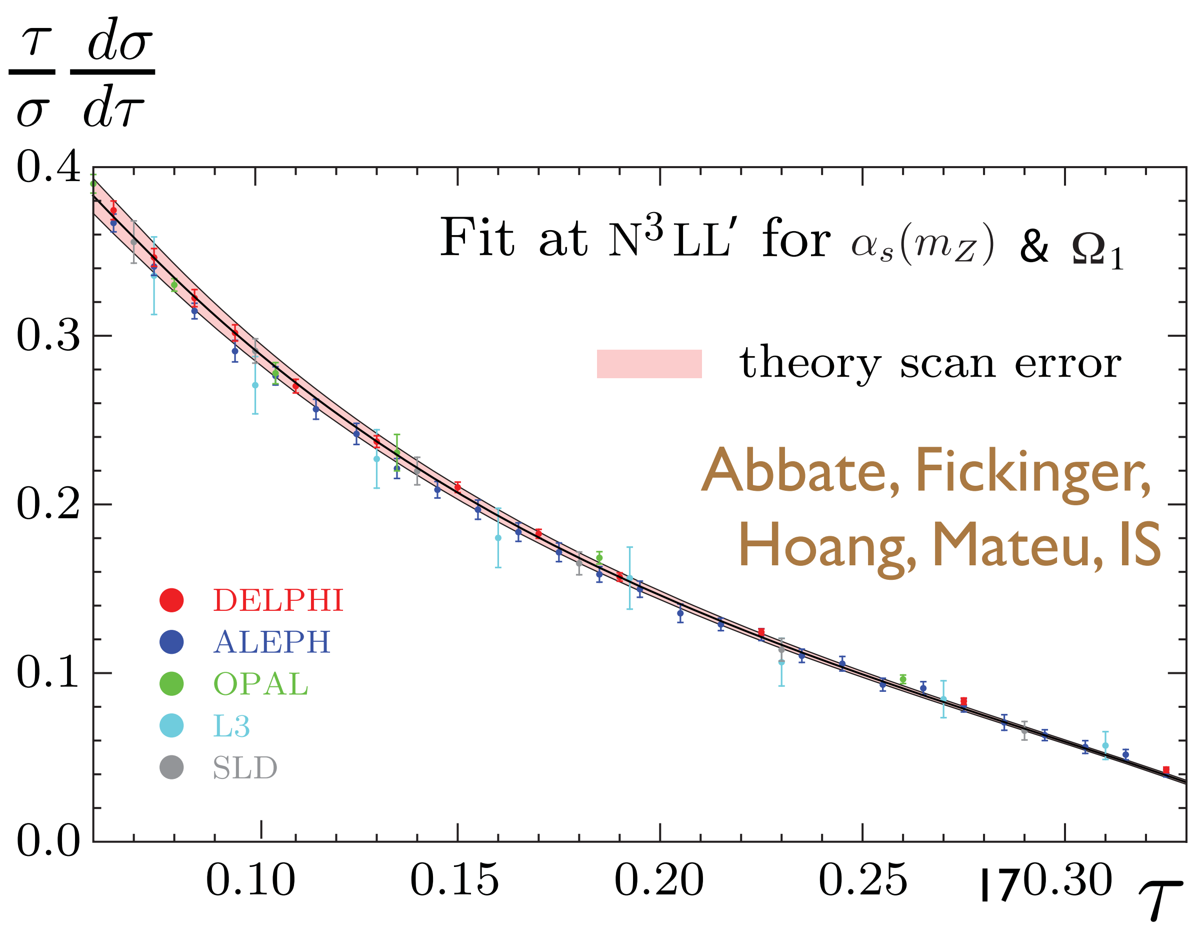
hard function — jet functions — perturbative soft function — non-perturbative soft function

(No PDFs, No Glaubers, Softs contribute)

thrust



$\mathrm{N^3LL'} + \mathcal{O}(\alpha_s^3)$

Two parameter fit:

$$\{\alpha_s(m_Z), \Omega_1\}$$

$$\frac{\chi^2}{\mathrm{dof}} = \frac{440}{485} = 0.91$$

- Higgs with a Jet Veto

$pp \to H + $ 0-jets

$p_T^{\rm jet} \le p_T^{\rm cut} \ll m_H$

$\Lambda_{\rm QCD} \ll p_T^{\rm cut}$

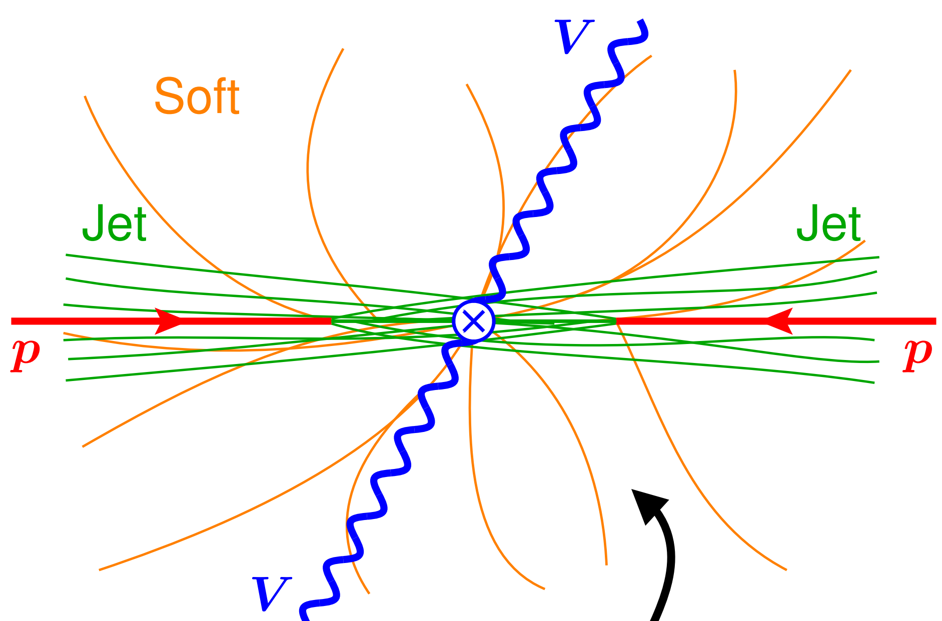
Berger, Marcantonini, IS
Tackmann, Waalewijn
Banfi, Salam, Zanderighi
Becher & Neubert
I.S., Tackmann, Walsh, Zuberi

$$\sigma_0(p_T^{\rm cut}) = H_{gg}(m_H) \times [B_g(m_H, p_T^{\rm cut}, R)]^2 \times S_{gg}(p_T^{\rm cut}, R)$$

(anti-kT jets, radius R)

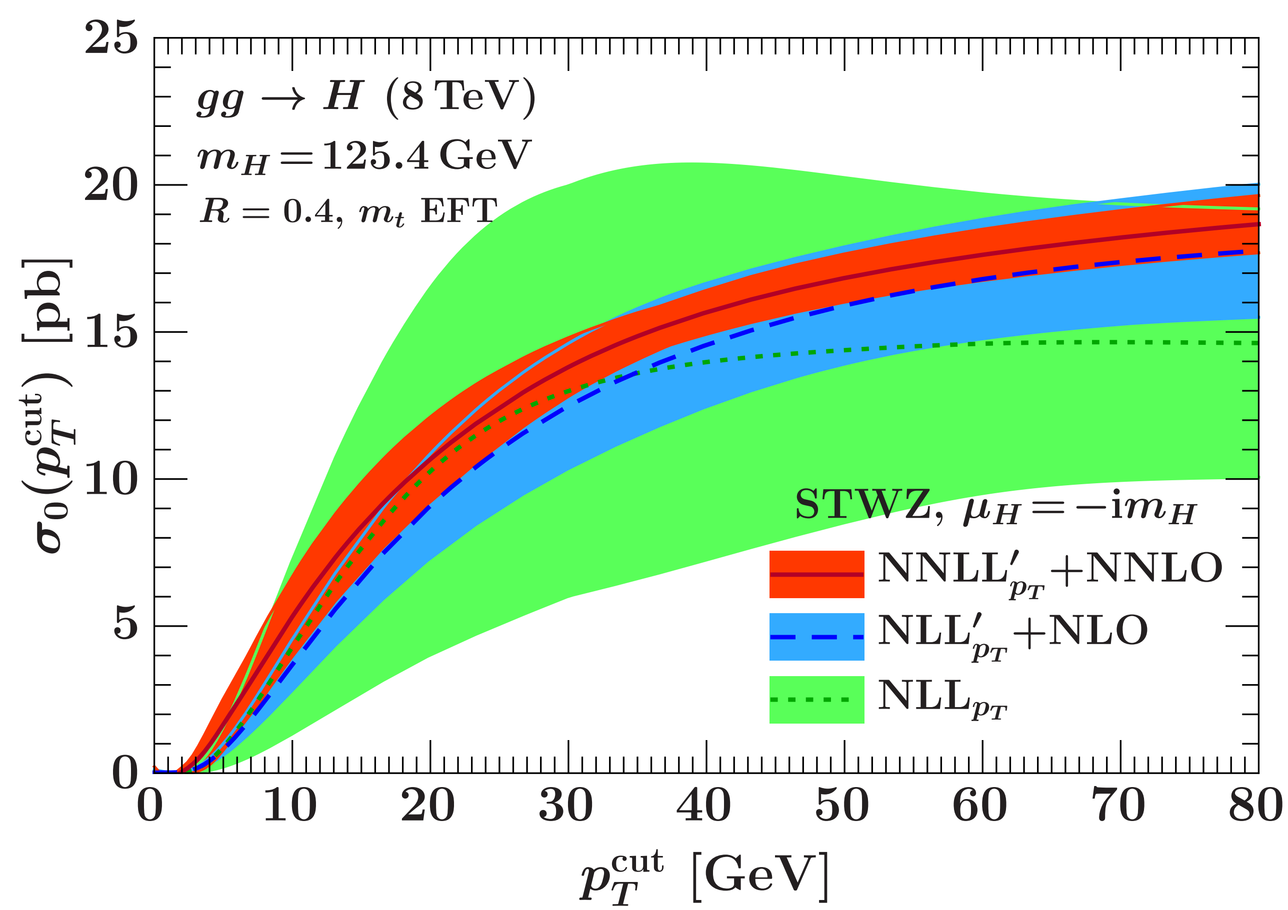
$$B_g = \mathcal{I}_{gj}(m_H, p_T^{\rm cut}, R) \otimes f_j$$

(PDFs and Softs contribute, Glaubers?)



V$= \gamma$, W, or Z

veto anti-kT jets here



I.S., Tackmann, Walsh, Zuberi

Factorization:

- Underlies all theoretical predictions for predictions of collisions. (Perturbative calculations & Monte Carlo)

- Allows us to distinguish functions which are

  perturbative: calculate with an expansion in $\alpha_s \ll 1$

  non-perturbative: extract from data exploiting universality, $\alpha_s \sim 1$

- Can exploit dependence of the functions on scales μ_i

  to sum series of large logarithms: $\sum_k a_k \alpha_s^k \ln^{2k}(z)\,, \quad \sum_k b_k \alpha_s^k \ln^k(z)\,,$

  $$\sum_k c_k \alpha_s^{k+2} \ln^{2k}(z)\,, \quad \ldots$$

- Has been tested experimentally for more processes than we have complete proofs.

Underlying Event?

- Radiation not described by primary hard scattering.
- Modeled by Multiple Particle Interactions (MPI) in Monte Carlos

No rigorous theoretical derivation in
a factorization framework.

New Tools for Hard Scattering

Jet Substructure

Multiple Variables

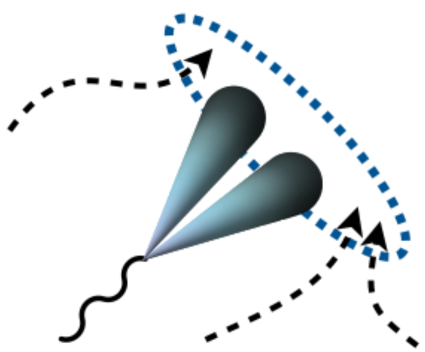
$\Bigg\}$ More Scales!

Jet Substructure:

key tools for:

- grooming jets
- tagging subjets

eg. W/Z tagging in 2016



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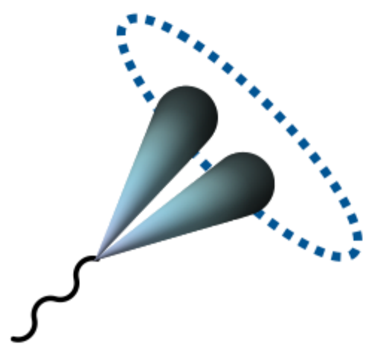
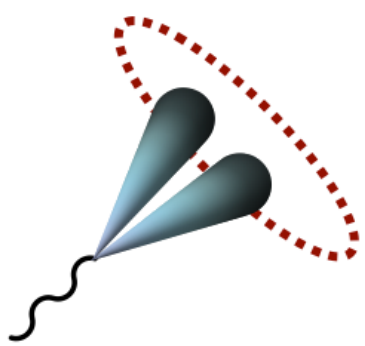


vs.



Soft Drop

Larkoski, Marzani, Soyez, Thaler

Trimming

Krohn, Thaler, Wang

N-subjettiness

Thaler, van Tilburg
(see also Stewart, Tackmann, Waalewijn)

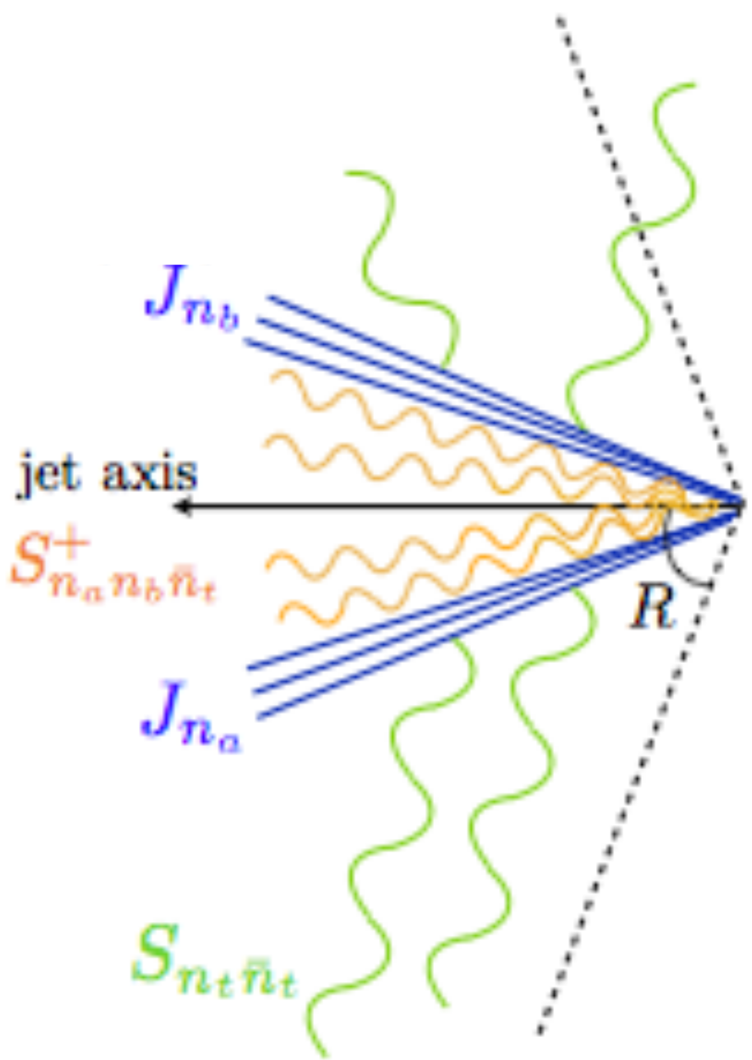
D_2

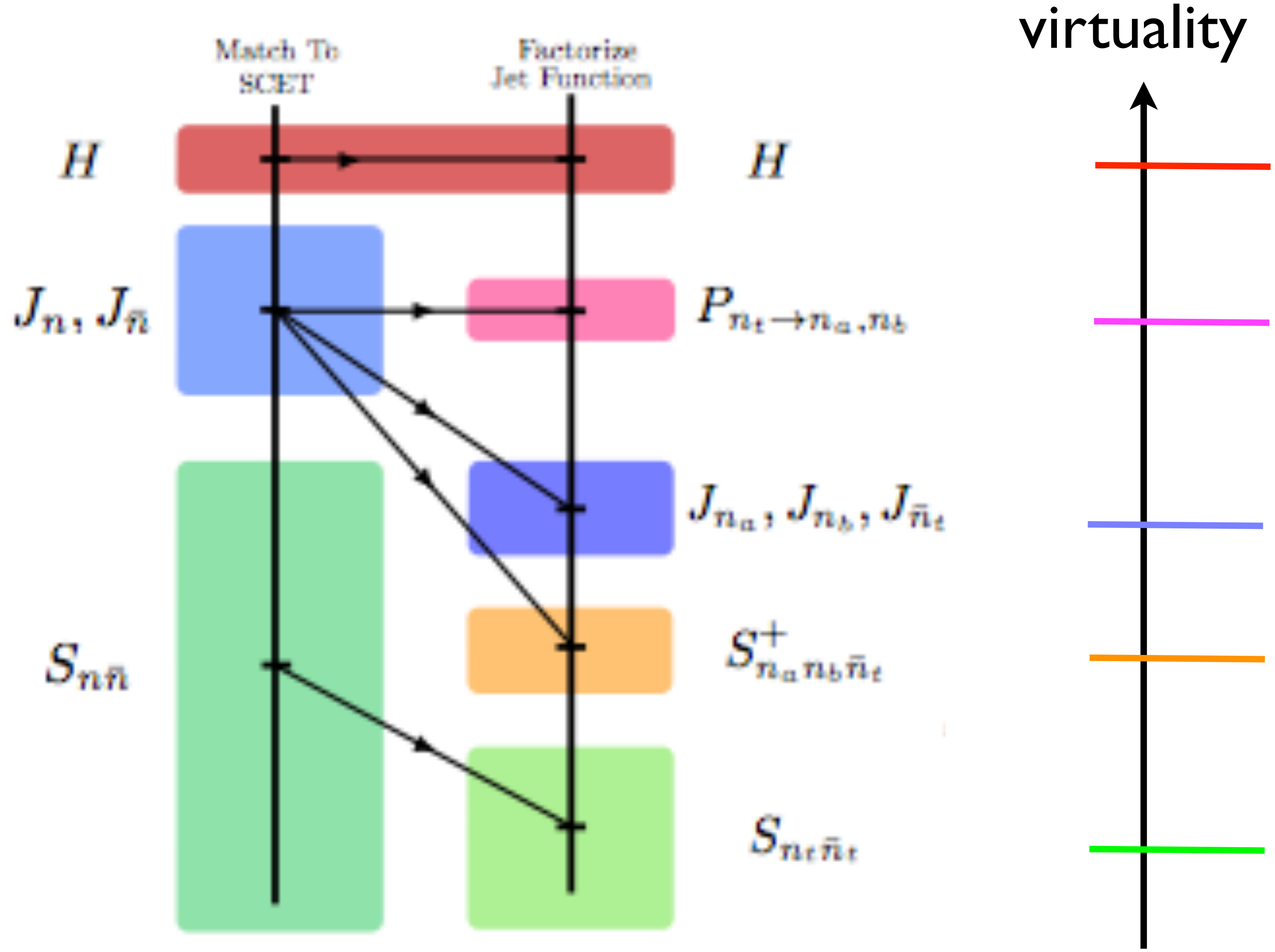
Larkoski, Moult, Neill

More scales:

Collinear Subjets

Bauer, Tackmann, Walsh, Zuberi 2012





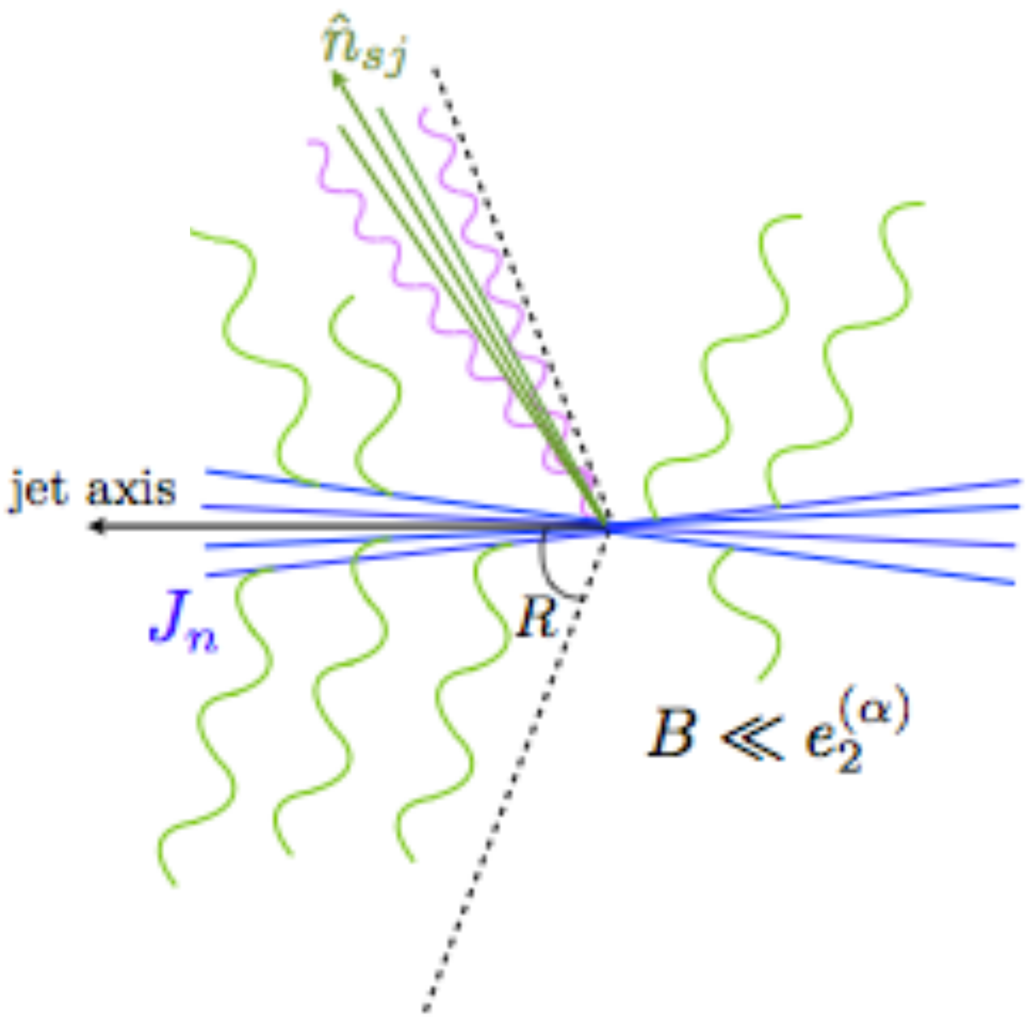
<u>also used for:</u>

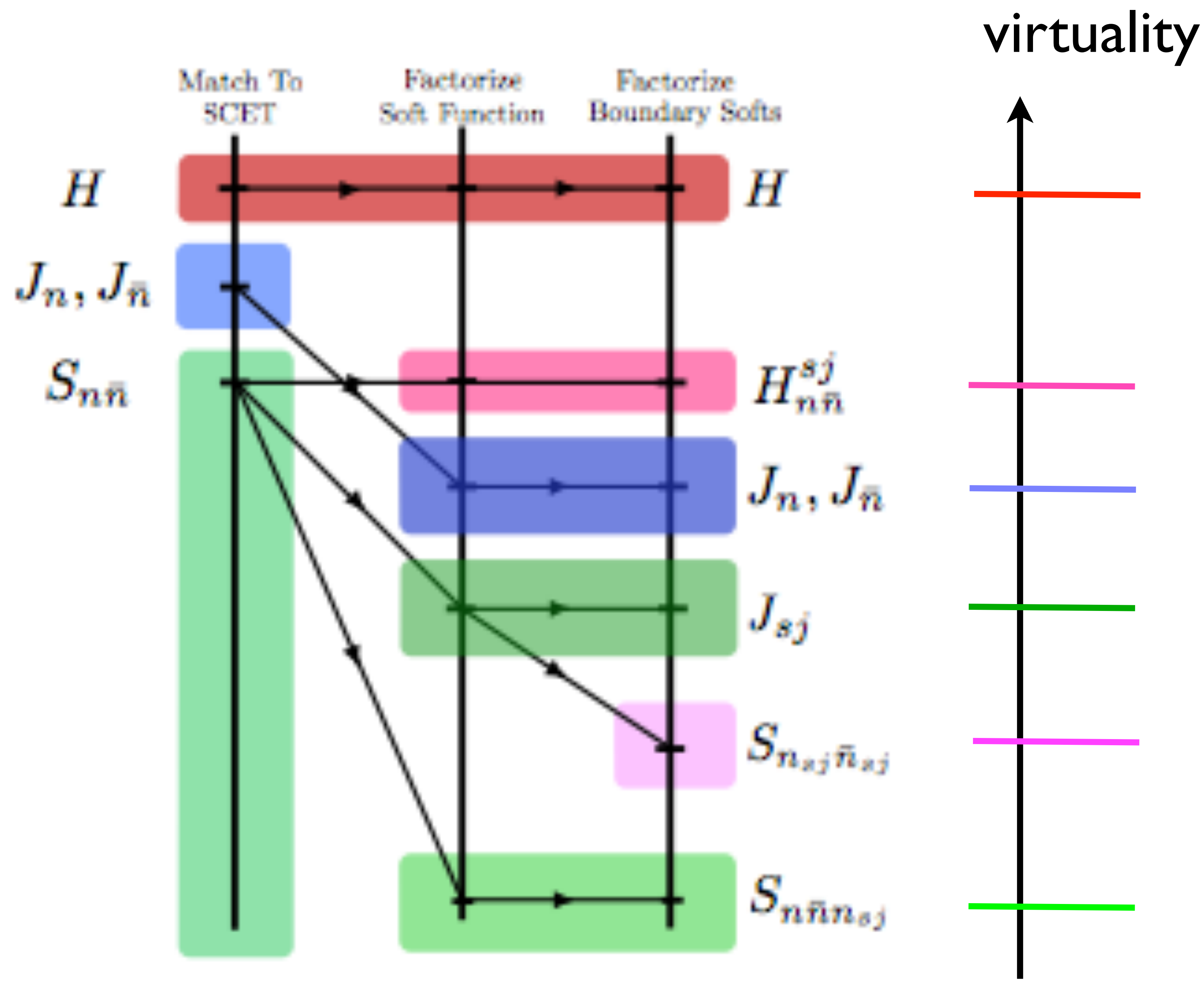
Multiple Measurements: Procura, Waalewijn, Zeune 2014

Sum Logs of Jet Radius, ln(R): Chien, Hornig, Lee; Becher, Neubert, Rothen, Shao; Hornig, Makris, Mehen; Kolodrubetz, Pietrulewicz, IS, Tackmann, Waalewijn, …

More scales:

Soft Subjet Larkoski, Moult, Neill





Factorization theorems for both collinear and soft subjects were use for for the calculation of D_2 by Larkoski, Moult, Neill

Soft Drop

Larkoski, Marzani, Soyez, Thaler 2014

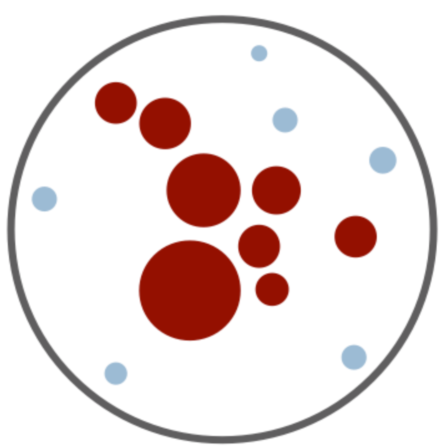
Grooms soft radiation from the jet

$$\frac{\min(p_{Ti}, p_{Tj})}{p_{Ti} + p_{Tj}} > z_{\mathrm{cut}} \left(\frac{\Delta R_{ij}}{R_0} \right)^{\beta}$$

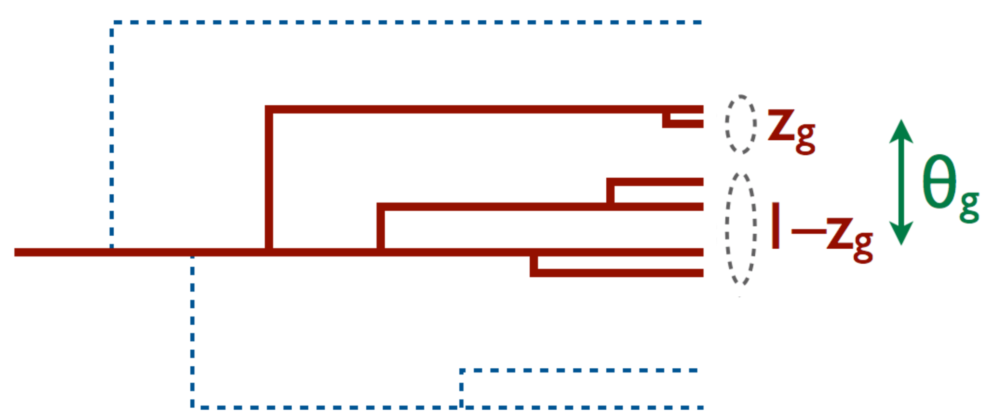
$$z > z_{\mathrm{cut}}\, \theta^{\beta}$$

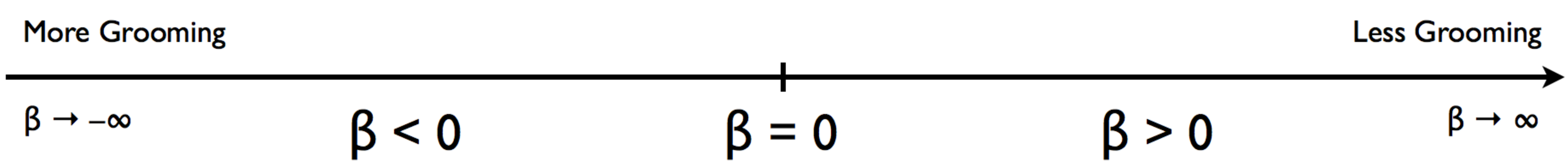
two grooming parameters

Groomed Jet



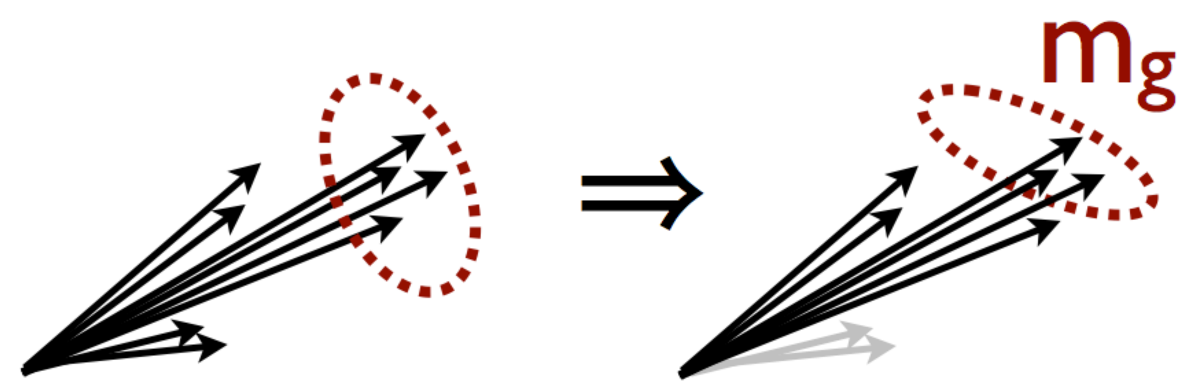
Groomed Clustering Tree



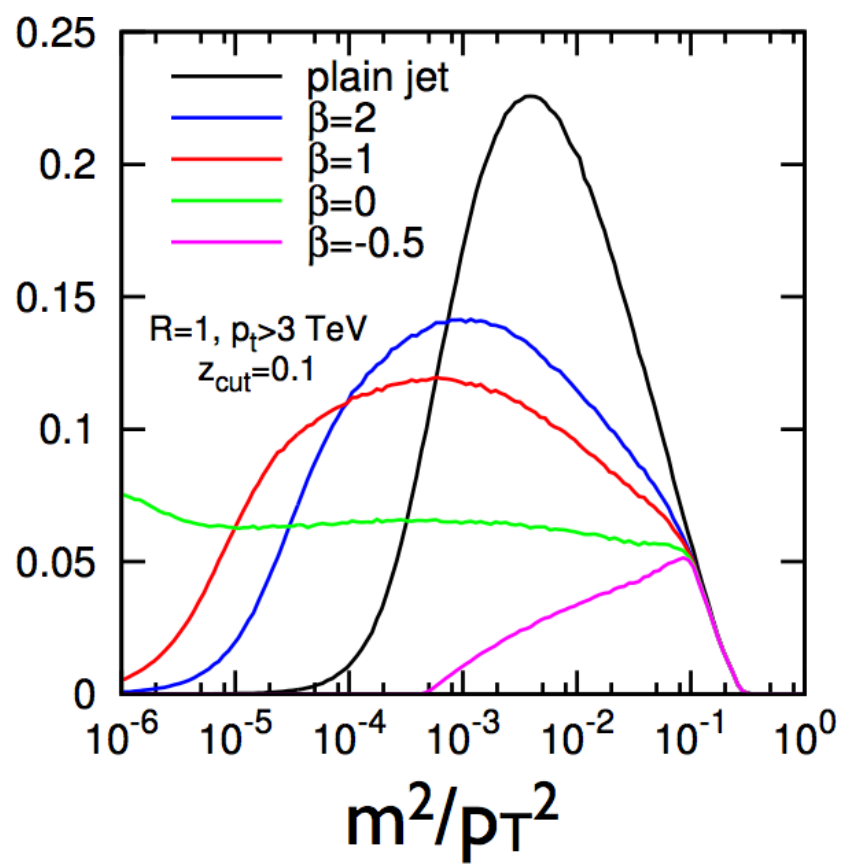


Calculating Mass?

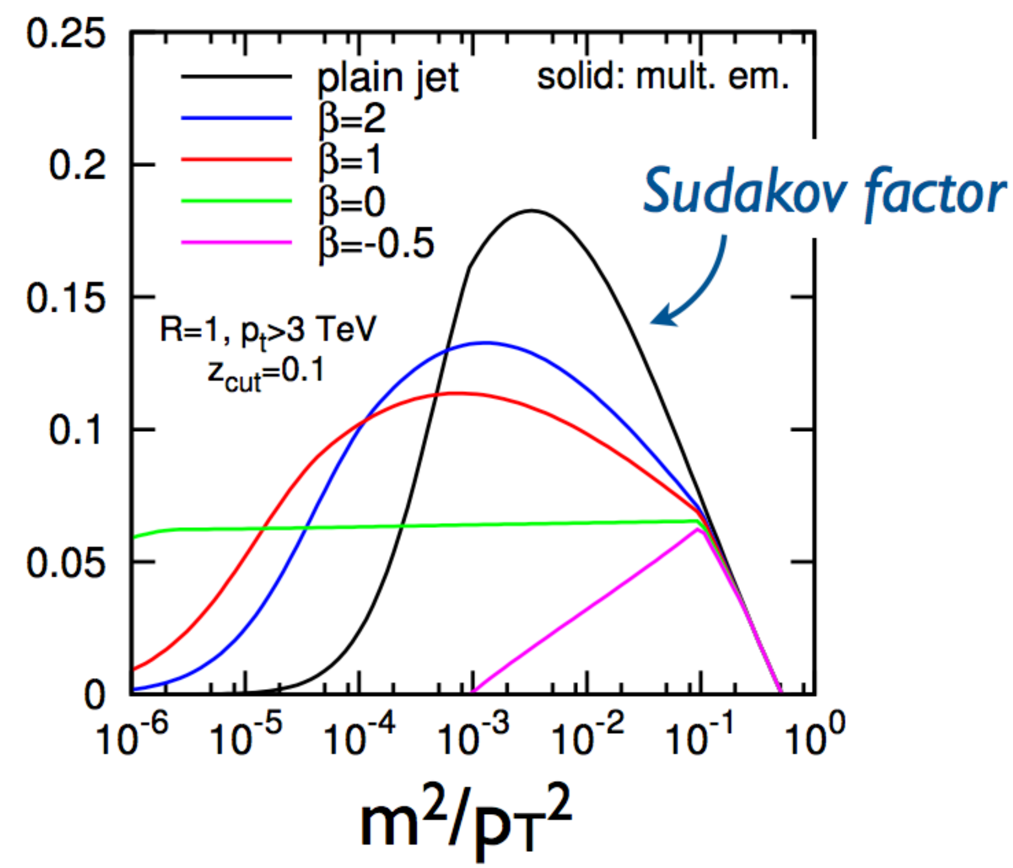
Larkoski, Marzani, Soyez, Thaler 2014

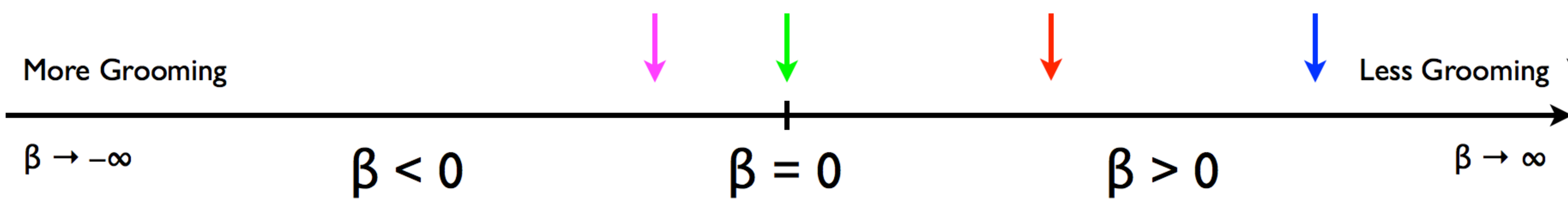


Pythia 8, partonic



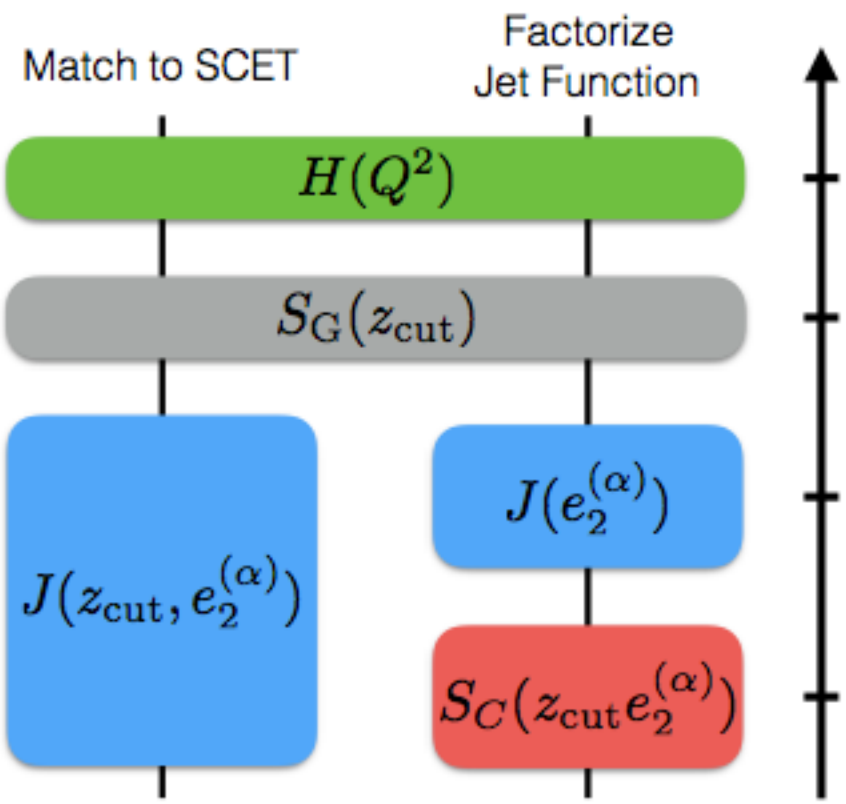
Pert. QCD at $\simeq$NLL

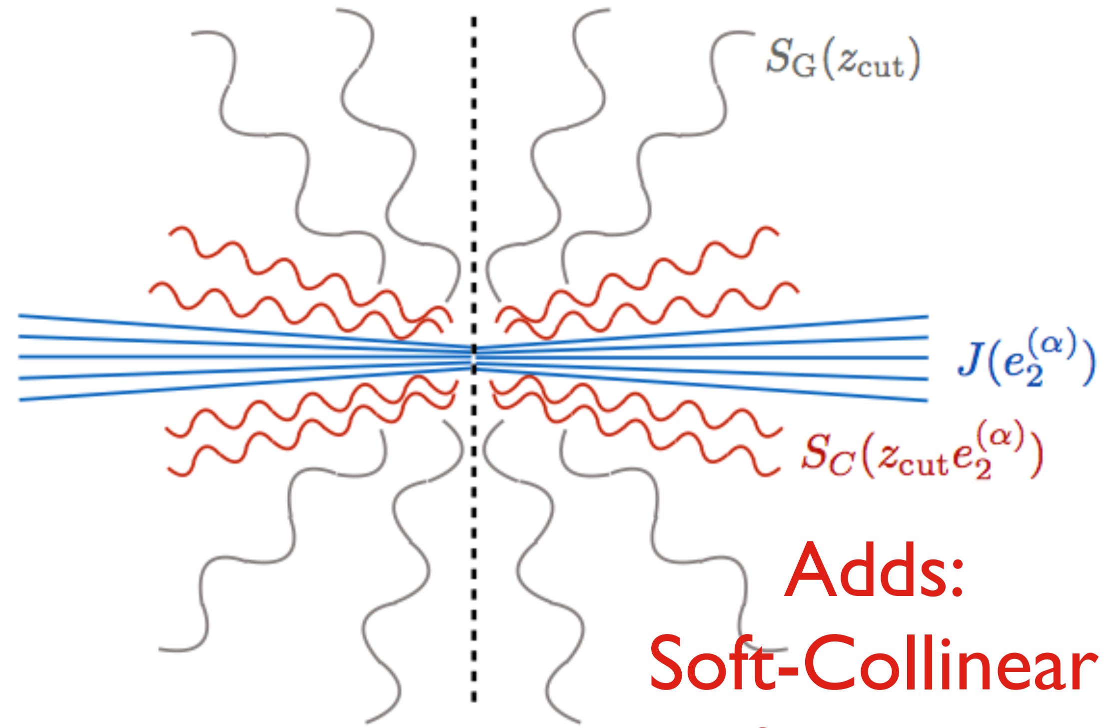




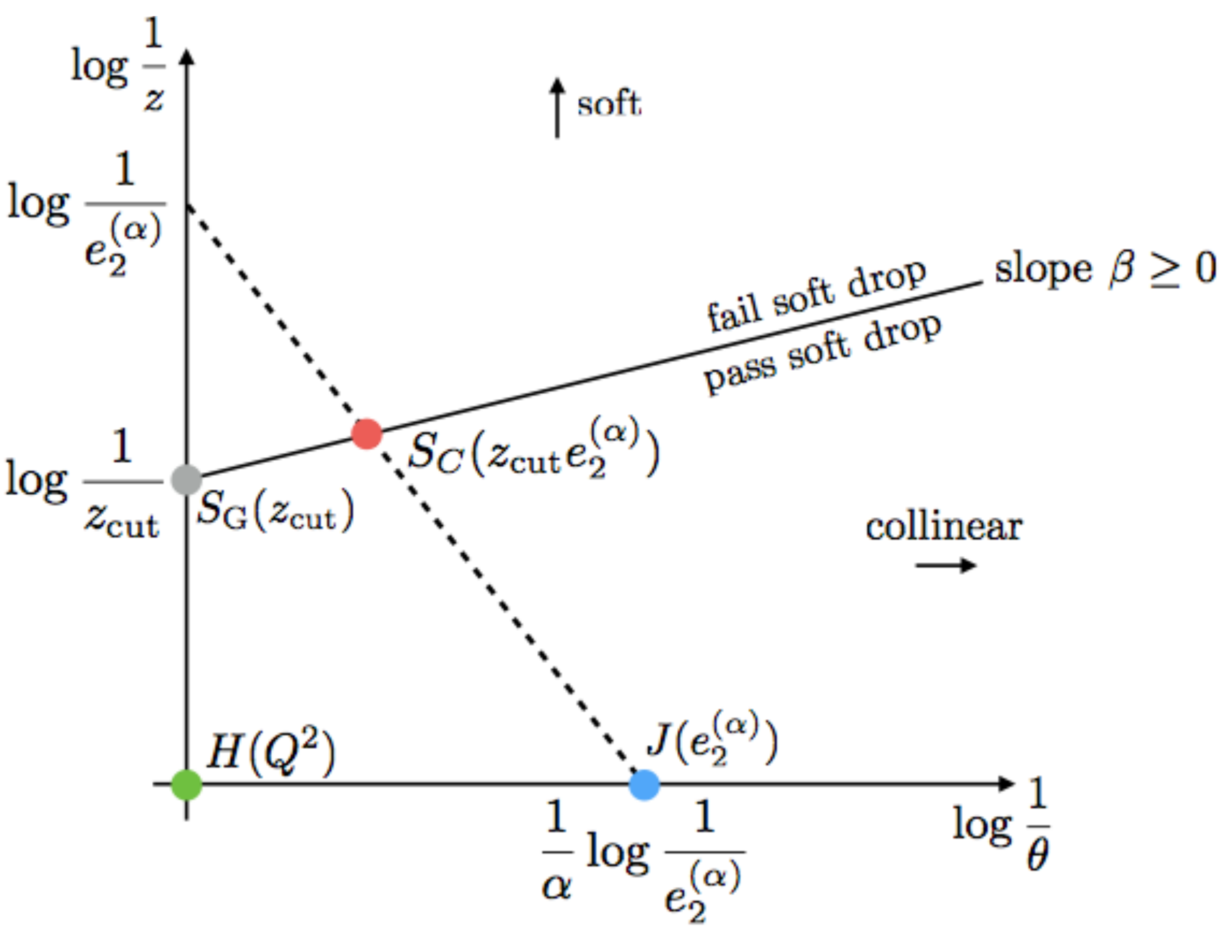
Soft Drop Factorization

Frye, Larkoski, Schwartz, Yan 2016





Adds:
Soft-Collinear
function S_C



$$\frac{d\sigma}{de_2 \ldots} = H(Q^2) S_G(z_{cut}, \beta) \times \left[S_C(e_2, z_{cut}, \beta) \otimes J(e_2) \right]$$

isolates measurement
achieve NNLL precision

Extracting a Short Distance Top Mass at the LHC

$$\text{CMS:} \quad m_t^{\rm MC} = 172.44 \pm 0.49 \qquad \text{ATLAS:} \quad m_t^{\rm MC} = 172.84 \pm 0.70$$

To improve on the current experimental measurements:

- must use a kinematically sensitive LHC observable
- theoretically tractable (factorization at Hadron level), to obtain a measurement in a precise mass scheme
- control contamination (ISR, Underlying Event, …)

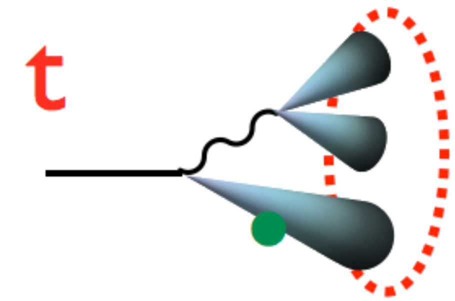
or calibrate the $m_t^{\rm MC}$ parameter in Monte Carlo with Hadron level theory predictions (not discussed today)

Butenschoen, Dehnadi, Hoang, Mateu, Preisser, IS 2016

Top Jet Mass with Soft Drop $p_T \gg m_T \gg \Gamma_t > \Lambda_{\mathrm{QCD}}$

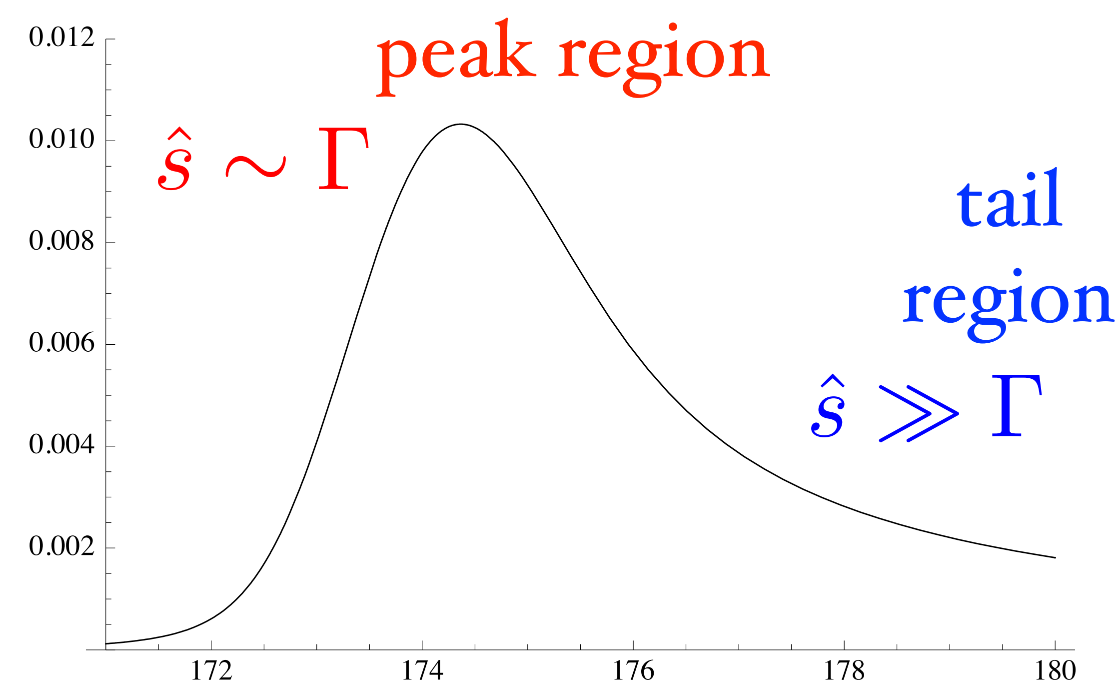
A. Hoang, S. Mantry, **A. Pathak**, IS (to appear)

- Boosted Tops $p_T \gg m_t$ retain top decay products



- Fat Jets $R \gg \dfrac{m_t}{p_T}$
- Sensitivity $\hat{s} \sim \Gamma_t$ for measurement of jet-mass m_J

$$\hat{s} = \frac{m_J^2 - m_t^2}{m_t}$$



- Grooming z_{cut}, β
- Jet Veto $\mathcal{T}^{\mathrm{cut}}$

(Perturbative and Nonperturbative effects give $\Gamma > \Gamma_t$)

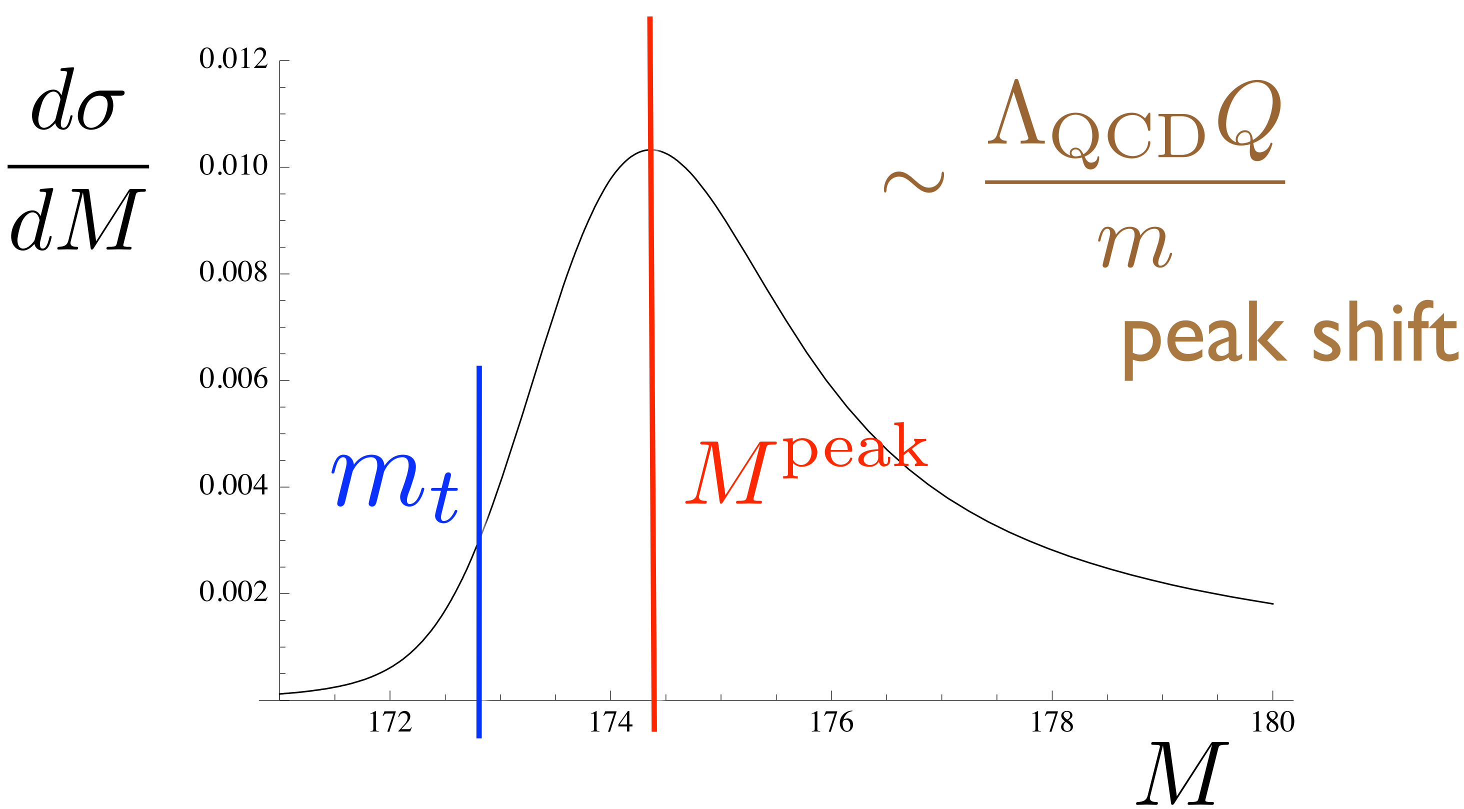
Without Soft Drop: $e^+e^- \to t\bar{t}$

QCD → SCET → HQET

- Factorization Thm. derived with hemisphere masses. Fleming, Hoang, Mantry, IS 2007

$$\frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2} = H_Q(Q) H_m(m, Q/m) \int d\ell d\ell' dk dk'$$
$$\times J_t\left(\hat{s}_t - \frac{Q\ell}{m}, \Gamma_t, \delta m\right) J_t\left(\hat{s}_{\bar{t}} - \frac{Q\ell'}{m}, \Gamma_t, \delta m\right)$$
$$\times S(\ell - k, \ell' - k') F(k, k')$$

control over mass scheme



Without Soft Drop: $pp \to t\bar{t}$

QCD → SCET → HQET

- Can be extended to pp (using N-jettiness)

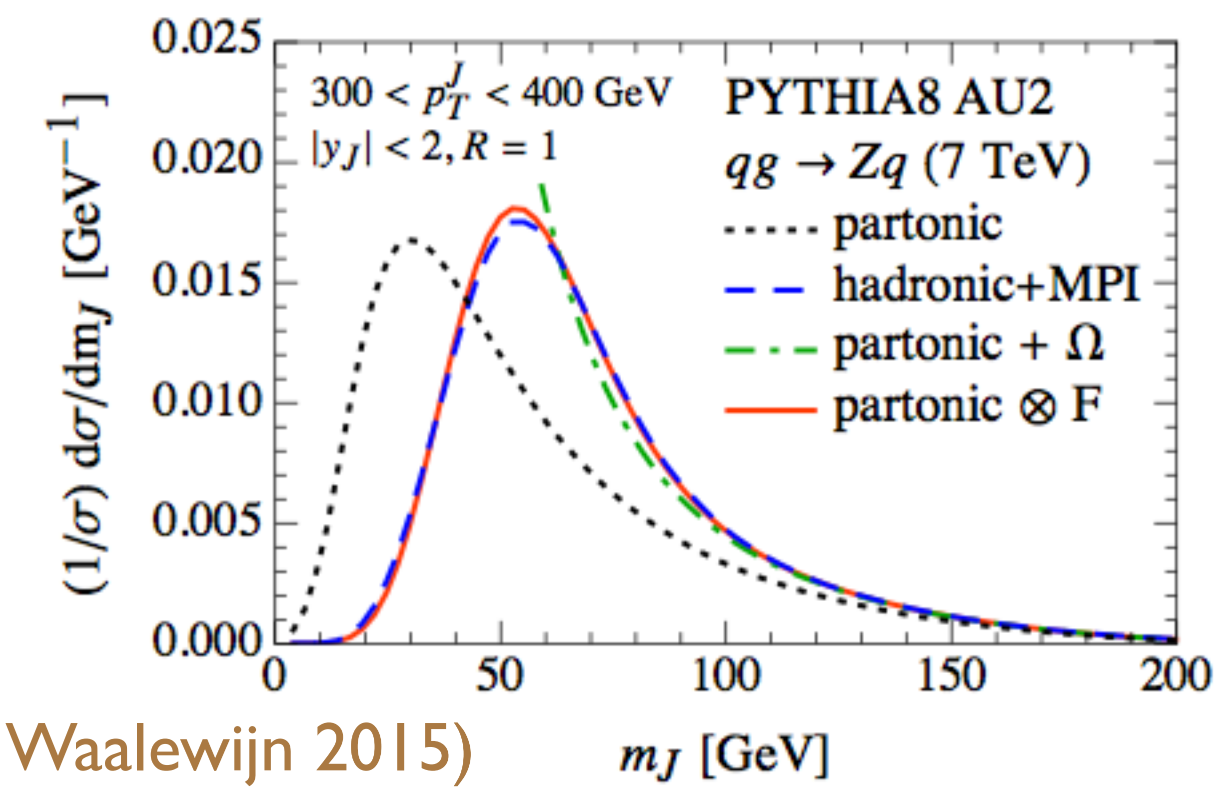
A. Hoang, S. Mantry, A. Pathak, IS

$$\frac{d^2\sigma}{dM_{J1}^2 dM_{J2}^2 d\mathcal{T}^{\rm cut}} = \mathrm{tr}\big[\hat{H}_{Qm}\hat{S}(\mathcal{T}^{\rm cut}, R, \ldots)\otimes F\big]\otimes J_t \otimes J_t \otimes \mathcal{I}\mathcal{I} \otimes f f$$

same jet functions! includes PDFs, multiple channels, color correlations, Jet Radius R, Jet veto, ISR, hadronization

- BUT control of underlying event is model dependent (a factorization violating effect).

Simple one parameter function does give a reasonable model



(IS, Tackmann, Waalewijn 2015)

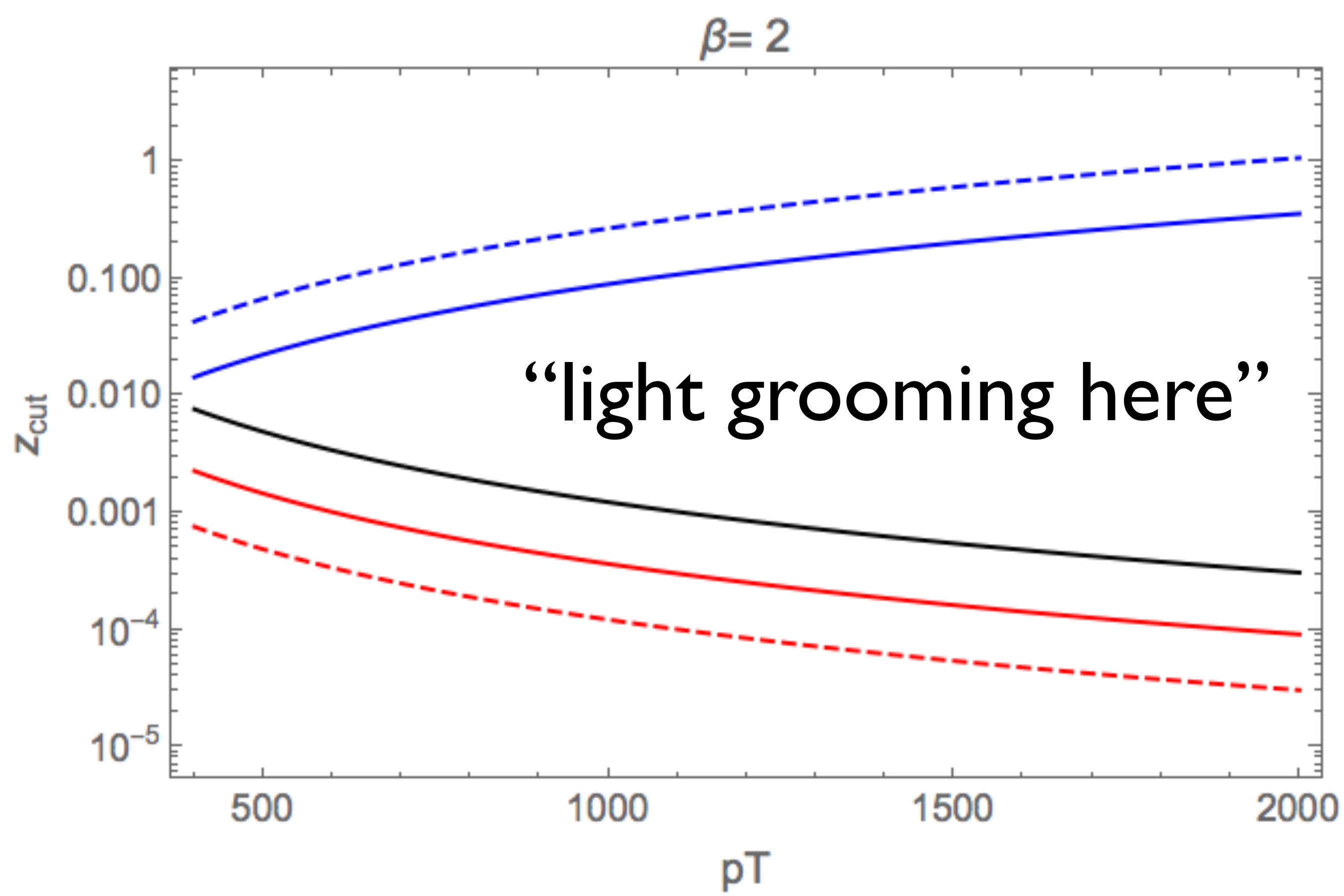
With Soft Drop on one (or both) jets:

Restricted range, can only apply a "light soft drop" for tops:

$$\frac{\Gamma_t}{m}\left(\frac{Q}{2m}\right)^{\beta} \gg z_{\rm cut} \gg \frac{2m\Gamma_t}{Q^2}$$

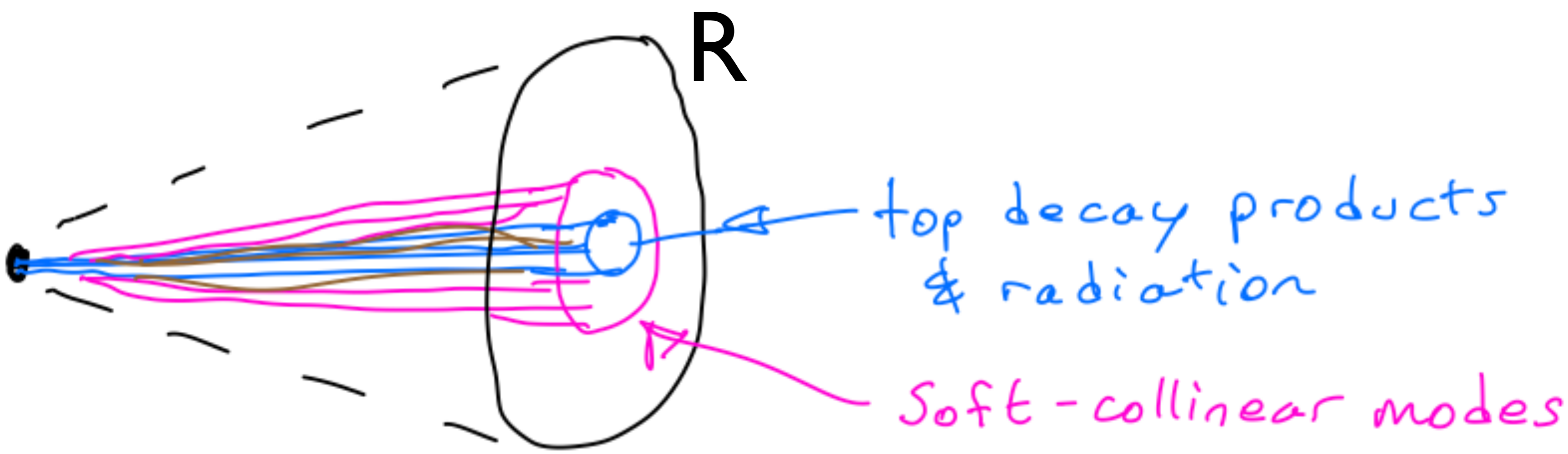
Ensure soft drop does not touch J_t

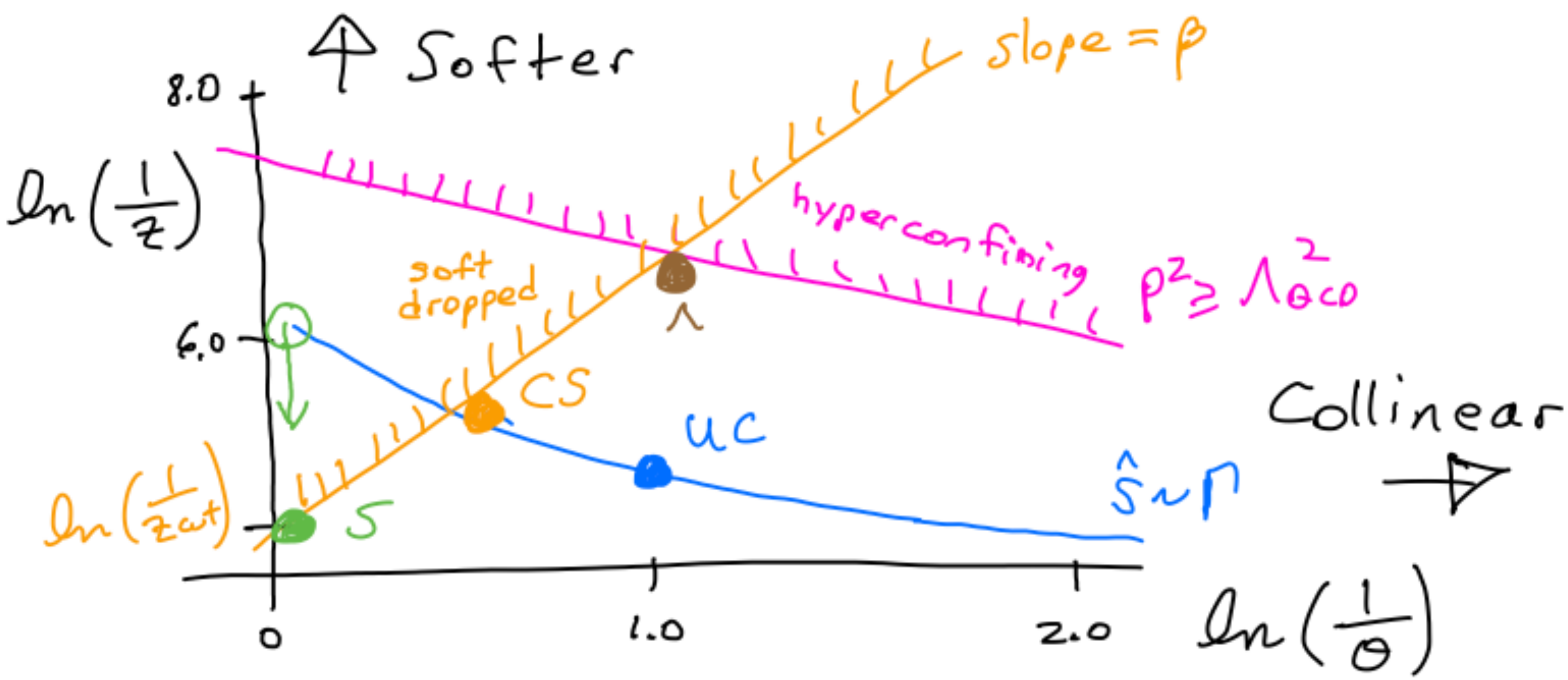
Ensure soft drop removes global soft radiation from measurement



$$\frac{d^2\sigma}{dM_J^2 d\mathcal{T}^{\rm cut}} = {\rm tr}\big[\hat{H}_{Qm}\hat{S}(\mathcal{T}^{\rm cut}, Qz_{\rm cut}, \beta, \ldots)\otimes F\big]\otimes J_t \otimes \mathcal{I}\mathcal{I}\otimes f f$$

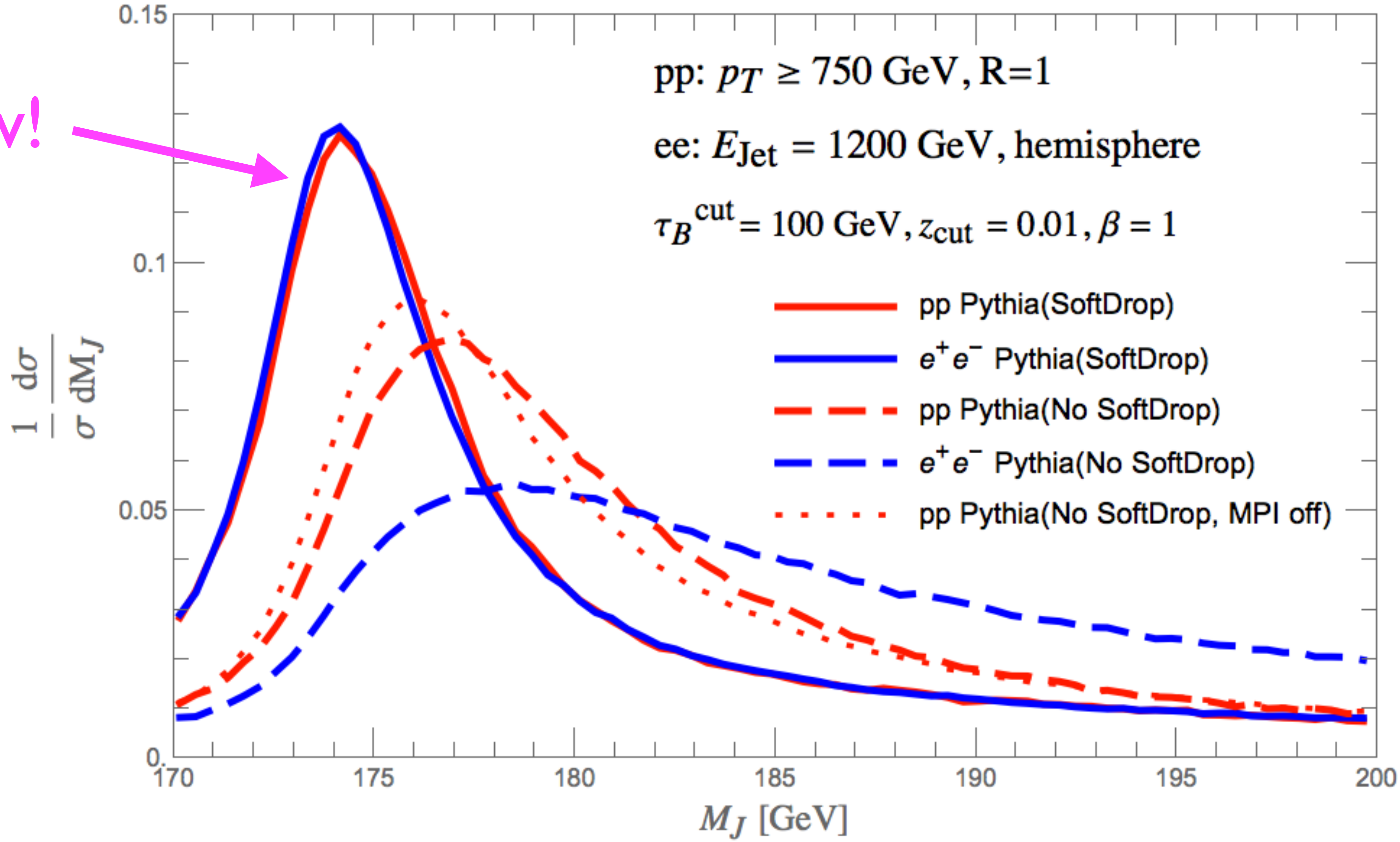
$$\times\left\{\int d\ell J_t\left(\hat{s}_t - \frac{Q\ell}{m}, \Gamma_t, \delta m\right)\otimes S_C\left[\ell - \left(\frac{k^{2+\beta}}{2^\beta Q z_{\rm cut}}\right)^{\frac{1}{1+\beta}}, Qz_{\rm cut}, \beta\right] F(k)\right\}$$


R
top decay products
& radiation
soft-collinear modes

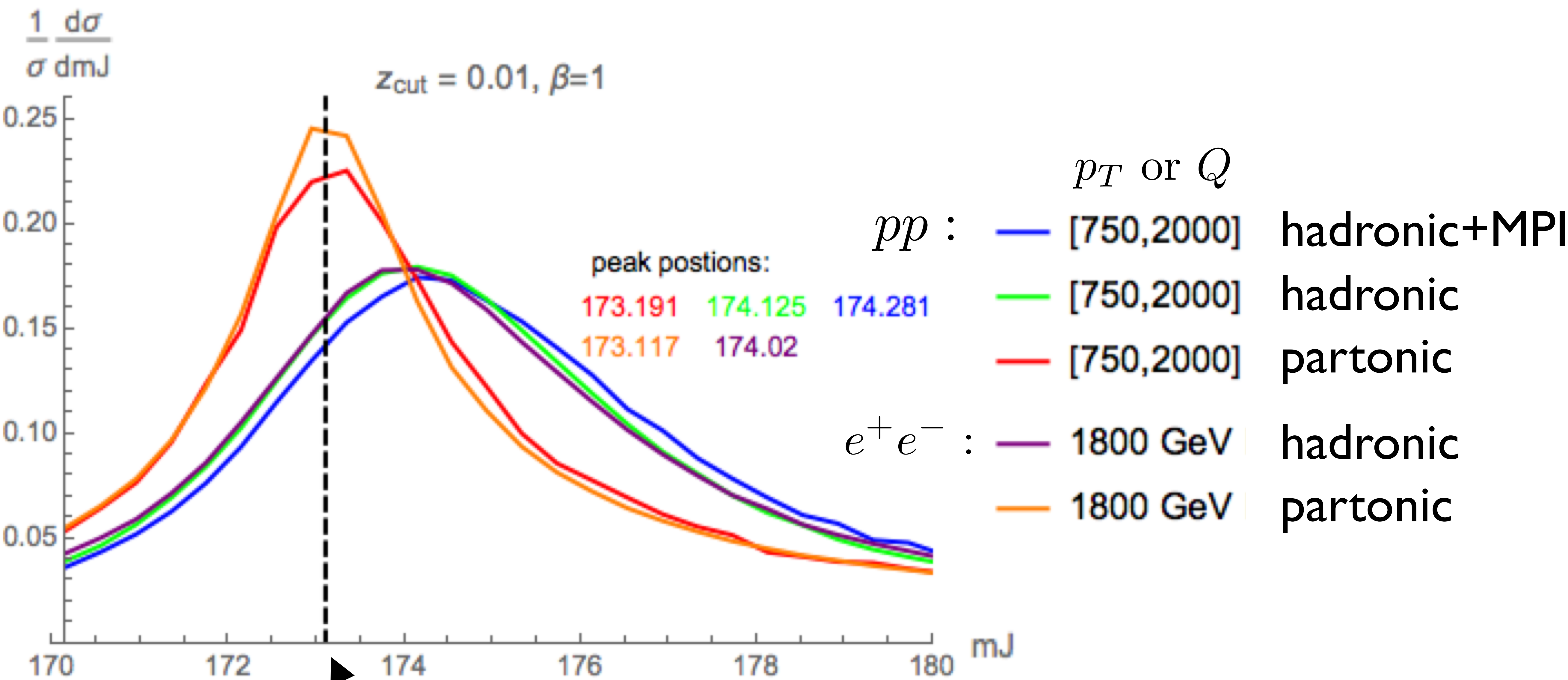

Softer
8.0
ln(1/z)
6.0
ln(1/zcut)
slope = β
hyperconfining
ρ² ≥ Λ²QCD
soft dropped
Λ
CS
UC
S
ŝ ~ ρ
Collinear
0
1.0
2.0
ln(1/θ)

Pythia (Hadronic e+e-) versus (Hadronic+MPI pp)

Wow!

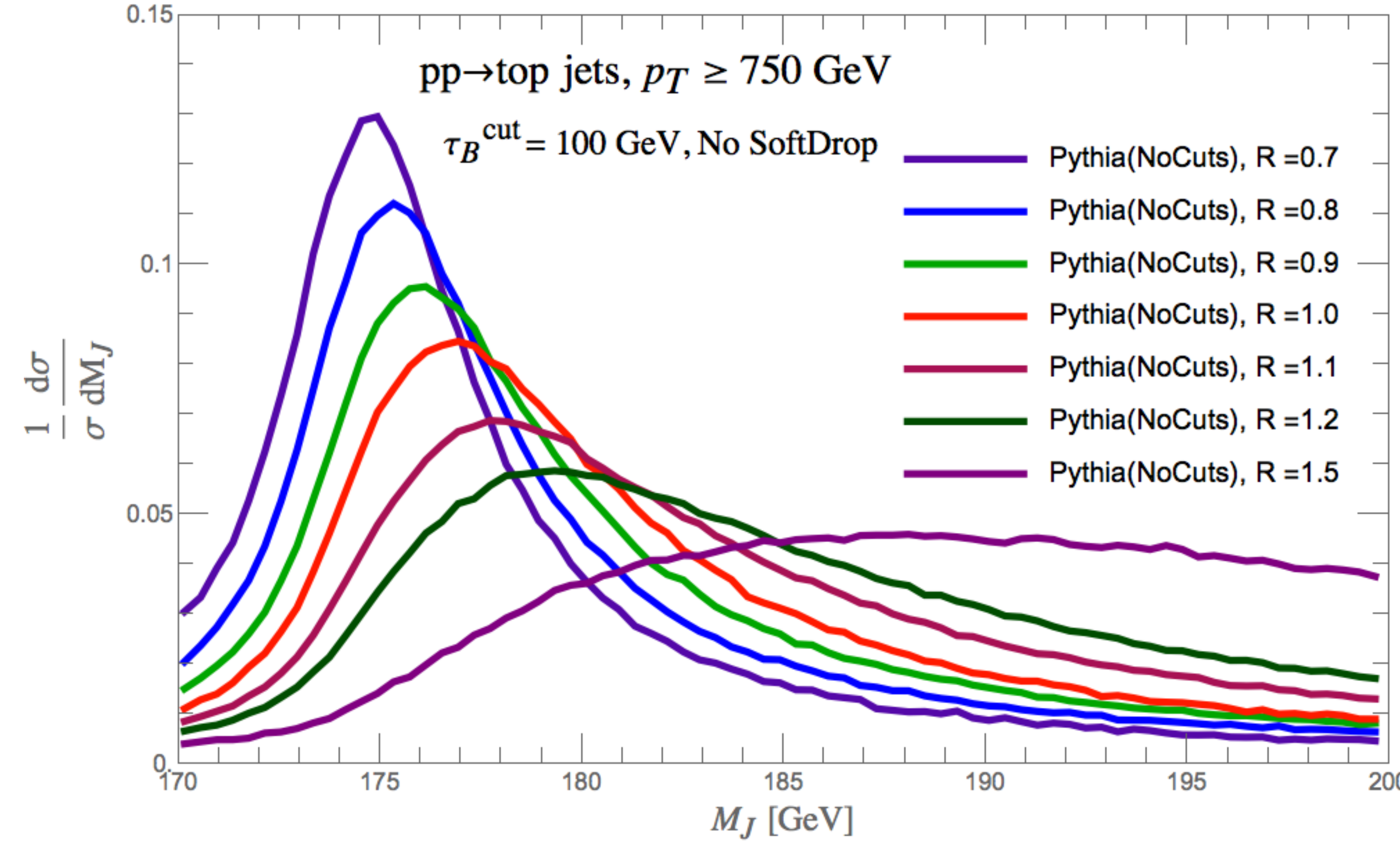


e+e- comparison with pp: MPI and Hadronization effects
(All curves with SoftDrop)

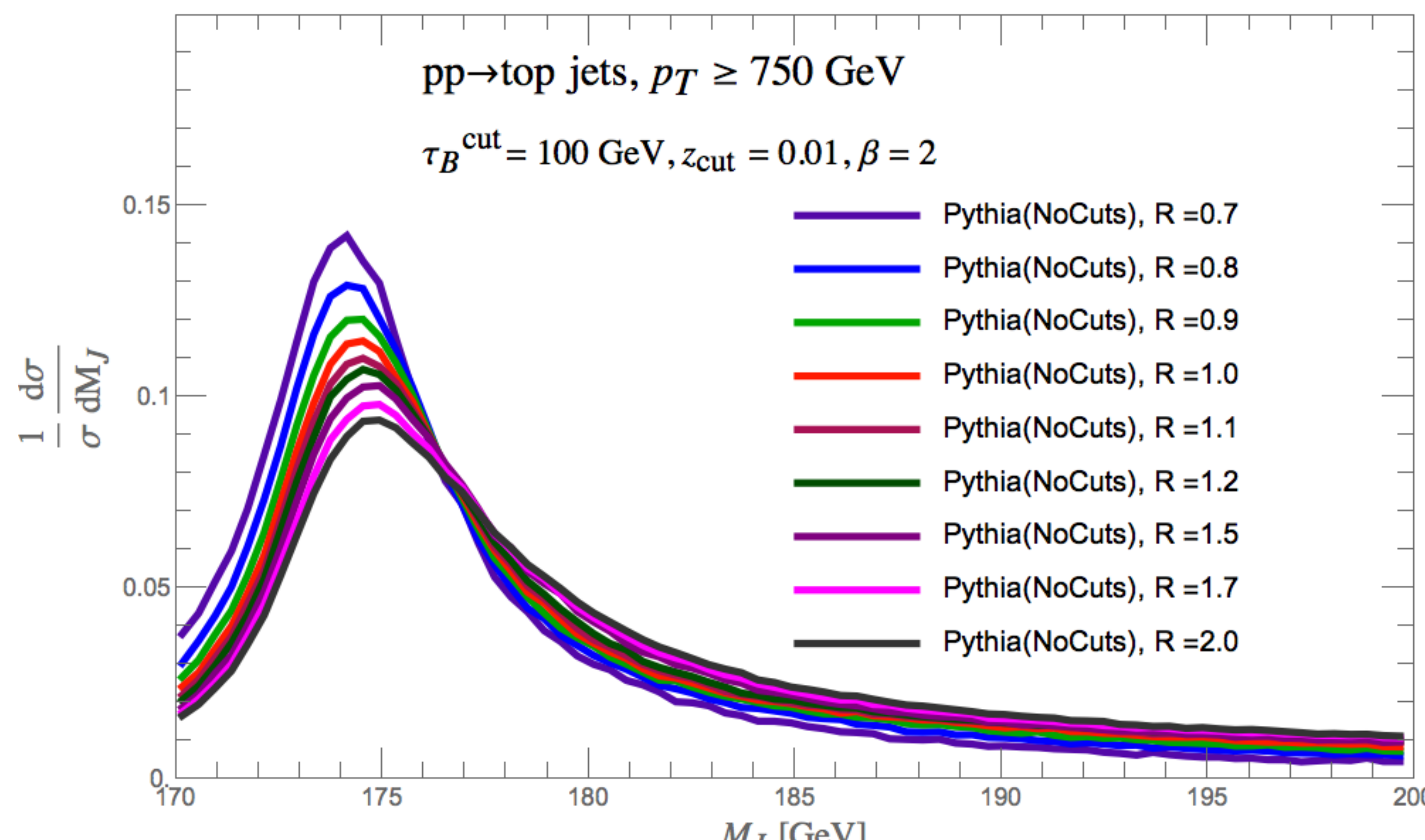


input mass in
Pythia =173.1 GeV

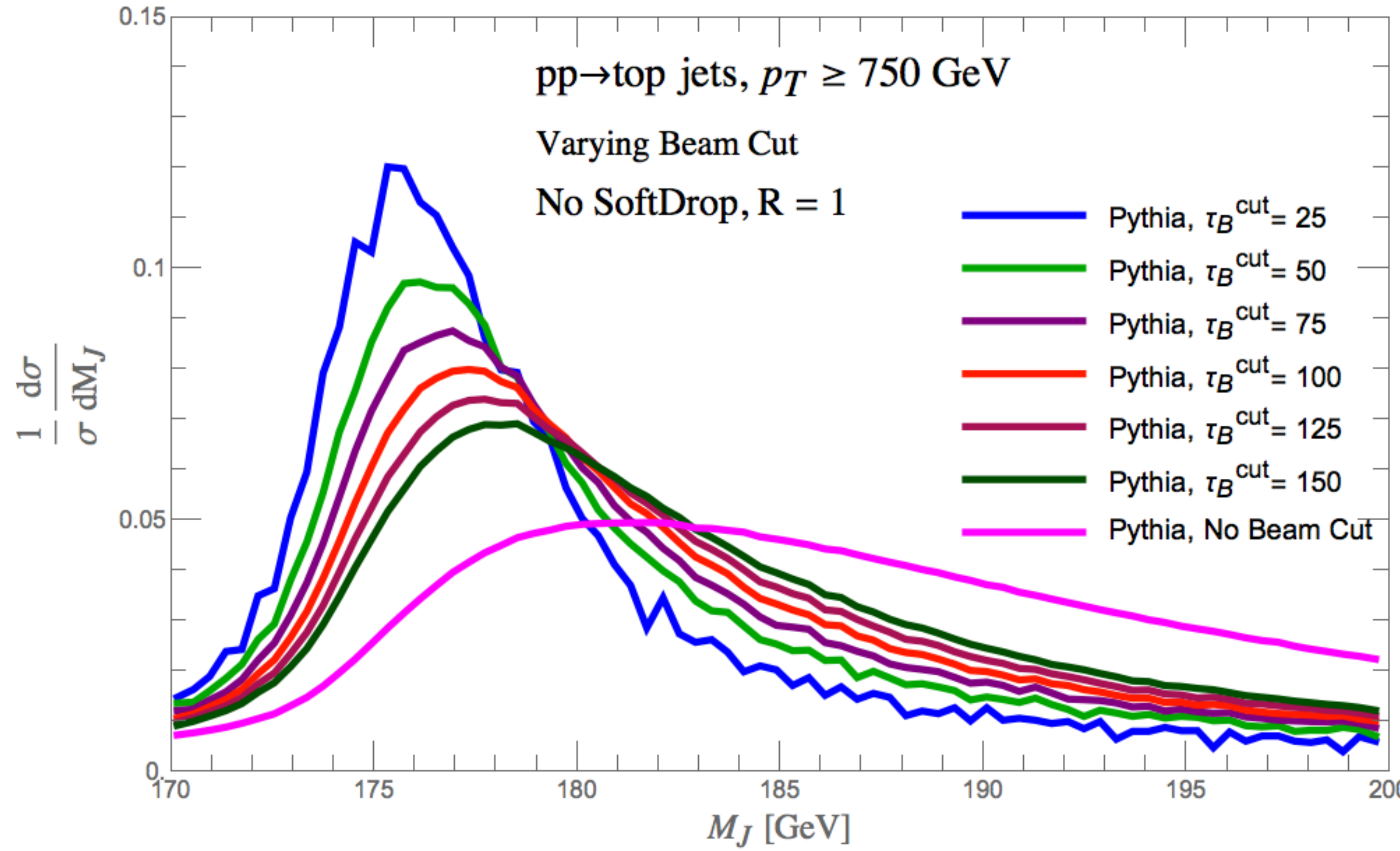
Jet Radius Dependence

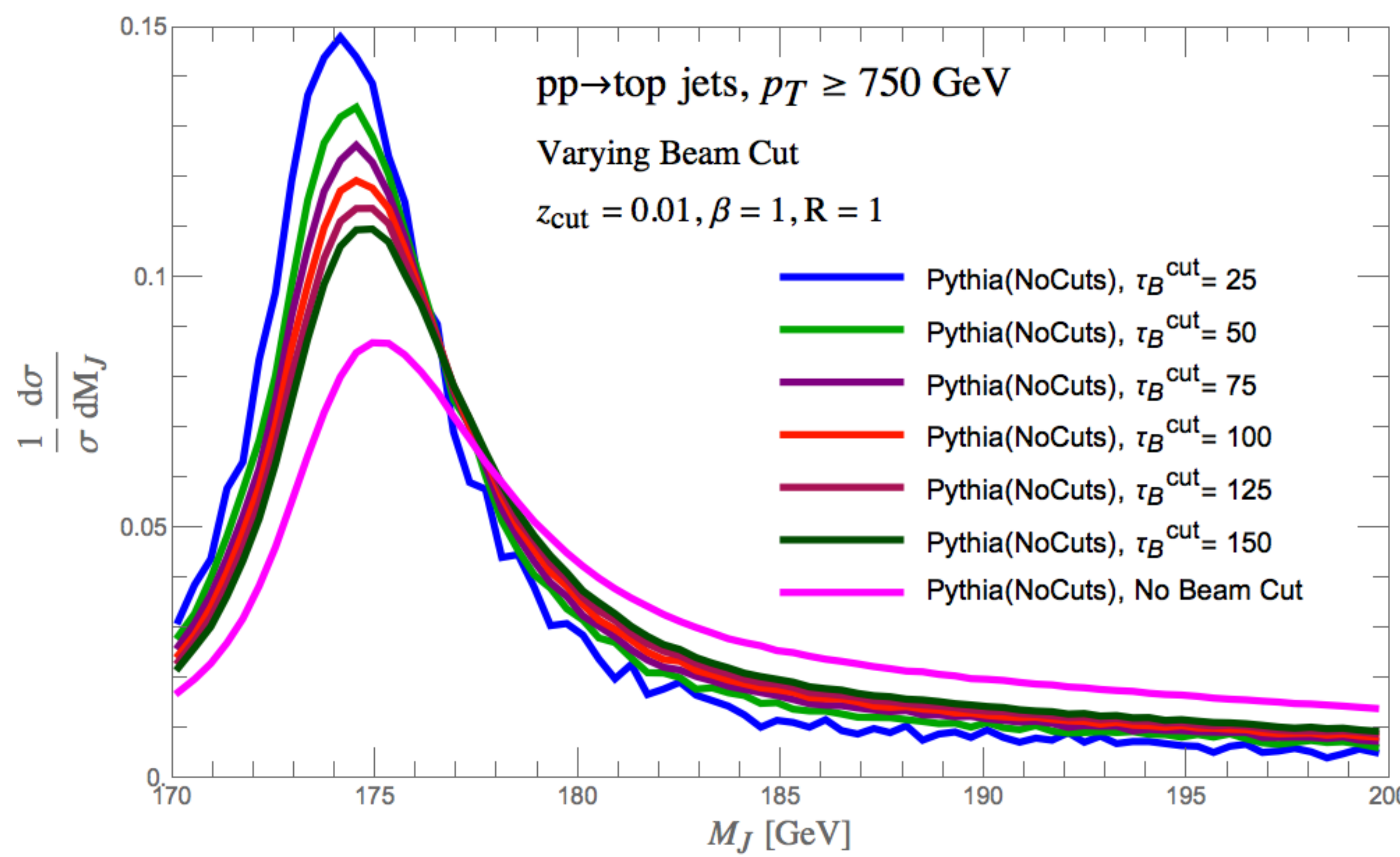


residual dependence
~ 200 MeV (this pT)



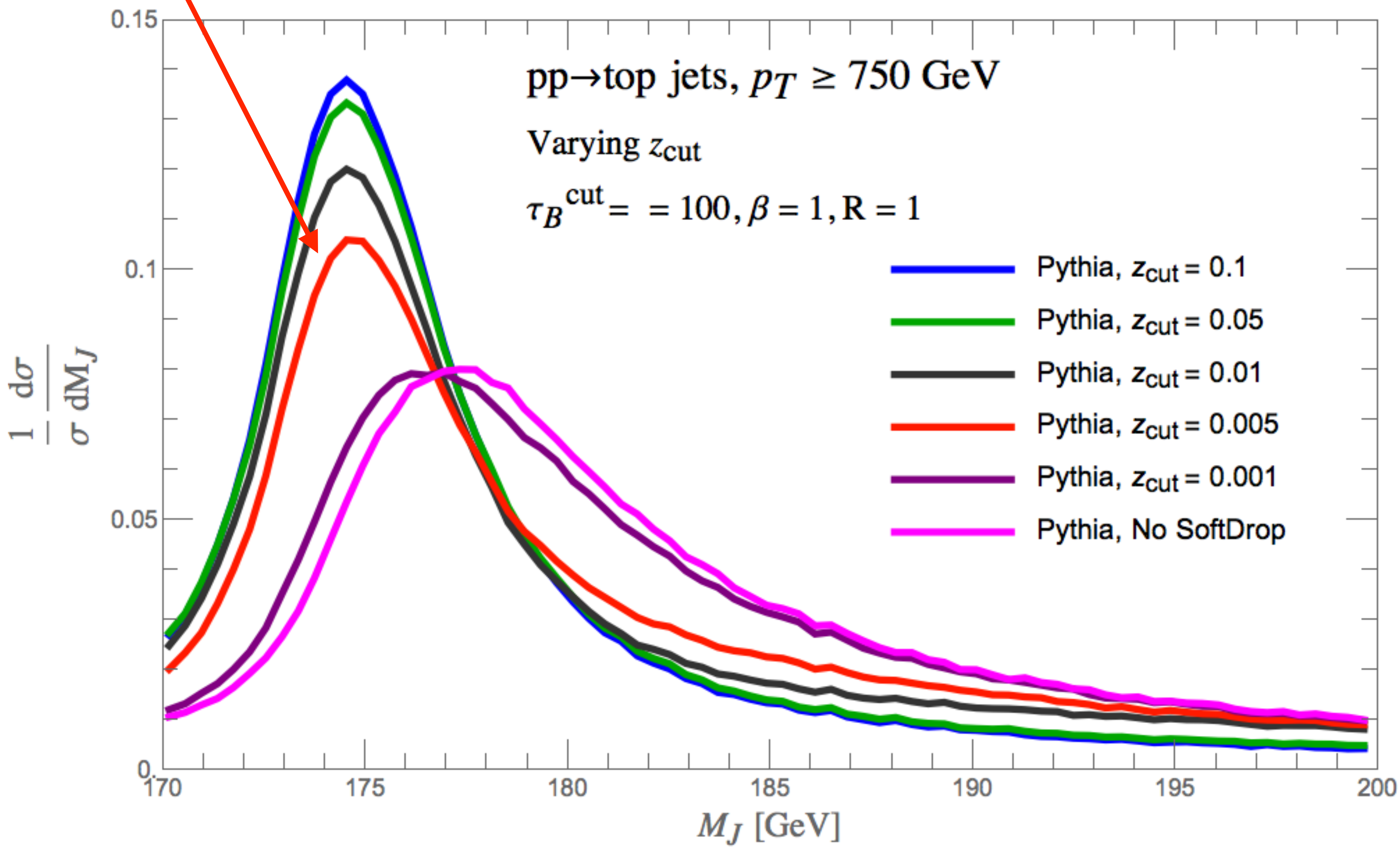
Beam Cut Dependence





z_{cut} dependence

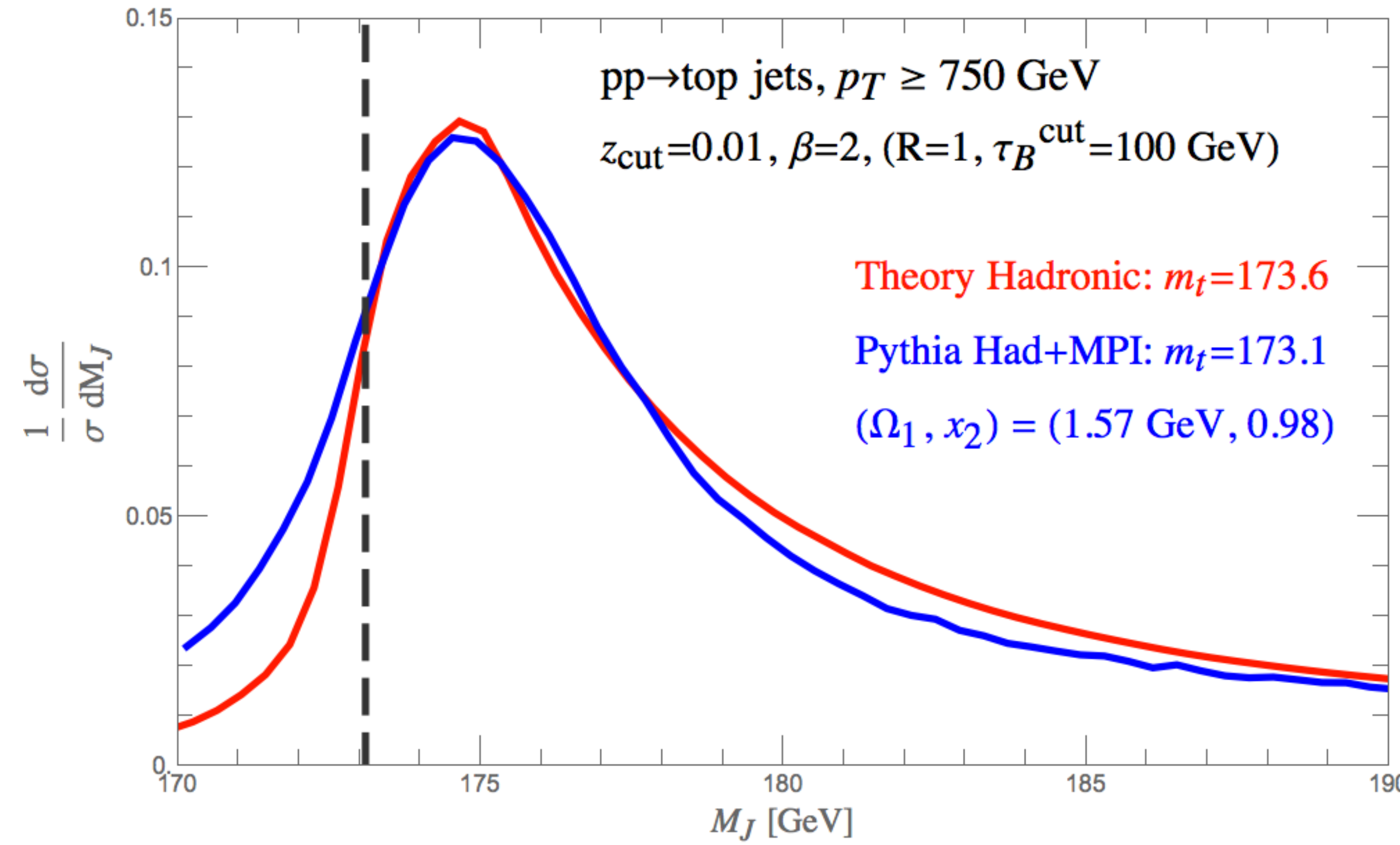
Transition for "light grooming" as predicted by factorization!



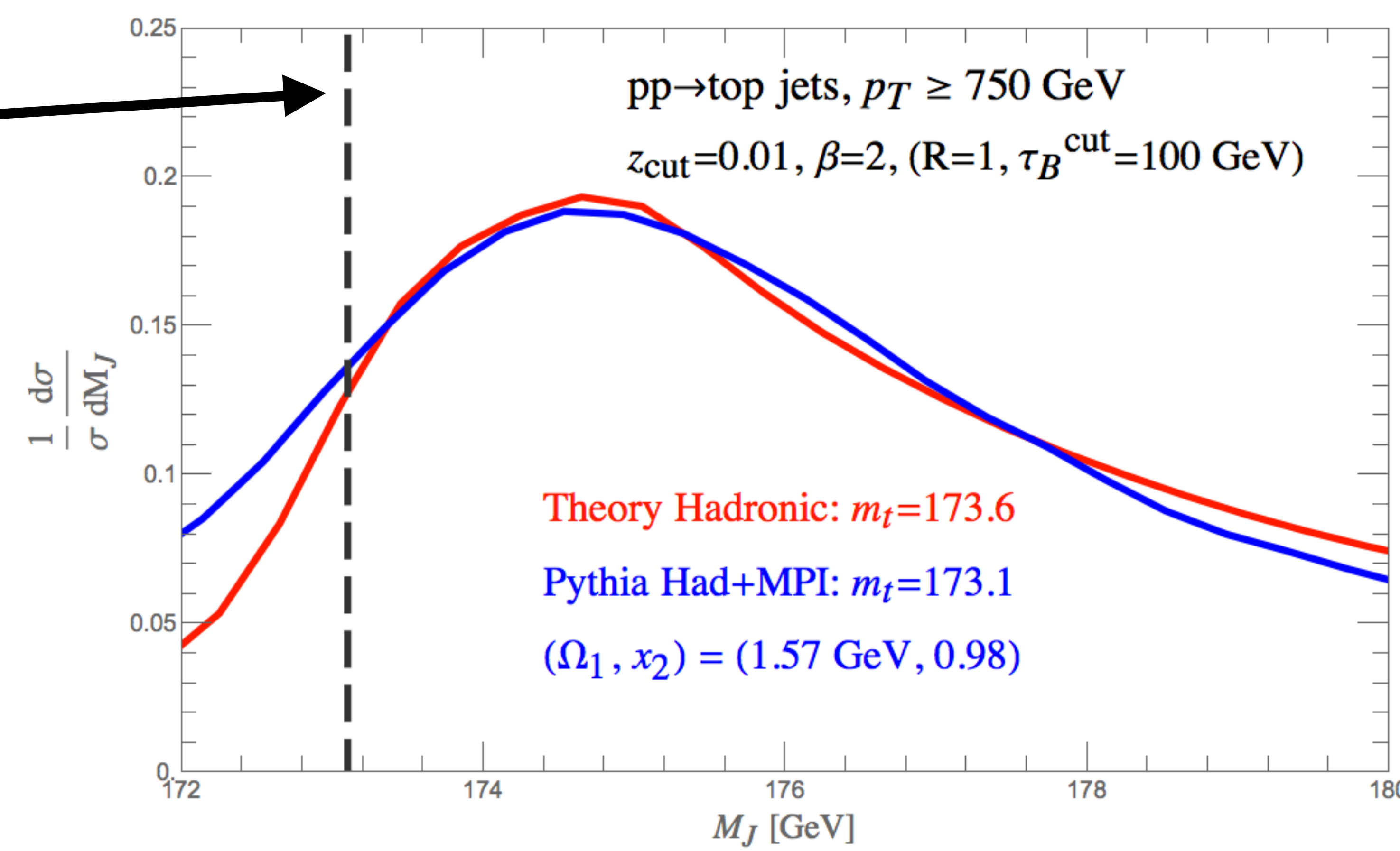
Pythia vs. Factorization

Pythia vs. Factorization with SoftDrop

include:
- MPI,
- Hadronization

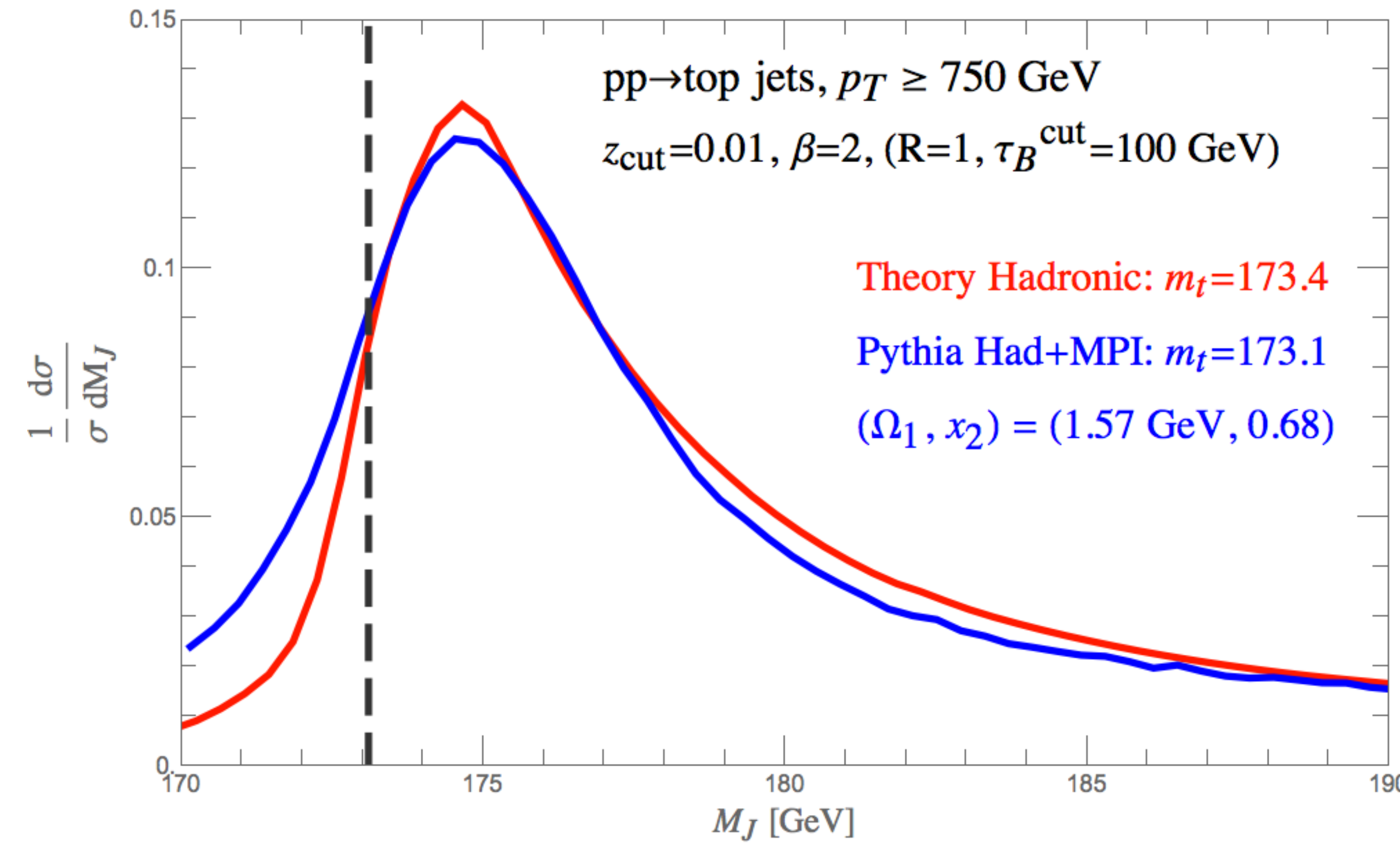


input mass in Pythia =173.1 GeV

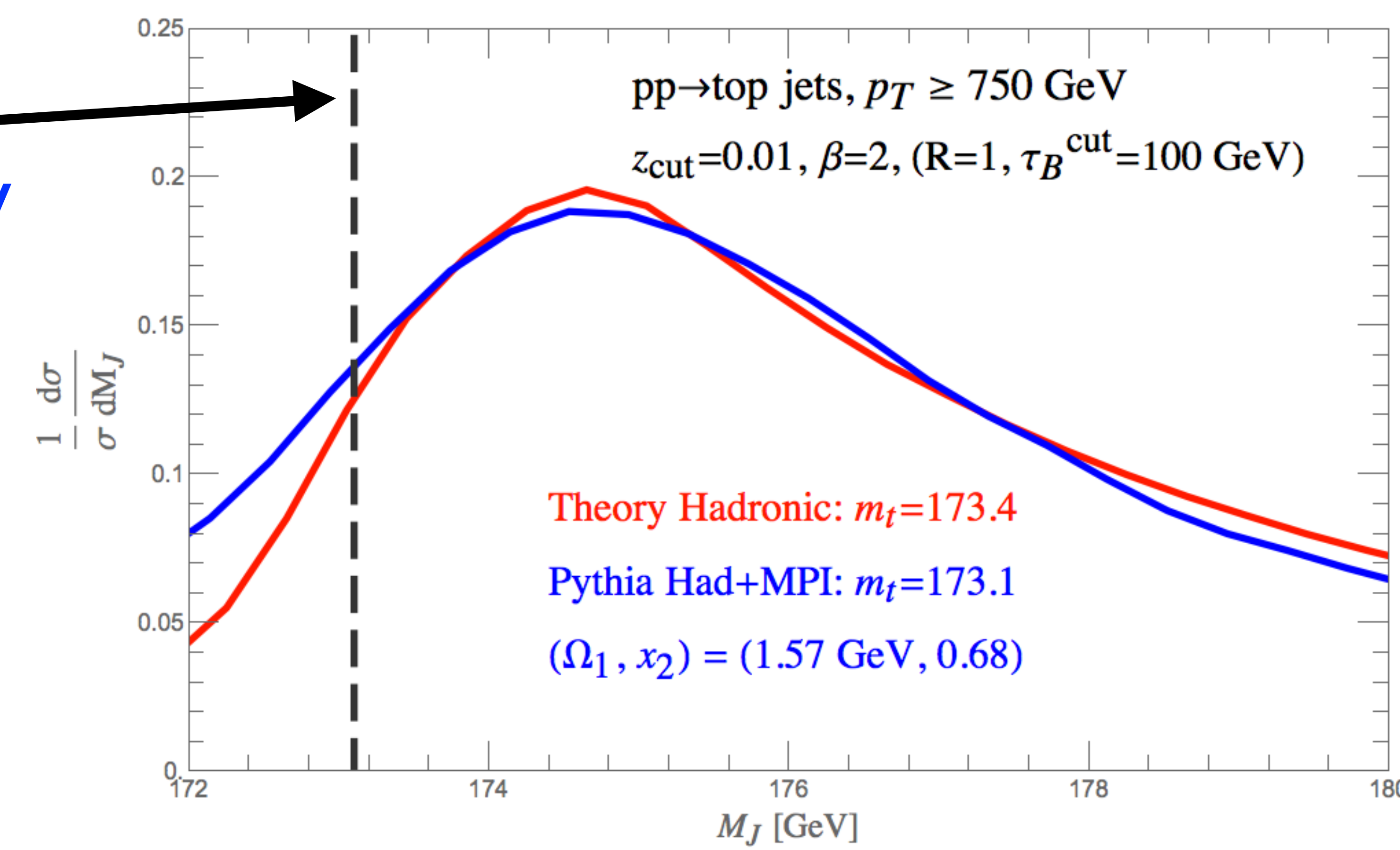


Pythia vs. Factorization with SoftDrop

include:
- MPI,
- Hadronization



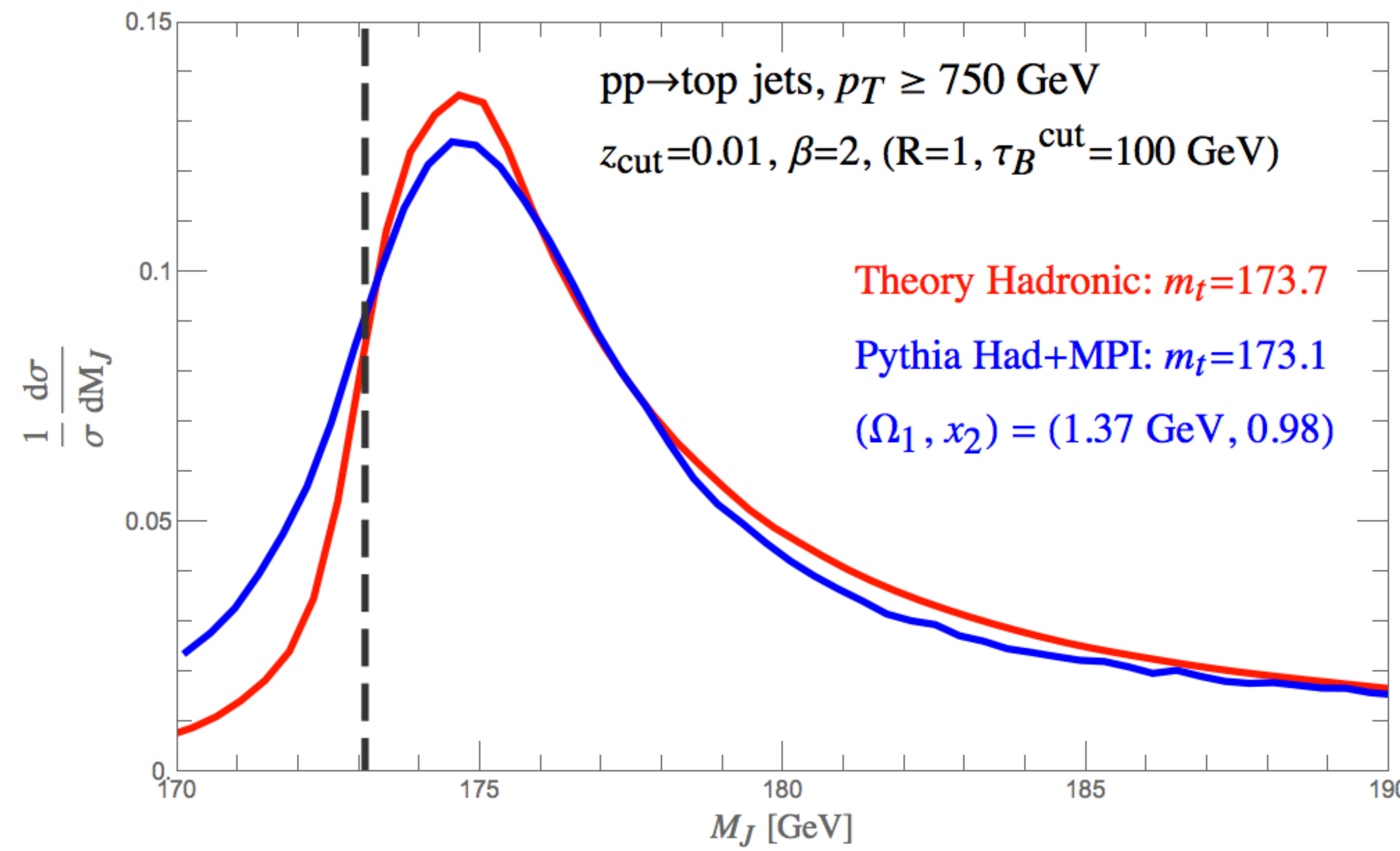
input mass in Pythia =173.1 GeV

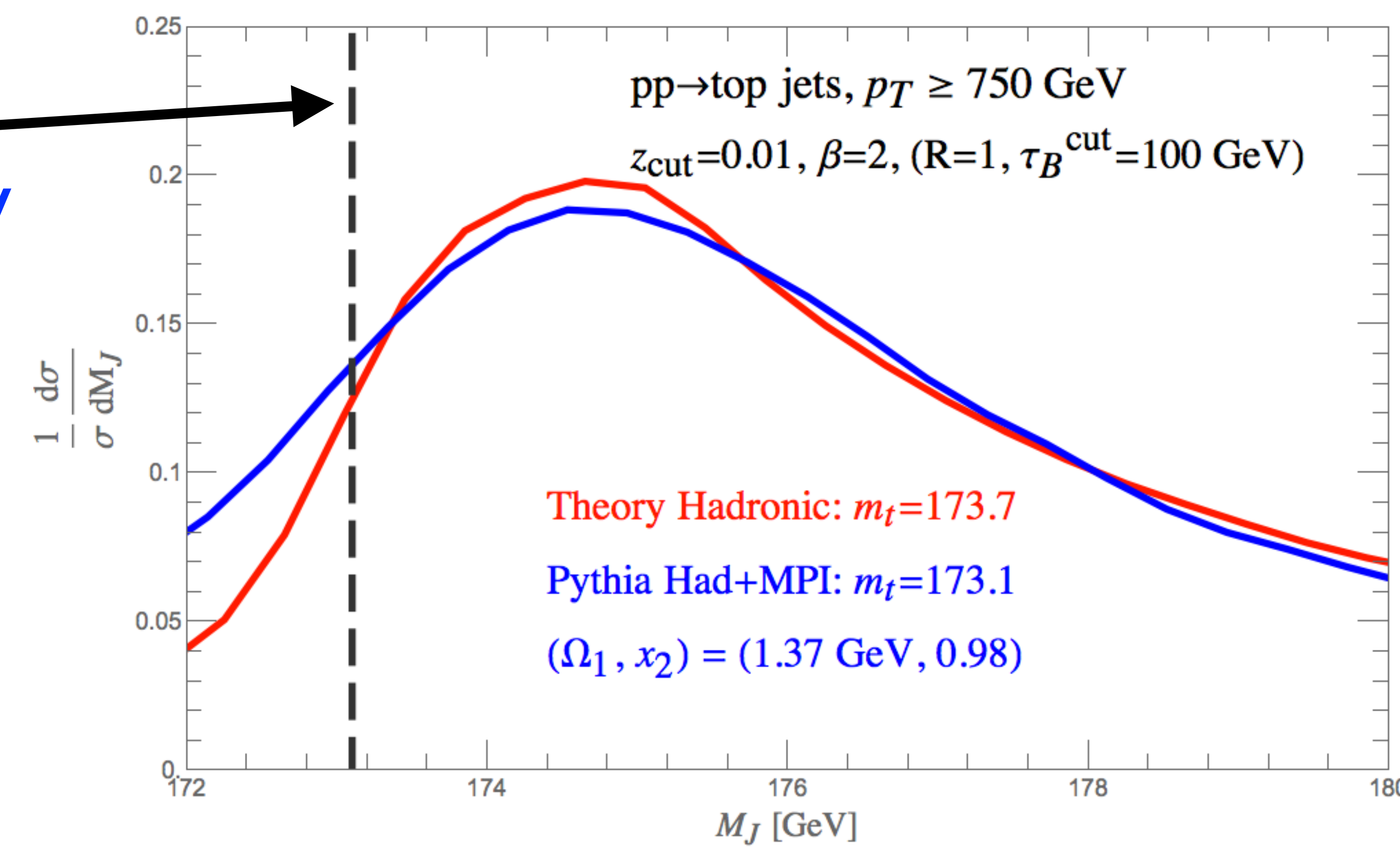


Pythia vs. Factorization with SoftDrop

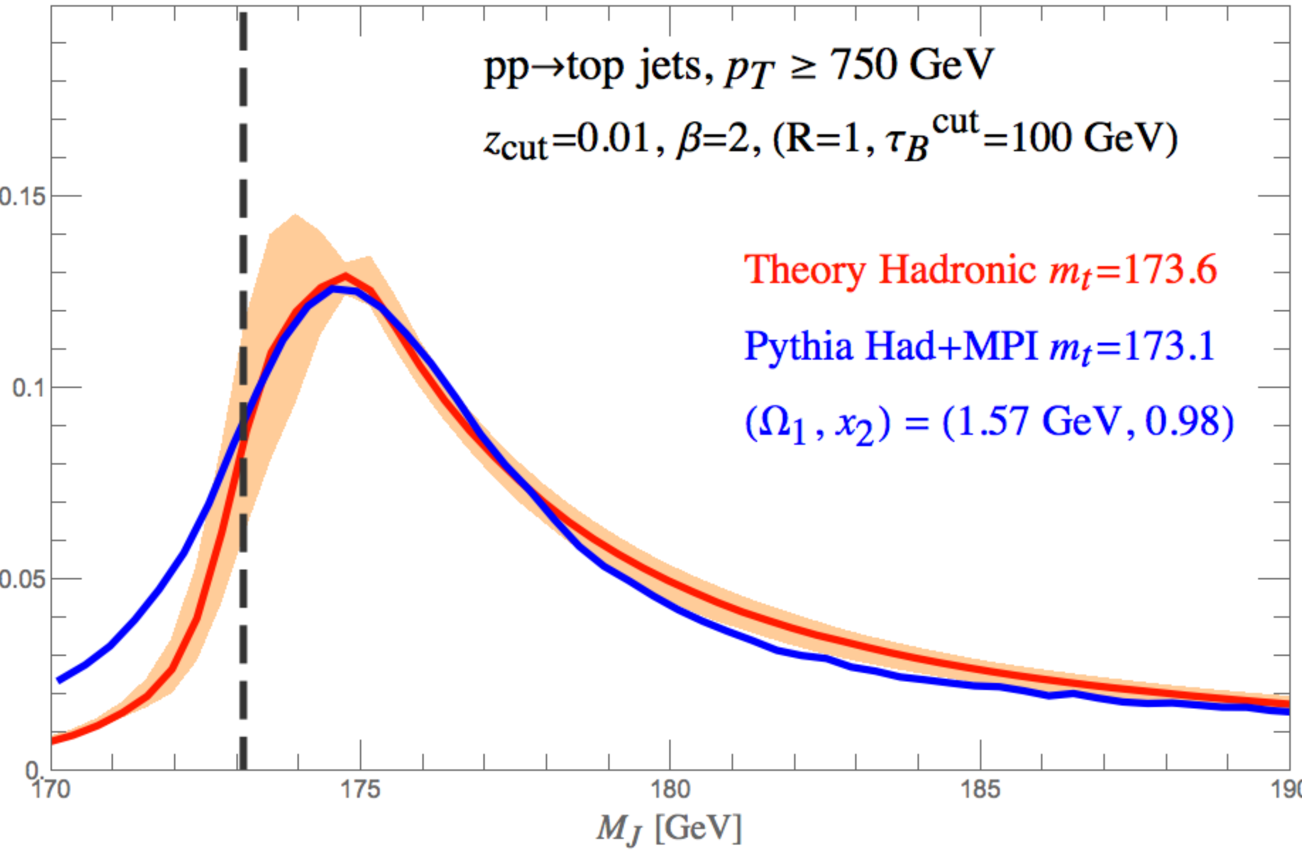
include:
- MPI,
- Hadronization

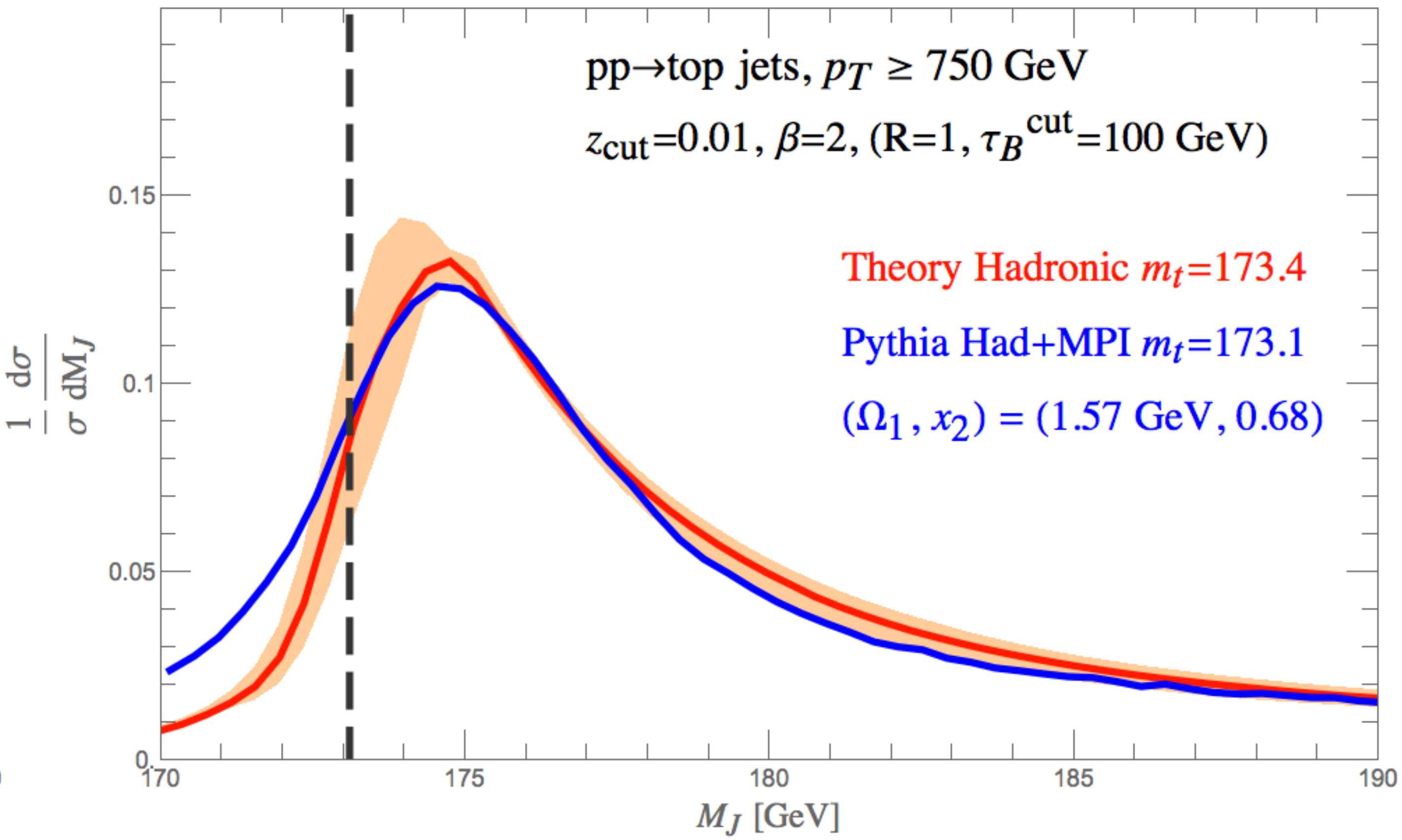
input mass in Pythia =173.1 GeV

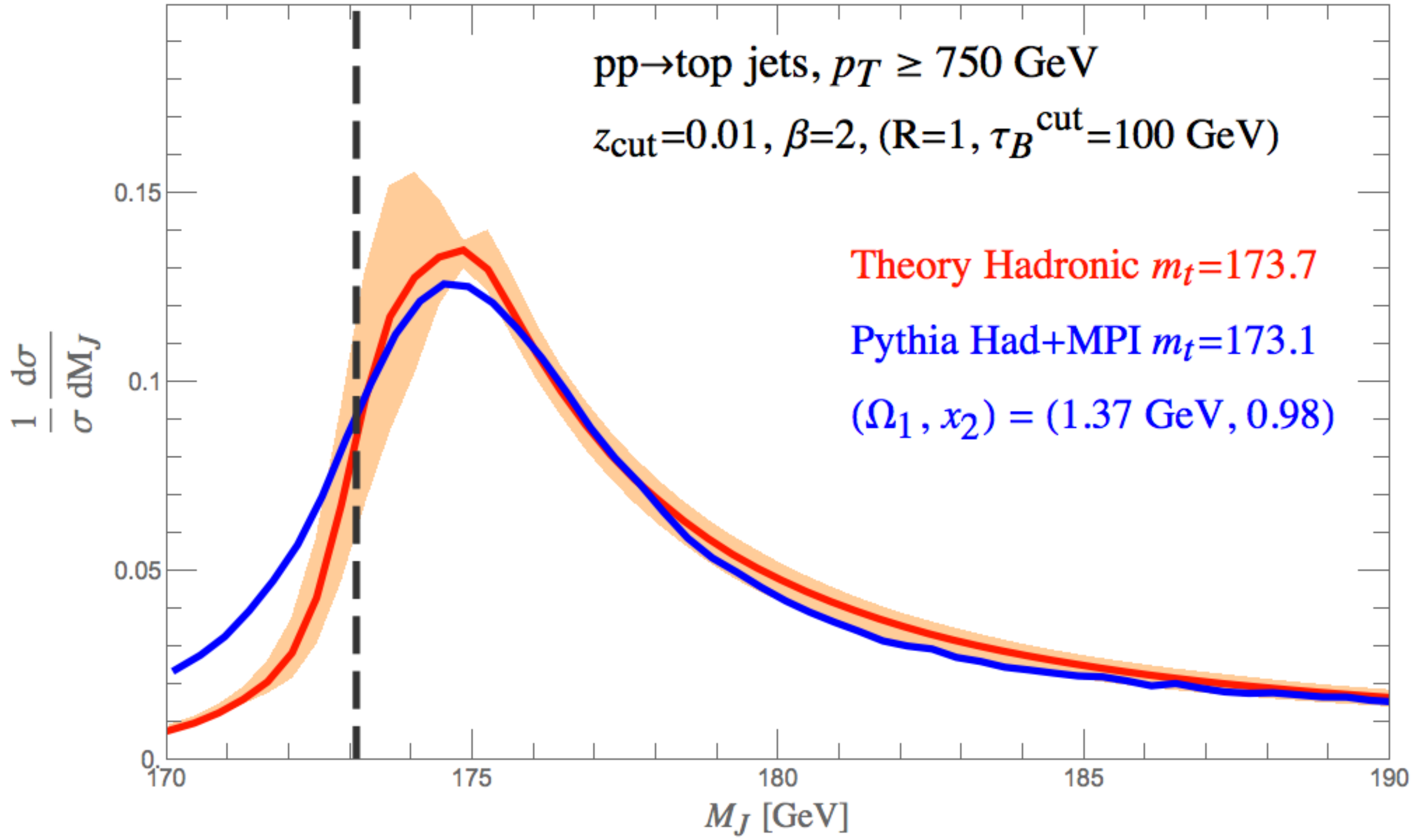




Adding NLL uncertainties







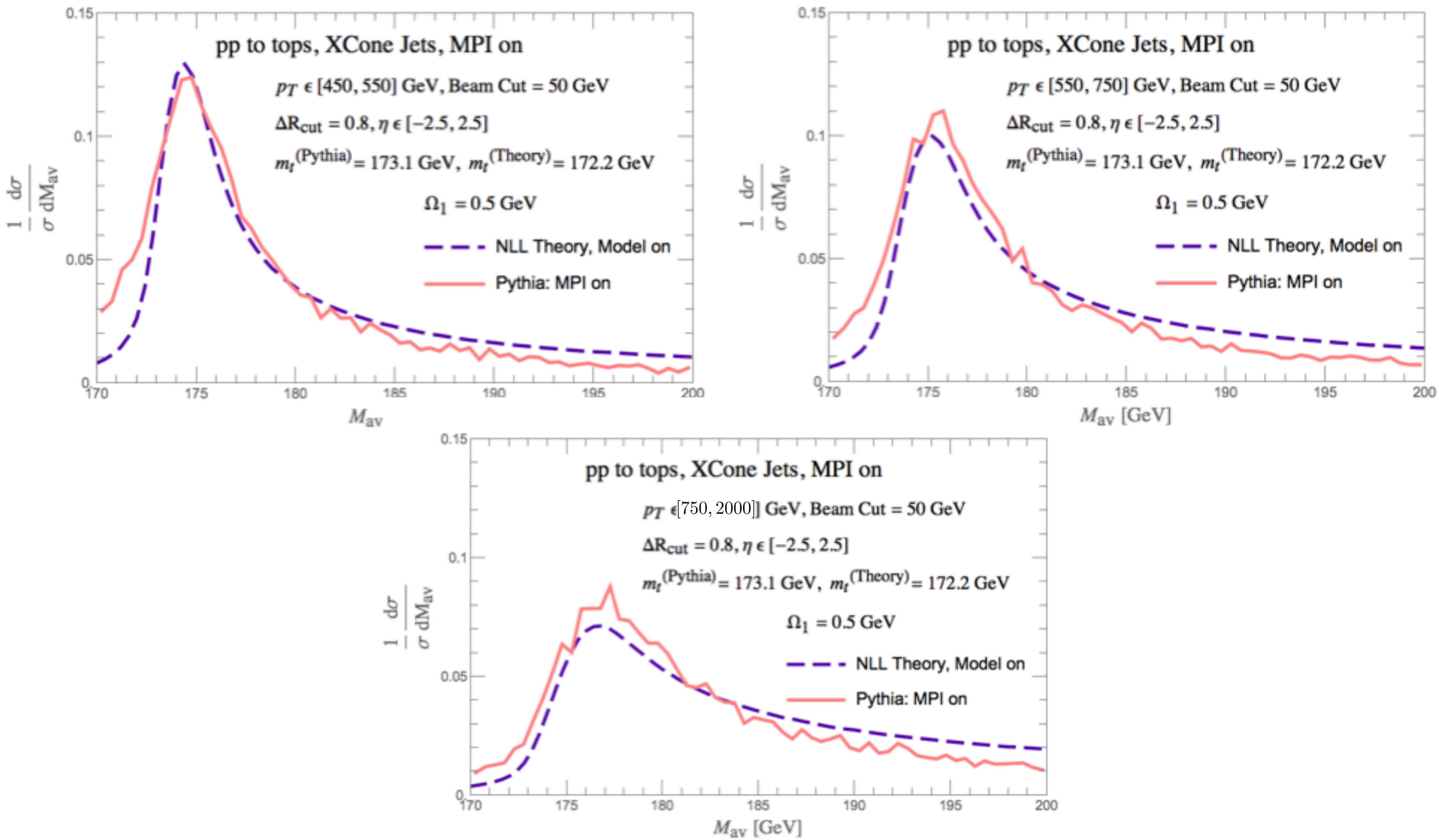
Looks very promising.

But note that this was high pT.
Not yet clear whether lower pT values can be predicted with SoftDrop.

(Pythia: curves do not change for lower pT with R=1)

Pythia vs. Factorization, no SoftDrop

various pT (include MPI & Had., stronger beam cut)



Determine Glauber Lagrangian

$$\mathcal{L}^{(0)}_{\mathrm{SCET_{II}}} = \mathcal{L}^{(0)}_{\mathrm{SCET_{II},S,\{n_i\}}} + \mathcal{L}^{(0)}_{G}(\psi_S, A_S, \xi_{n_i}, A_{n_i})$$

IS, Rothstein arXiv:1601.04695

"Factorization Violation"

Phrase is used in different ways.

Factorization formula is invalid.

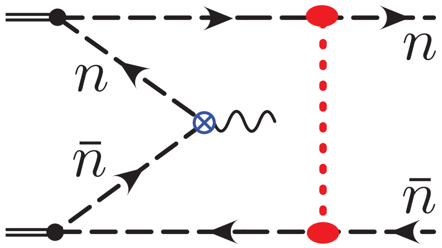
Reasons Factorization could fail:

- Measurement doesn't factor: no simple factorization with universal functions. (eg. Jade algorithm)
- Divergent convolutions, not controlled by ones regulation procedures. (Requires more careful definition of functions.)

$$\int_0^1 \frac{dx}{x^2}\,\phi_\pi(x,\mu)$$

- Interactions that couple other modes and spoil factorization.

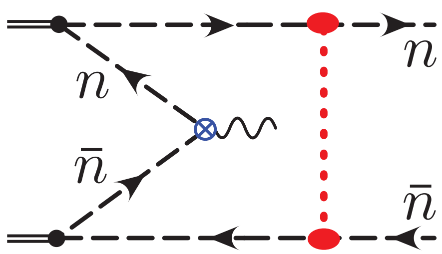
Glauber exchange



spectator-spectator cancel in proof for Drell-Yan

All examples of factorization violation I know of that has been studied in the literature are related to Glauber exchange.

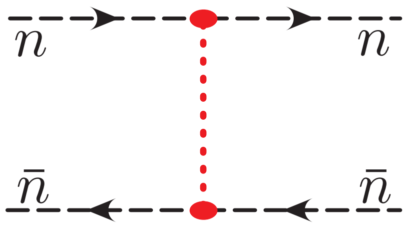
Glauber Exchange could violate factorization:



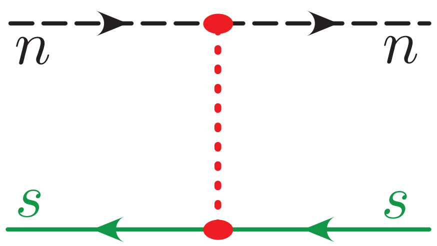
couples n-collinear, n̄-collinear, and soft modes

Glauber's dominate Forward Scattering:

n-$\bar{n}$
fwd. scattering



n-s
fwd. scattering



(small-x logs, reggeization, BFKL, BK/BJMWLK, ...)

Modes:

$\lambda \ll 1 \qquad \text{large } Q$

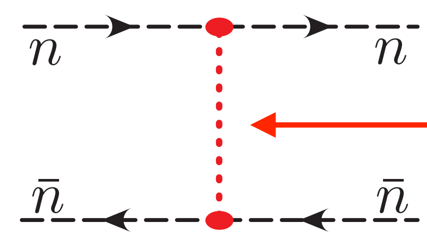
can do calculations with back-to-back collinear particles, then generalize

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n-collinear "jet"	onshell
$\bar{n}$-collinear	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$	$(\bar{n}\cdot p, n\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	$\bar{n}$-collinear "jet"	onshell
soft	$\psi_S,\ A_S^\mu$	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{us},\ A_{us}^\mu$	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda),\ a+b>2$ (here $\{a,b\} = \{2,2\}, \{2,1\}, \{1,2\}$)	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

Integrate out

Need 3-types of Glauber momenta: $(+, -, \perp)$

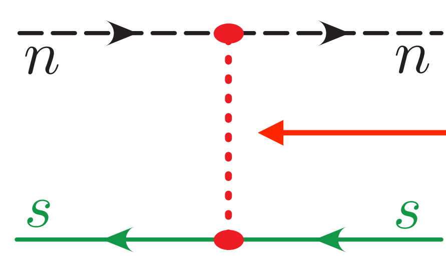
n-$\bar{n}$
fwd. scattering



$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda)$

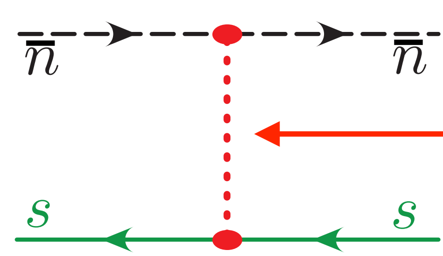
$s \gg t$

n-s
fwd. scattering



$p^\mu \sim Q(\lambda^2, \lambda, \lambda)$

$\bar{n}$-s
fwd. scattering



$p^\mu \sim Q(\lambda, \lambda^2, \lambda)$

(also scatter forward gluons)

Modes:

$\lambda \ll 1 \qquad \text{large } Q$

can do calculations with back-to-back collinear particles, then generalize

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar n\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n-collinear "jet"	onshell
$\bar n$-collinear	$\xi_{\bar n},\ A_{\bar n}^\mu$	$(\bar n\cdot p, n\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	$\bar n$-collinear "jet"	onshell
soft	$\psi_S,\ A_S^\mu$	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{us},\ A_{us}^\mu$	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda),\ a+b>2$ (here $\{a,b\} = \{2,2\},\{2,1\},\{1,2\}$)	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

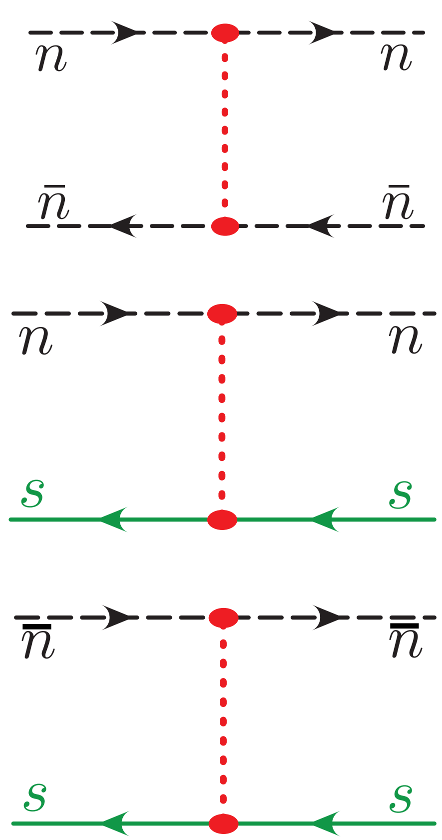
Integrate out

Need 3-types of Glauber momenta:

n-$\bar n$
fwd. scattering

n-s
fwd. scattering

$\bar n$-s
fwd. scattering



- $\dfrac{1}{k_\perp^2}$ potentials
- instantaneous in x^+, x^- (t and z)

Goals for treating Glauber Operator in EFT:

- Hard Scattering and Forward Scattering in single framework
- Distinct Infrared Modes in Feyn. Graphs + Power Counting → derive when eikonal approximation is relevant
- $\overline{\rm MS}$ style renormalization for rapidity divergences (counterterms, renormalization group equations, ...)
- Sum Large Logs: $\ln\left(\frac{Q^2}{m^2}\right)\,, \quad \ln(x)$
- Valid to all orders in α_s & clear path to study subleading power amplitudes with Glauber effects (subleading ops & Lagrangians)
- Factorization violating interactions may also have factorization formulae (could predict things about UE, etc.)
- Framework to (re)derive factorization theorems via $\cancel{+\mathcal{L}_G^{(0)}}$

Full Leading Power Glauber Lagrangian:

$$\mathcal{L}_G^{\mathrm{II}(0)} = \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

(3 rapidity sectors)

(2 rapidity sectors)

sum pairwise on all collinears

sum on all collinears

- Interactions with more sectors are given by T-products
- No Wilson coefficients ie. no new structures at loop level.

Uses SCET building blocks:

$$\chi_n = W_n^\dagger \xi_n \qquad \psi_s^n = S_n^\dagger \psi_s$$

$$\mathcal{B}_{n\perp}^\mu = \frac{1}{g}\left[W_n^\dagger i D_{n\perp}^\mu W_n\right] \qquad \mathcal{B}_{S\perp}^{n\mu} = \frac{1}{g}\left[S_n^\dagger i D_{S\perp}^\mu S_n\right] \qquad \widetilde{\mathcal{B}}_{S\perp}^{nAB} = -i f^{ABC} \mathcal{B}_{S\perp}^{nC}$$

$$\widetilde{G}_s^{\mu\nu\,AB} = -i f^{ABC} G_s^{\mu\nu\,A}$$

Full Leading Power Glauber Lagrangian:

$$\mathcal{L}_G^{\mathrm{II}(0)} = \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

(3 rapidity sectors)

(2 rapidity sectors)

sum pairwise on all collinears

sum on all collinears

- Interactions with more sectors are given by T-products
- No Wilson coefficients ie. no new structures at loop level.

$$\mathcal{O}_n^{qB} = \overline{\chi}_n T^B \frac{\bar{n}\!\!\!/}{2} \chi_n \qquad \mathcal{O}_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^C \frac{\bar{n}}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{n\perp}^{D\mu}$$

$$\mathcal{O}_{\bar{n}}^{qB} = \overline{\chi}_{\bar{n}} T^B \frac{n\!\!\!/}{2} \chi_{\bar{n}} \qquad \mathcal{O}_{\bar{n}}^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{\bar{n}\perp\mu}^C \frac{n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{\bar{n}\perp}^{D\mu}$$

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \Big\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g\widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} - \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu} \mathcal{P}_\mu^\perp - g\widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g\widetilde{\mathcal{B}}_{S\perp\mu}^{\bar{n}} - \frac{n_\mu \bar{n}_\nu}{2} \mathcal{S}_n^T ig\widetilde{G}_s^{\mu\nu} \mathcal{S}_{\bar{n}} \Big\}^{BC}$$

$$\mathcal{O}_s^{q_n B} = 8\pi\alpha_s \left(\bar{\psi}_S^n \, T^B \frac{\bar{n}\!\!\!/}{2} \psi_S^n \right) \qquad \mathcal{O}_s^{g_n B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{nC} \frac{n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{nD\mu} \right)$$

$$\mathcal{O}_s^{q_{\bar{n}} B} = 8\pi\alpha_s \left(\bar{\psi}_S^{\bar{n}} \, T^B \frac{n\!\!\!/}{2} \psi_S^{\bar{n}} \right) \qquad \mathcal{O}_s^{g_{\bar{n}} B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{\bar{n}C} \frac{\bar{n}}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{\bar{n}D\mu} \right)$$

Soft $\mathcal{O}_s^{BC}$ Operator

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \sum_i C_i O_i^{BC}$$

basis of $\mathcal{O}(\lambda^2)$ operators allowed by symmetries:

$$O_1 = \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar n} \mathcal{P}_{\perp\mu}, \qquad O_2 = \mathcal{P}_\perp^\mu \mathcal{S}_{\bar n}^T \mathcal{S}_n \mathcal{P}_{\perp\mu},$$

$$O_3 = \mathcal{P}_\perp \cdot (g\widetilde{\mathcal{B}}_{S\perp}^n)(\mathcal{S}_n^T \mathcal{S}_{\bar n}) + (\mathcal{S}_n^T \mathcal{S}_{\bar n})(g\widetilde{\mathcal{B}}_{S\perp}^{\bar n})\cdot \mathcal{P}_\perp, \qquad O_4 = \mathcal{P}_\perp \cdot (g\widetilde{\mathcal{B}}_{S\perp}^{\bar n})(\mathcal{S}_{\bar n}^T \mathcal{S}_n) + (\mathcal{S}_{\bar n}^T \mathcal{S}_n)(g\widetilde{\mathcal{B}}_{S\perp}^{n})\cdot \mathcal{P}_\perp,$$

$$O_5 = \mathcal{P}_\mu^\perp (\mathcal{S}_n^T \mathcal{S}_{\bar n})(g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu}) + (g\widetilde{\mathcal{B}}_{S\perp}^{n\mu})(\mathcal{S}_n^T \mathcal{S}_{\bar n})\mathcal{P}_\mu^\perp, \qquad O_6 = \mathcal{P}_\mu^\perp (\mathcal{S}_{\bar n}^T \mathcal{S}_n)(g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}) + (g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu})(\mathcal{S}_{\bar n}^T \mathcal{S}_n)\mathcal{P}_\mu^\perp,$$

$$O_7 = (g\widetilde{\mathcal{B}}_{S\perp}^{n\mu})\mathcal{S}_n^T \mathcal{S}_{\bar n}(g\widetilde{\mathcal{B}}_{S\perp\mu}^{\bar n}), \qquad O_8 = (g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu})\mathcal{S}_{\bar n}^T \mathcal{S}_n(g\widetilde{\mathcal{B}}_{S\perp\mu}^{n}),$$

$$O_9 = \mathcal{S}_n^T n_\mu \bar n_\nu (ig\widetilde{G}_s^{\mu\nu})\mathcal{S}_{\bar n}, \qquad O_{10} = \mathcal{S}_{\bar n}^T n_\mu \bar n_\nu (ig\widetilde{G}_s^{\mu\nu})\mathcal{S}_n,$$

octet Wilson line

octet reps

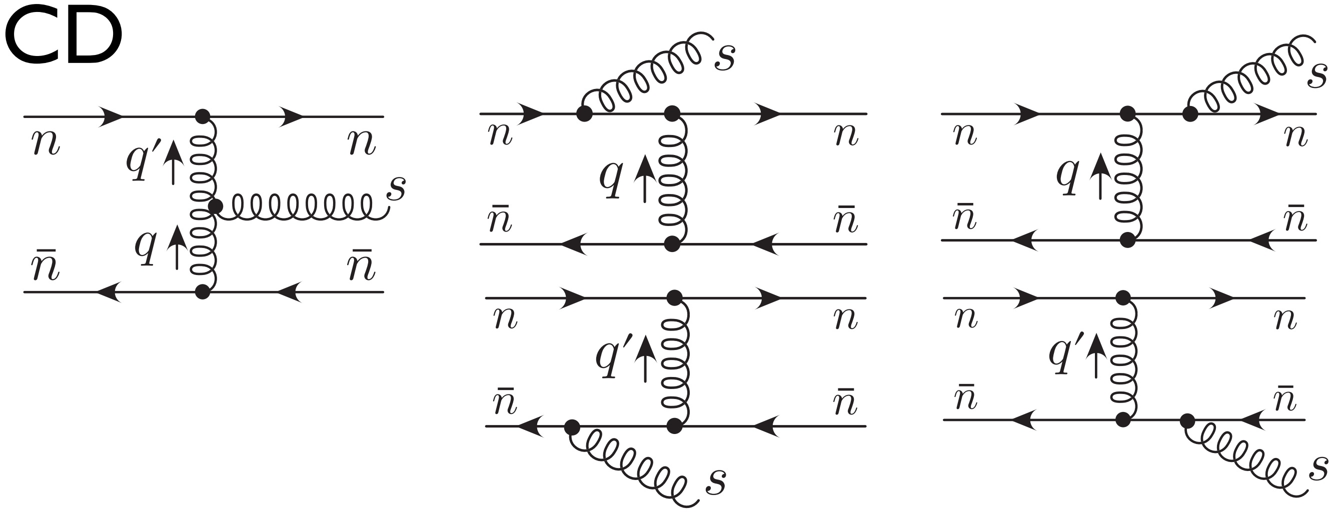
Restricted by: Hermiticity $O_i^\dagger\big|_{n\leftrightarrow\bar n} = O_i$, one $\mathcal{S}_n$, one $\mathcal{S}_{\bar n}$

operator identities: eg. $[\mathcal{P}_\perp^\mu(\mathcal{S}_n^T\mathcal{S}_{\bar n})] = -g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}(\mathcal{S}_n^T\mathcal{S}_{\bar n}) + (\mathcal{S}_n^T\mathcal{S}_{\bar n})g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu}$

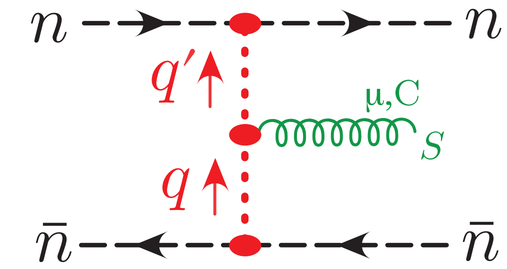
Matching with up to 2 soft gluons fixes all coefficients

One Soft Gluon:

QCD



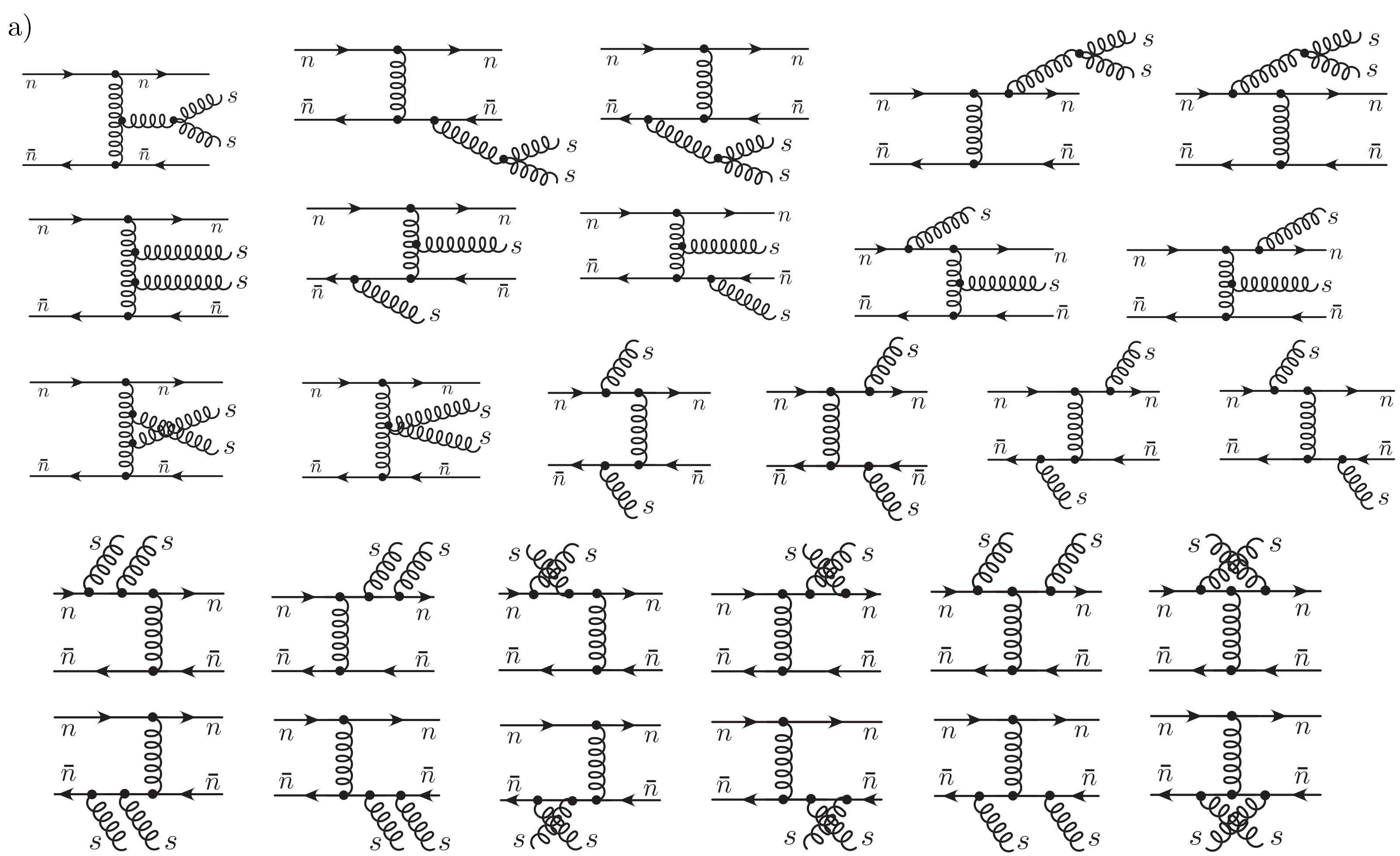
EFT



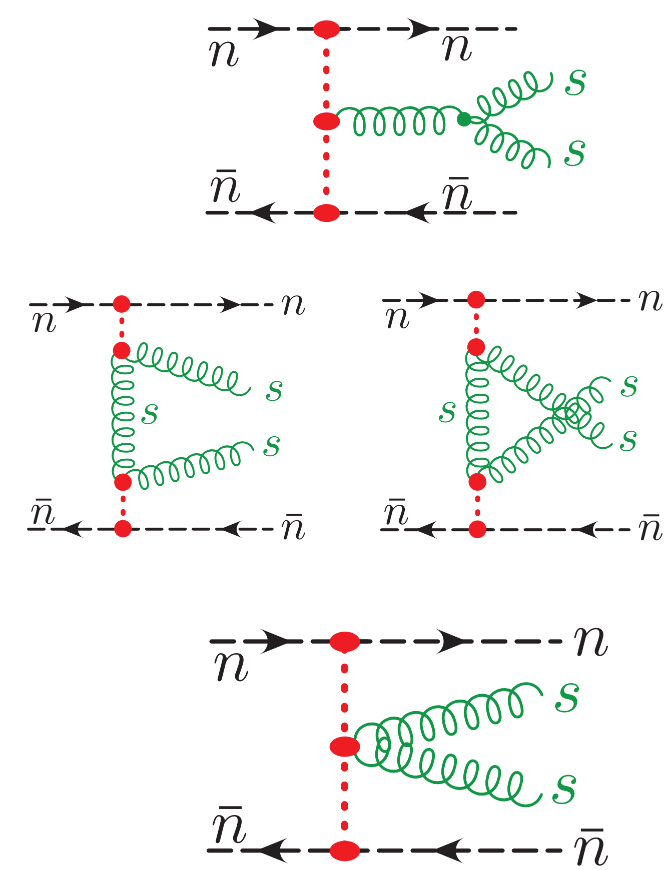
Lipatov vertex

Two Soft Gluons:

QCD



EFT



2-gluon vertex

Find:

$$\boxed{C_2 = C_4 = C_5 = C_6 = C_8 = C_{10} = 0},$$

$$\boxed{C_1 = -C_3 = -C_7 = +1\,, \qquad C_9 = -\frac{1}{2}}$$

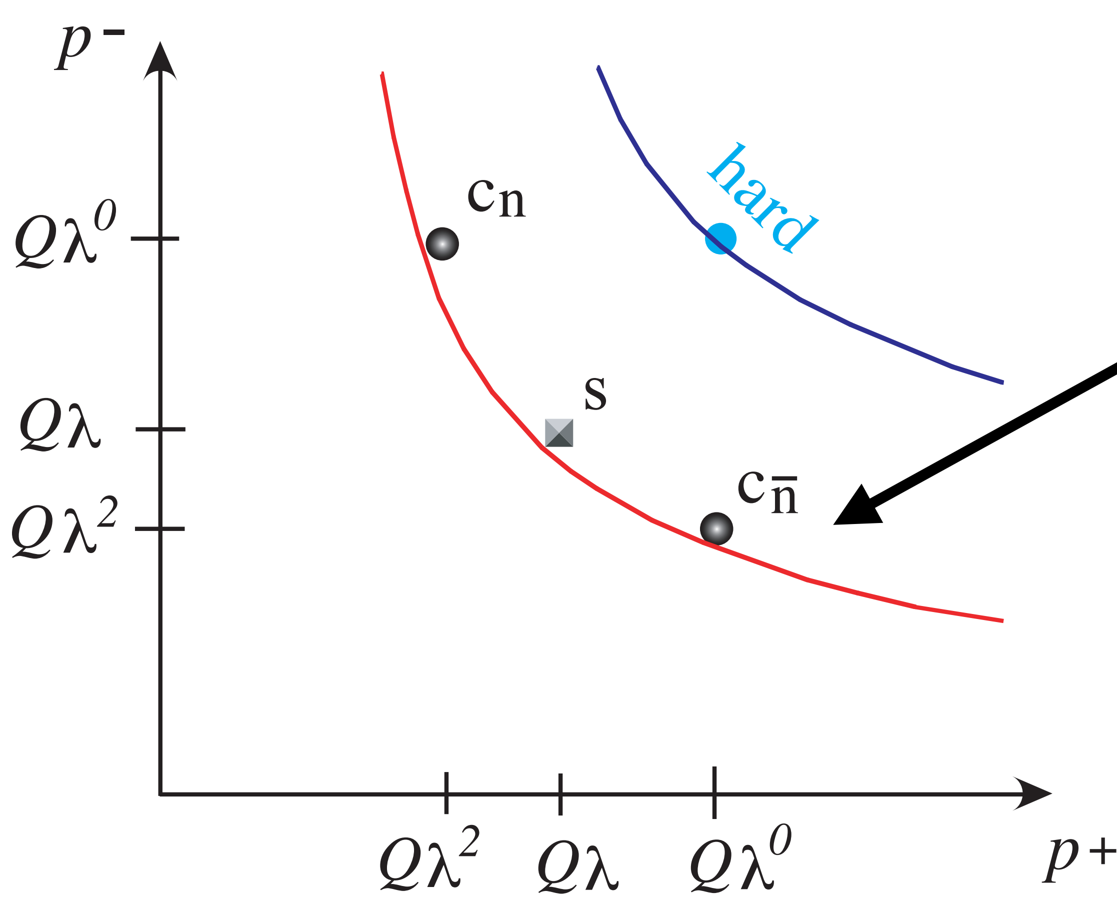
$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \Big\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g\widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} - \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu} \mathcal{P}_\mu^\perp - g\widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g\widetilde{\mathcal{B}}_{S\perp\mu}^{\bar{n}} - \frac{n_\mu \bar{n}_\nu}{2} \mathcal{S}_n^T ig\widetilde{G}_s^{\mu\nu} \mathcal{S}_{\bar{n}} \Big\}^{BC} .$$

- Requires rapidity regulator for Glauber potential $|2k^z|^{-\eta}\nu^\eta$ and for Wilson lines

$$S_n = \sum_{\text{perms}} \exp\left\{\frac{-g}{n\cdot\mathcal{P}}\left[\frac{w|2\mathcal{P}^z|^{-\eta/2}}{\nu^{-\eta/2}} n\cdot A_s\right]\right\} \qquad W_n = \sum_{\text{perms}} \exp\left\{\frac{-g}{\bar{n}\cdot\mathcal{P}}\left[\frac{w^2|\bar{n}\cdot\mathcal{P}|^{-\eta}}{\nu^{-\eta}} \bar{n}\cdot A_n\right]\right\}$$

(ala Chiu, Jain, Neill, Rothstein)

$$\nu\frac{\partial}{\partial\nu}w^2(\nu) = -\eta\, w^2(\nu)\,, \qquad \lim_{\eta\to 0} w(\nu) = 1$$



Modes distinguished by rapidity

RG scale: ν

(Zero-bin subtractions, avoid double counting IR regions)

Glauber Applications

Gluon Reggeization

$$\nu \sim \sqrt{s}$$

$$\mathcal{O}_n^A \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_S^{AB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^B$$

$$\nu \sim \sqrt{-t}$$

Consider separate rapidity renormalization of soft & collinear component operators

Either run collinear operators from $\nu \sim \sqrt{s}$ to $\nu \sim \sqrt{-t}$, or run soft operator.

$$\nu \frac{d}{d\nu}(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) = \gamma_{n\nu}(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})$$

$$\gamma_{n\nu} = \frac{\alpha_s(\mu) C_A}{2\pi} \ln\left(\frac{-t}{m^2}\right)$$

(IR divergent)

gives: $\left(\frac{s}{-t}\right)^{-\gamma_{n\nu}}$

virtual anom.dim. is Regge exponent for gluon

Forward Scattering & BFKL

Expand time evolution, do soft-collinear factorization term by term:

$$T \exp i\int d^4x\, \mathcal{L}_G^{\rm II(0)}(x) = \Big[1 + i\int d^4y_1\, \mathcal{L}_G^{\rm II(0)}(y_1) + \frac{i^2}{2!}T\int d^4y_1\, d^4y_2\, \mathcal{L}_G^{\rm II(0)}(y_1)\mathcal{L}_G^{\rm II(0)}(y_2) + \ldots\Big]$$

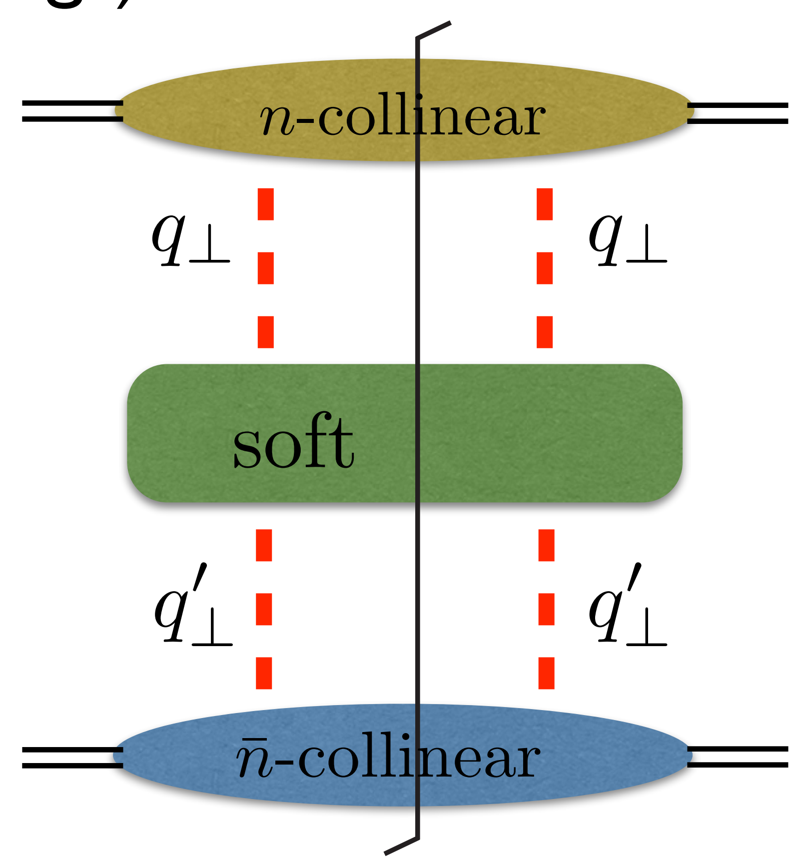
$$\sim 1 + T\sum_{k=1}^{\infty}\sum_{k'=1}^{\infty}\Big[\mathcal{O}_n^{jA_i}(q_{i\perp})\Big]^k\Big[\mathcal{O}_{\bar n}^{j'B_{i'}}(q_{i'\perp})\Big]^{k'} \otimes O_{s(k,k')}^{A_1\cdot A_k,B_1\cdots B_{k'}}(q_{\perp 1},\ldots,q_{\perp k'})$$

$$\equiv 1 + \sum_{k=1}^{\infty}\sum_{k'=1}^{\infty} U_{(k,k')}$$

Consider (linearized) forward scattering with one Glauber exchange, but all orders in other interactions (eg. leading logs):

$$T_{(1,1)} = \frac{1}{V_4}\sum_X \langle pp'|U_{(1,1)}^\dagger|X\rangle\langle X|U_{(1,1)}|pp'\rangle \quad = \ldots$$

$$= \int d^2q_\perp d^2q'_\perp\, C_n(q_\perp, p^-)S_G(q_\perp, q'_\perp)C_{\bar n}(q'_\perp, p'^+)$$

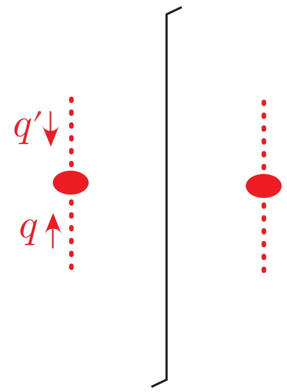


after rapidity renormalization:

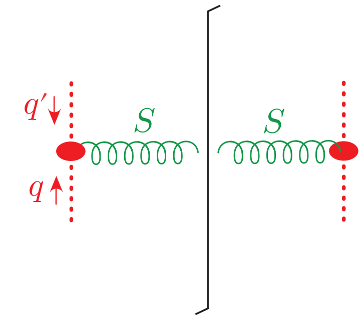
$$T_{(1,1)} = \int d^2q_\perp d^2q'_\perp\, C_n(q_\perp, p^-,\nu)S_G(q_\perp, q'_\perp,\nu)C_{\bar n}(q'_\perp, p'^+,\nu)$$

collinear and soft functions

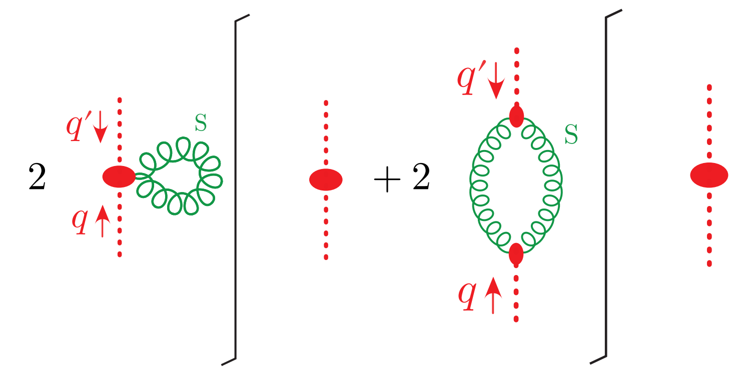
Consider rapidity renormalization for soft function that appears here:



$$= S_G^{(0)}(q_\perp, q'_\perp)$$
$$= (8\pi\alpha_s)^2 \delta^{AA}\,(2\pi)^2 \delta^2(\vec{q}_\perp + \vec{q}\,'_\perp)$$



rapidity divergent
real emission



rapidity divergent
virtual

cancel $1/\eta$ divergences

$$S_G(\vec{q}_\perp, \vec{q}\,'_\perp, \nu) = \int d^2k_\perp \; Z_{S_G}(q_\perp, k_\perp)\; S_G^{\rm bare}(k_\perp, q'_\perp)$$

$$0 = \nu \frac{d}{d\nu} S_G^{\rm bare}(q_\perp, q'_\perp)$$

$$\nu \frac{d}{d\nu} S_G(q_\perp, q'_\perp, \nu) = \int d^2k_\perp \; \gamma_{S_G}(q_\perp, k_\perp)\; S_G(k_\perp, q'_\perp, \nu)$$

$$= \frac{2C_A \alpha_s(\mu)}{\pi^2} \int d^2k_\perp \left[\frac{S_G(k_\perp, q'_\perp, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^{\,2}\; S_G(q_\perp, q'_\perp, \nu)}{2\vec{k}_\perp^{\,2}\,(\vec{k}_\perp - \vec{q}_\perp)^2} \right]$$

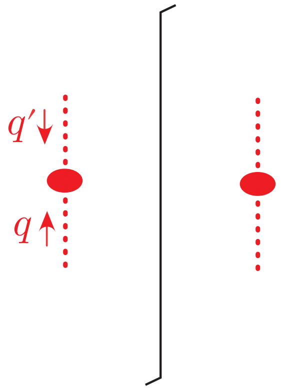
evolution
given by
BFKL equation

(see also work
by S. Fleming)

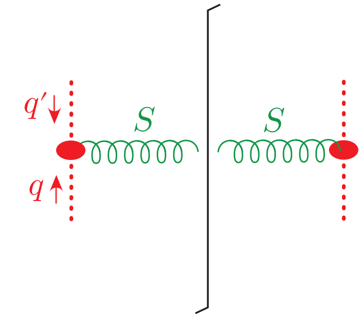
Sum usual LL: $\alpha_s^k \ln^k\left(\frac{s}{-t}\right)$ in forward cross-section

[or ln(x) in DIS]

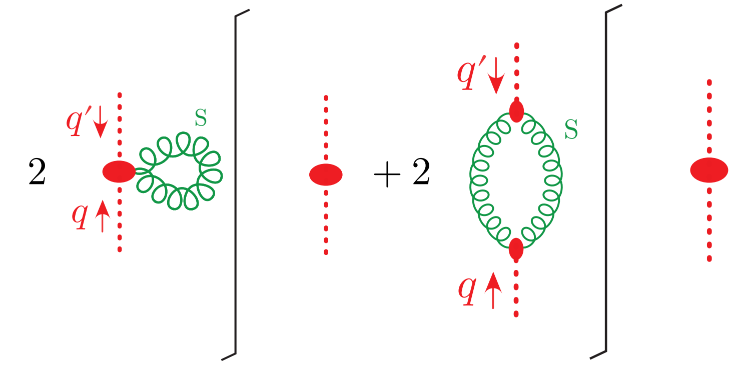
Consider rapidity renormalization for soft function that appears here:



$$= S_G^{(0)}(q_\perp, q'_\perp)$$
$$= (8\pi\alpha_s)^2 \delta^{AA} \, (2\pi)^2 \delta^2(\vec{q}_\perp + \vec{q}'_\perp)$$



rapidity divergent
real emission



rapidity divergent
virtual

$$S_G(\vec{q}_\perp, \vec{q}'_\perp, \nu) = \int d^2k_\perp \; Z_{S_G}(q_\perp, k_\perp) \; S_G^{\rm bare}(k_\perp, q'_\perp) \qquad\qquad 0 = \nu \frac{d}{d\nu} S_G^{\rm bare}(q_\perp, q'_\perp)$$

$$\Rightarrow \quad \nu \frac{d}{d\nu} S_G(q_\perp, q'_\perp, \nu) = \int d^2k_\perp \; \gamma_{S_G}(q_\perp, k_\perp) \; S_G(k_\perp, q'_\perp, \nu)$$

$$= \frac{2C_A\alpha_s(\mu)}{\pi^2} \int d^2k_\perp \left[\frac{S_G(k_\perp, q'_\perp, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^{\,2} \; S_G(q_\perp, q'_\perp, \nu)}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right]$$

evolution
given by
BFKL equation

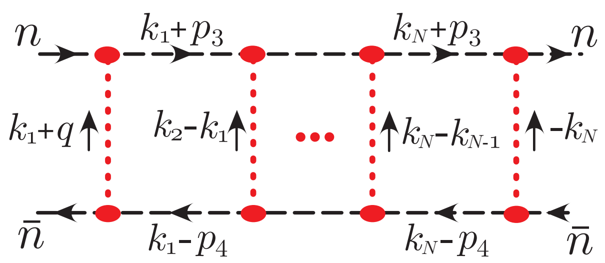
RGE consistency:

$$\nu \frac{d}{d\nu} C_n(q_\perp, p^-, \nu) = -\frac{C_A\alpha_s}{\pi^2} \int d^2k_\perp \left[\frac{C_n(k_\perp, p^-, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^{\,2} \; C_n(q_\perp, p^-, \nu)}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right] \qquad -\frac{1}{2}(\text{BFKL})$$

Eikonal Scattering

Rapidity regulator consistent with eikonal phase

Sum up Glauber Boxes



$$= i(-2g^2)^{N+1}\mathcal{S}^{n\bar{n}}_{(N+1)}\, I^{(N)}(q_\perp)\int \frac{\text{đ}k_1^z\cdots \text{đ}k_N^z\,\left|2k_1^z(2k_1^z-2k_2^z)\cdots(2k_{N-1}^z-2k_N^z)2k_N^z\right|^{-\eta}\nu^{N\eta}}{2^N(-k_1^z+\Delta_1+i0)\cdots(-k_N^z+\Delta_N+i0)}$$

Fourier transform k_i^z :

$$= 2(-ig^2)^{N+1}\mathcal{S}^{n\bar{n}}_{(N+1)}\, I^{(N)}(q_\perp)\Big(\kappa_\eta\frac{\eta}{2}\Big)^{N+1}\int_{-\infty}^{+\infty}\Big[\prod_{j=1}^{N+1} dx_j\,|x_j|^{-1+\eta}\Big]\;\theta(x_2-x_1)\theta(x_3-x_2)\cdots\theta(x_{N+1}-x_N)\exp\Big[\sum_{m=1}^{N} i\Delta_m(x_{m+1}-x_m)\Big]$$

need $x_j \to 0$

ordered collapse to equal longitudinal postion

$$= -2(ig^2)^{N+1}\mathcal{S}^{n\bar{n}}_{(N+1)} I_\perp^{(N)}(q_\perp)\;\frac{1}{(N+1)!}\Big[1+\mathcal{O}(\eta)\Big]$$

Fourier transform $q_\perp$:
$$\int \text{đ}^{d-2}q_\perp\, e^{i\vec{q}_\perp\cdot\vec{b}_\perp}\sum_{N=0}^{\infty}\text{G.Box}_N^{2\to 2}(q_\perp) = (\tilde{G}(b_\perp)-1)2\mathcal{S}^{n\bar{n}}$$

gives classic eikonal scattering result:
$$\boxed{\tilde{G}(b_\perp) = e^{i\phi(b_\perp)}}$$

$$\phi(b_\perp) = -\mathbf{T}_1^A\otimes\mathbf{T}_2^A\, g^2(\mu)\int\frac{\text{đ}^{d-2}q_\perp\,(\iota^\epsilon\mu^{2\epsilon})}{\vec{q}_\perp^{\,2}}\,e^{i\vec{q}_\perp\cdot\vec{b}_\perp}$$

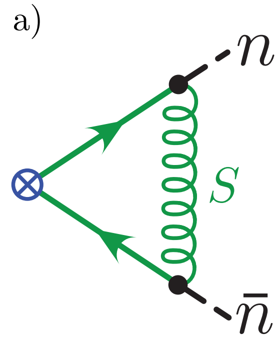
Hard Scattering $\otimes$

The Cheshire Glauber

e.g. $J_\Gamma = (\bar{\xi}_n W_n) S_n^\dagger \Gamma S_{\bar{n}} (W_{\bar{n}}^\dagger \xi_{\bar{n}})$ **Active-Active and Soft Overlap**

naive soft:

a)



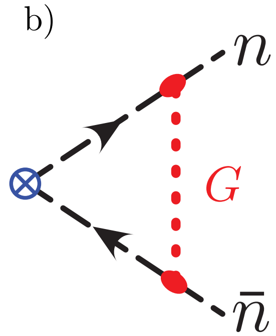
$$\tilde{S} = -2ig^2 C_F \mathcal{S}_\Gamma \int d\!\!{}^-{}^d k \, \frac{(\iota^\epsilon \mu^{2\epsilon} \, |k_z|^{-\eta} \, \nu^\eta)}{[k^2 - m^2][n\cdot k + i0][\bar{n}\cdot k - i0]}$$

$$= \mathcal{S}_\Gamma \frac{C_F \alpha_s}{2\pi} \Big\{ \Big[\frac{-2h(\epsilon, \mu^2/m^2)}{\eta} + \ln\frac{\mu^2}{\nu^2}\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\Big) + \frac{1}{\epsilon^2} - \frac{1}{2}\ln^2\frac{\mu^2}{m^2} - \frac{\pi^2}{12}\Big] + \Big[(i\pi)\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\Big)\Big]\Big\}$$

with physical directions for soft Wilson lines in hard scattering

true soft: includes 0-bin subtraction $S = \tilde{S} - S^{(G)}$ has no $i\pi$ term

b)



Glauber:

$$G = S^{(G)} = \bar{u}_n \Gamma v_{\bar{n}} \frac{C_F \alpha_s}{2\pi} \Big[(i\pi)\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\Big)\Big]$$

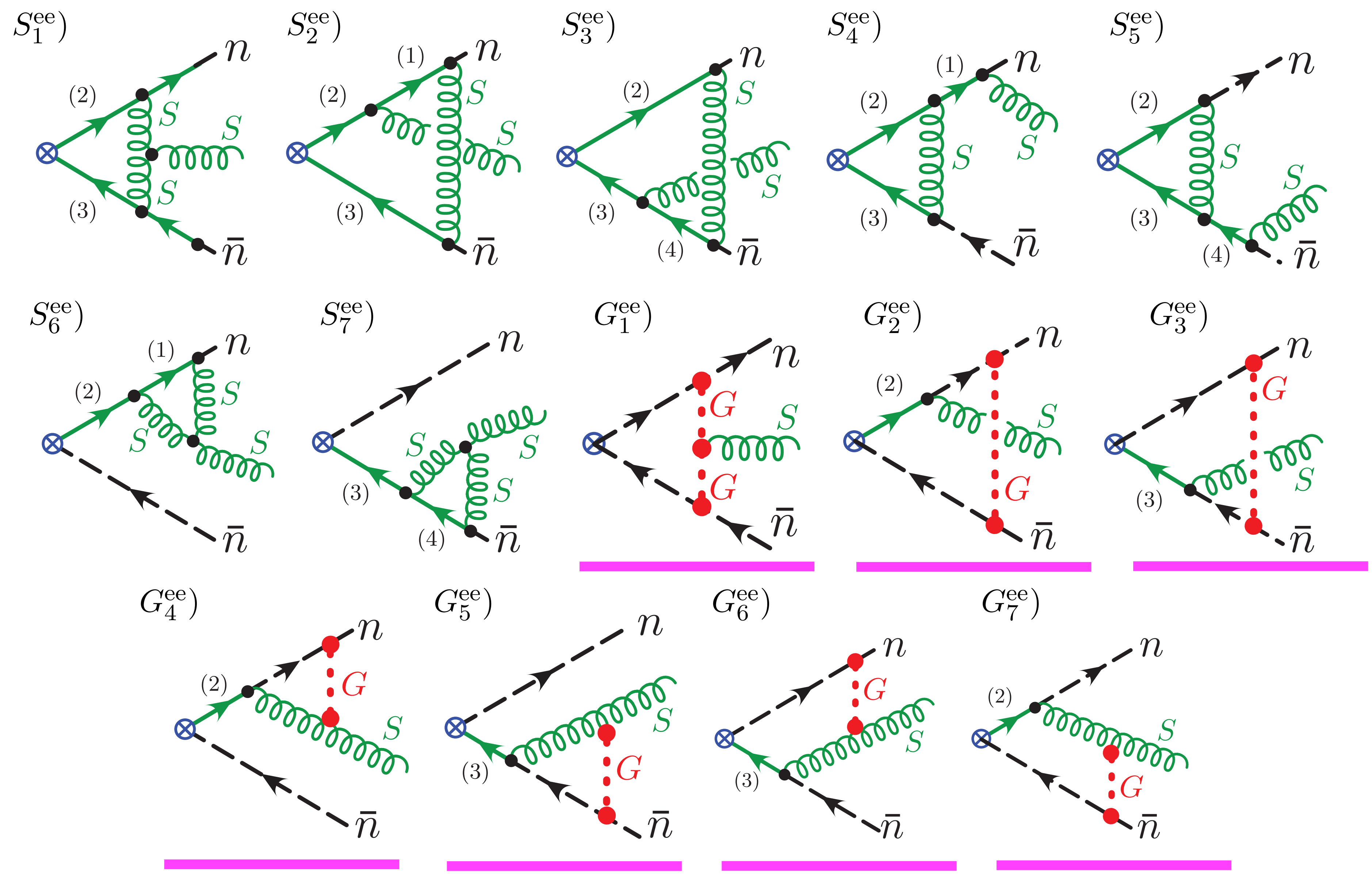
Glaubers give $(i\pi)$ terms

BUT $(\tilde{S} - S^{(G)}) + G = \tilde{S}$

- so we don't see Glauber in Hard Matching
- can <u>absorb this Glauber</u> into Soft Wilson lines if they have proper directions

Also true in the presence of additional emissions:

e^+e^-

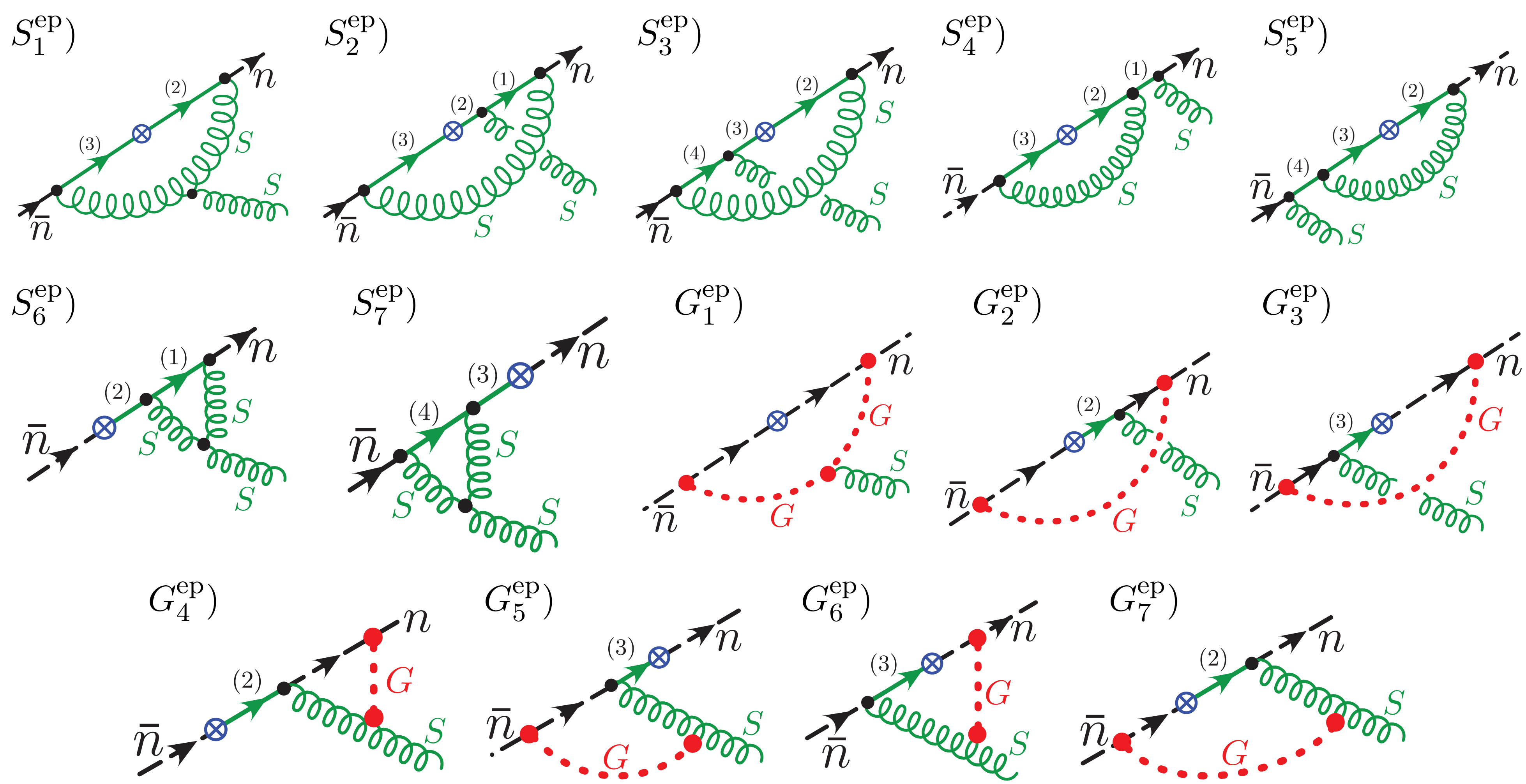


physical:

Glauber again gives all $(i\pi)$ terms here.

Also true in the presence of additional emissions:

$e^- p$

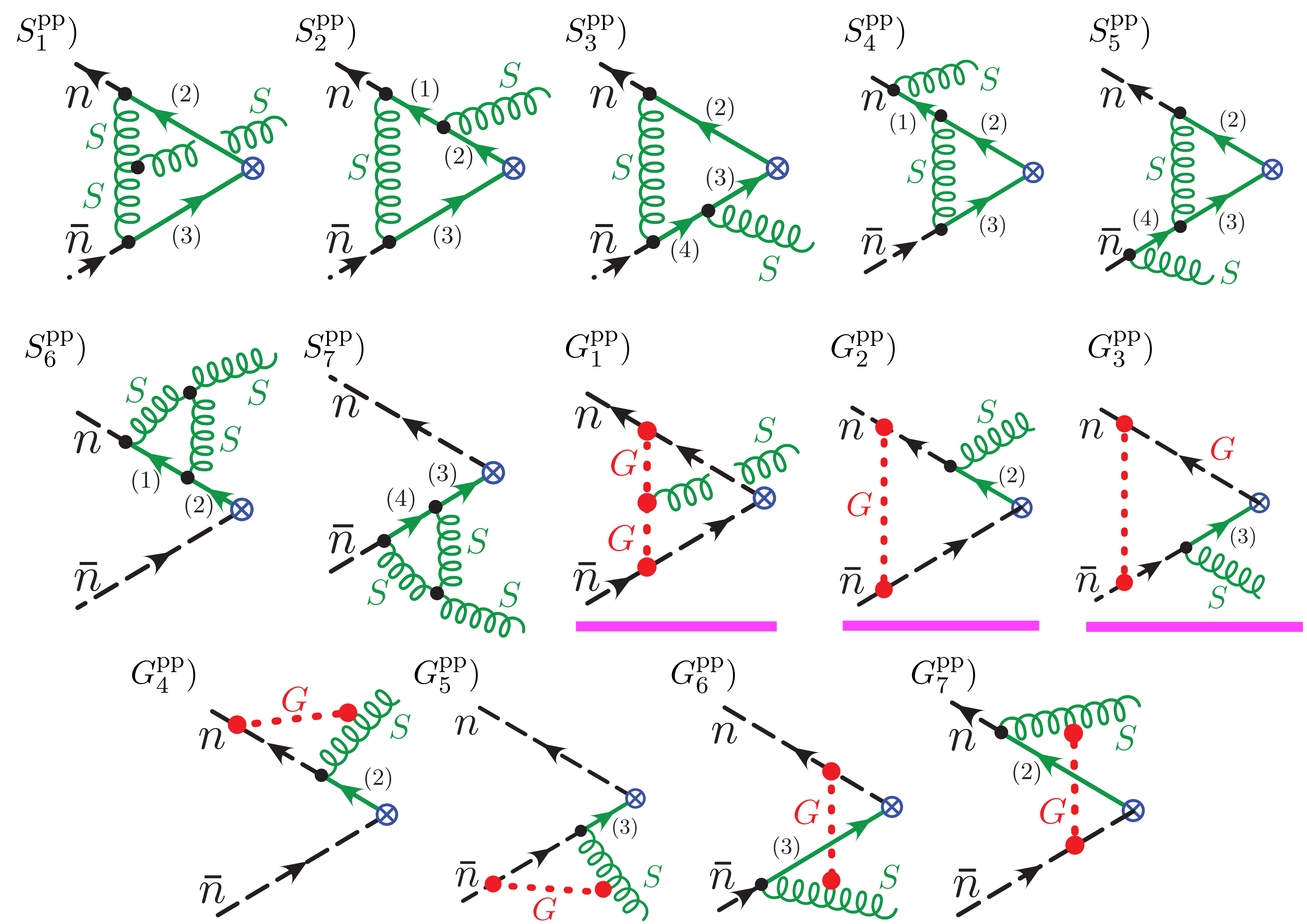


physical:

Glauber again gives all $(i\pi)$ terms here.

Also true in the presence of additional emissions:

pp

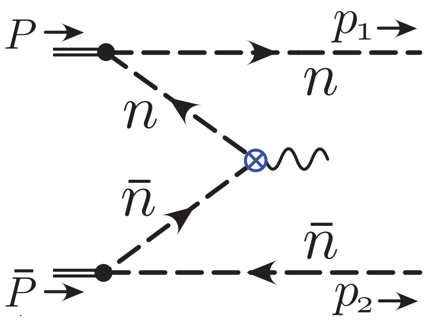


physical:

Glauber again gives all $(i\pi)$ terms here.

Hadron Scattering

Add interpolating fields for initial state hadrons.



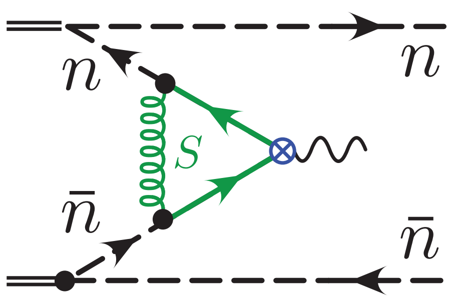
$$
= \mathcal{S}^{\gamma}\left[\frac{1}{\vec{p}_{1\perp}^{\,2}}\,\frac{1}{\vec{p}_{2\perp}^{\,2}}\right]\left[\frac{\bar{n}\cdot p_1\,\bar{n}\cdot(P-p_1)}{\bar{n}\cdot P}\,\frac{n\cdot p_2\,n\cdot(\bar{P}-p_2)}{n\cdot\bar{P}}\right]
$$

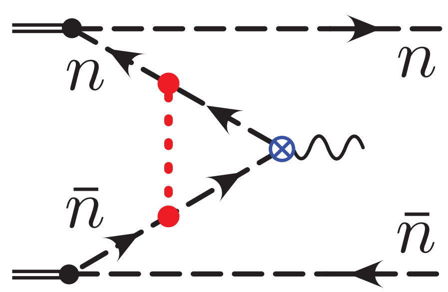
$$
\equiv \mathcal{S}^{\gamma}\,E(p_{1\perp},p_{2\perp}),
$$

$$
\mathcal{S}^{\gamma} = \bar{u}_n \gamma_{\perp}^{\mu} v_{\bar{n}}^{*}
$$

"an end E"

Active-Active





Same correspondence:

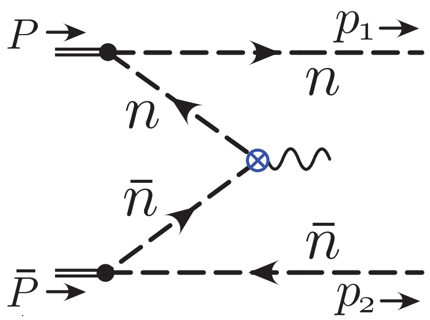
$$
S = \tilde{S} - S^{(G)}
$$

$$
G = S^{(G)}
$$

$$
(\tilde{S} - S^{(G)}) + G = \tilde{S}
$$

Hadron Scattering

Add interpolating fields for initial state hadrons.

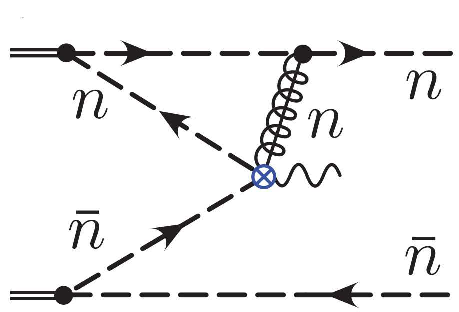


$$
\begin{aligned}
&= \mathcal{S}^\gamma \left[\frac{1}{\vec{p}_{1\perp}^{\,2}} \frac{1}{\vec{p}_{2\perp}^{\,2}}\right] \left[\frac{\bar{n}\cdot p_1\, \bar{n}\cdot(P-p_1)}{\bar{n}\cdot P} \frac{n\cdot p_2\, n\cdot(\bar{P}-p_2)}{n\cdot \bar{P}}\right] \\
&\equiv S^\gamma\, E(p_{1\perp}, p_{2\perp}),
\end{aligned}
$$

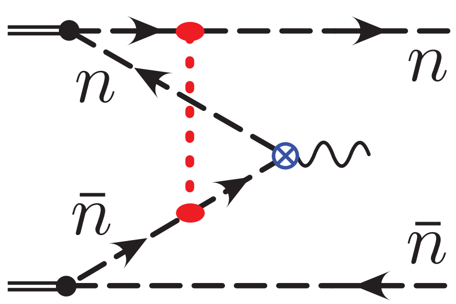
$$\mathcal{S}^\gamma = \bar{u}_n \gamma_\perp^\mu v_{\bar{n}}^*$$

"an end E"

Active-Spectator

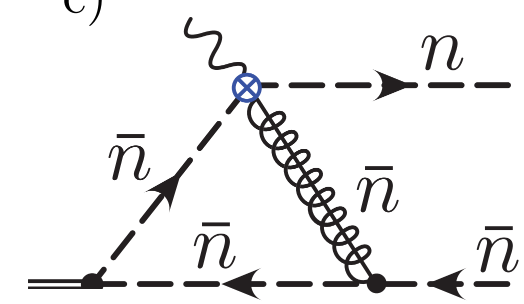


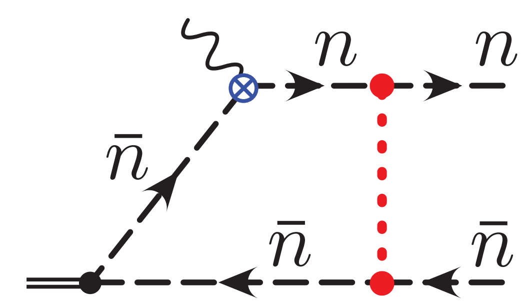
$$C_n = \tilde{C}_n - C_n^{(G)}$$



$$G = C_n^{(G)}$$

c)

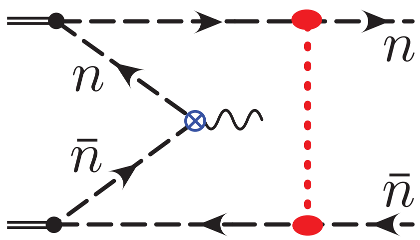




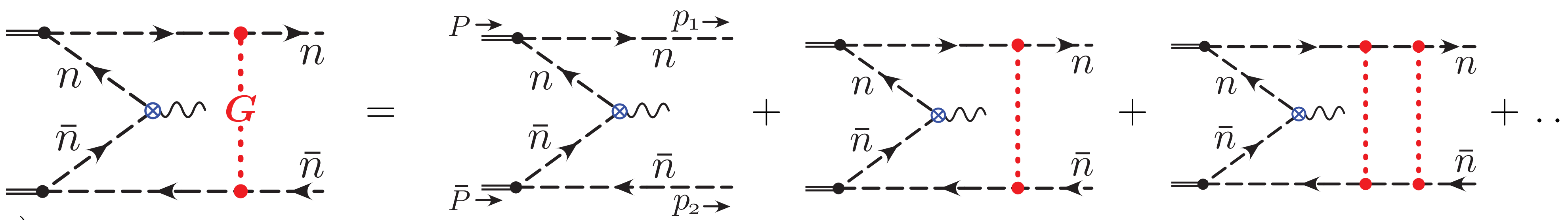
- can absorb this Glauber into the Collinear Wilson line with physical directions (note: connection to eikonalization)

$$J_\Gamma = (\bar{\xi}_n W_n) S_n^\dagger \Gamma S_{\bar{n}} (W_{\bar{n}}^\dagger \xi_{\bar{n}})$$

Spectator Scattering



no analogous soft or collinear diagrams at leading power



$$= -\mathcal{S}^\gamma \int \bar{d}^{d-2} k_\perp \, G(k_\perp)\, E(p_{1\perp} + k_\perp, p_{2\perp} - k_\perp)$$

$G(k_\perp) = \text{Fourier Transform of } e^{i\phi}$

$$= -\mathcal{S}^\gamma \int \bar{d}^{d-2} k_\perp \, G(k_\perp)\, E\Big(k_\perp - \Delta p_\perp - \frac{q_\perp}{2}, \Delta p_\perp - k_\perp - \frac{q_\perp}{2}\Big)$$

$$q_\perp = -p_{1\perp} - p_{2\perp}$$

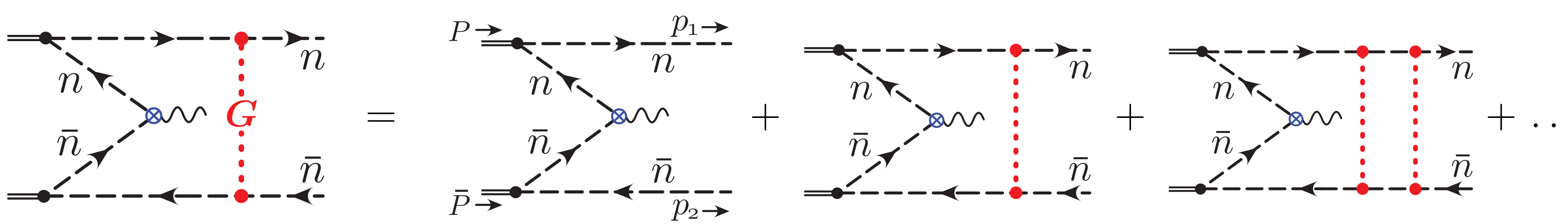
$$\Delta p_\perp = (p_{2\perp} - p_{1\perp})/2$$

$$\equiv -\mathcal{S}^\gamma \int \bar{d}^{d-2} k_\perp \, G(k_\perp)\, E'(\Delta p_\perp - k_\perp, q_\perp)$$

$$= -\mathcal{S}^\gamma \int \bar{d}^{d-2} k_\perp \int d^{d-2} b_\perp \, e^{-i\vec{k}_\perp \cdot \vec{b}_\perp} \, \tilde{G}(b_\perp) \int d^{d-2} b'_\perp \, e^{-i(\Delta\vec{p}_\perp - \vec{k}_\perp)\cdot \vec{b}'_\perp} \, \tilde{E}'(b'_\perp, q_\perp)$$

$$= -\mathcal{S}^\gamma \int d^{d-2} b_\perp \, e^{-i\Delta\vec{p}_\perp \cdot \vec{b}_\perp} \, \tilde{E}'(b_\perp, q_\perp)\, e^{i\phi(b_\perp)} \equiv \mathcal{A}_{SS}(\Delta p_\perp, q_\perp)$$

Spectator Scattering



$$q_\perp = -p_{1\perp} - p_{2\perp}$$

$$\Delta p_\perp = (p_{2\perp} - p_{1\perp})/2$$

$$= -\mathcal{S}^\gamma \int d^{d-2}b_\perp \, e^{-i\Delta\vec{p}_\perp\cdot\vec{b}_\perp} \; \tilde{E}'(b_\perp, q_\perp)\; e^{i\phi(b_\perp)} \;\; \equiv \;\; \mathcal{A}_{SS}(\Delta p_\perp, q_\perp)$$

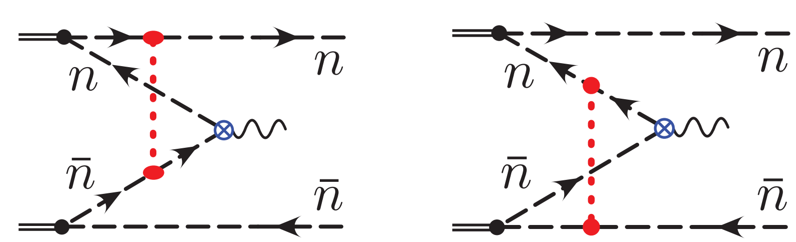
phase cancels IF we integrate over $\Delta p_\perp$

$$\int \bar{d}^{d-2}\Delta p_\perp \, \big|\mathcal{A}_{SS}(\Delta p_\perp, q_\perp)\big|^2$$

$$= |\mathcal{S}^\gamma|^2 \int \bar{d}^{d-2}\Delta p_\perp \int d^{d-2}b_\perp \, d^{d-2}b'_\perp \; e^{i\Delta\vec{p}_\perp\cdot(\vec{b}'_\perp - \vec{b}_\perp)} \; \tilde{E}'(b_\perp, q_\perp)\tilde{E}'^\dagger(b'_\perp, q_\perp) \; e^{i\phi(b_\perp) - i\phi(b'_\perp)}$$

$$= |\mathcal{S}^\gamma|^2 \int d^{d-2}b_\perp \, \big|\tilde{E}'(b_\perp, q_\perp)\big|^2$$

$$= |\mathcal{S}^\gamma|^2 \int \bar{d}^{d-2}\Delta p_\perp \, \big|E'(\Delta p_\perp, q_\perp)\big|^2$$



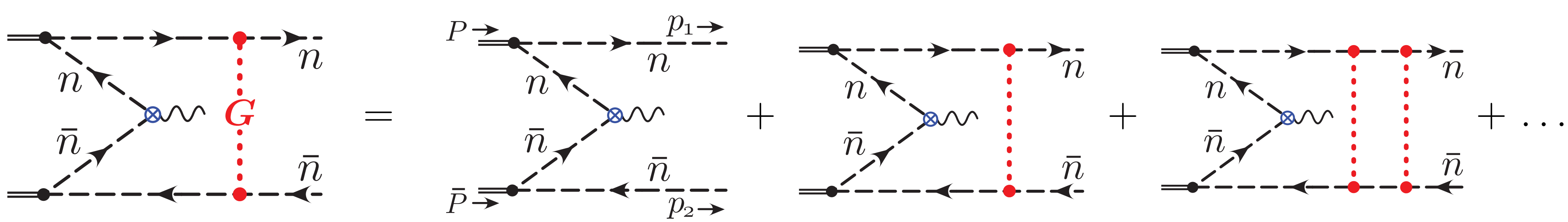
Active-Spectator and the Collinear Overlap

- cancel? $= \frac{1}{2} S^\gamma \int \bar{d}^{d'}k_\perp \, G^0(k_\perp) E(p_{1\perp} + k_\perp, p_{2\perp})$

now need to integrate over $p_{i\perp}$

(importance of Wilson line directions for TMDs)

Spectator Scattering



$$q_\perp = -p_{1\perp} - p_{2\perp}$$

$$\Delta p_\perp = (p_{2\perp} - p_{1\perp})/2$$

cancel IF we integrate over $\Delta p_\perp$

Measurements (like beam thrust & transverse thrust) that disrupt this integration can cause a non-cancellation.

(Gaunt; Zeng)

Single t-scale SCET:

$\Delta p_\perp \sim \Lambda_{\mathrm{QCD}} \ll \mathcal{T}$ — cancel as in inclusive DY, up to power corrections — $\dfrac{\Lambda_{\mathrm{QCD}}}{\mathcal{T}} \ll 1$

(Aybat & Sterman)

$\Delta p_\perp \sim \mathcal{T}, \sqrt{Q\mathcal{T}}$ — starts at $\mathcal{O}(\alpha_s^4)$, calculable factorization violation $(\mathcal{I}\mathcal{I}) \otimes f \otimes f$

(cf. Gaunt; Zeng)

Need a multi t-scale SCET for most interesting effects

Summary

- Promising new method to measure Top Quark Mass
- EFT formalism for $s \gg t$, Fwd. Scattering & Fact. Violation
- Universal Operators that can be used for many processes & problems
- Reggeization, BFKL, Shockwave picture, S-G & C-G overlaps, …

Future Directions

- More pT bins, NNLL, fits , combine SoftDrop & no SoftDrop, …
- pp Monte Carlo calibration
- Joint DGLAP(μ) and BFKL(ν) resummation for small-x
- Study and prove or disprove factorization for less inclusive processes
- Improve theoretical description of Underlying Event
- ….