Bay Area Particle Theory Seminar

March 13, 2015

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Non-Abelian Strings in Yang-Mills

★ Abrikosov strings (1950s);

★ Nielsen-Olesen (1973);

★ ★ Cosmic strings (Kibble 1970s, Witten 1985);

★ ★ ★ Non-Abelian strings in susy Yang-Mills (2003):

Bulk G×G→CF locking→(G_{diag}→H)→G/H coset model on the world sheet → (susy in bulk→susy on ws);

★ ★ ★ ★ Non-Abelin strings in CM phenomena

★ Hanany-Tong, 2003

★ ★ Auzzi et al., 2003

★ ★ ★ Shifman-Yung, 2003 - …

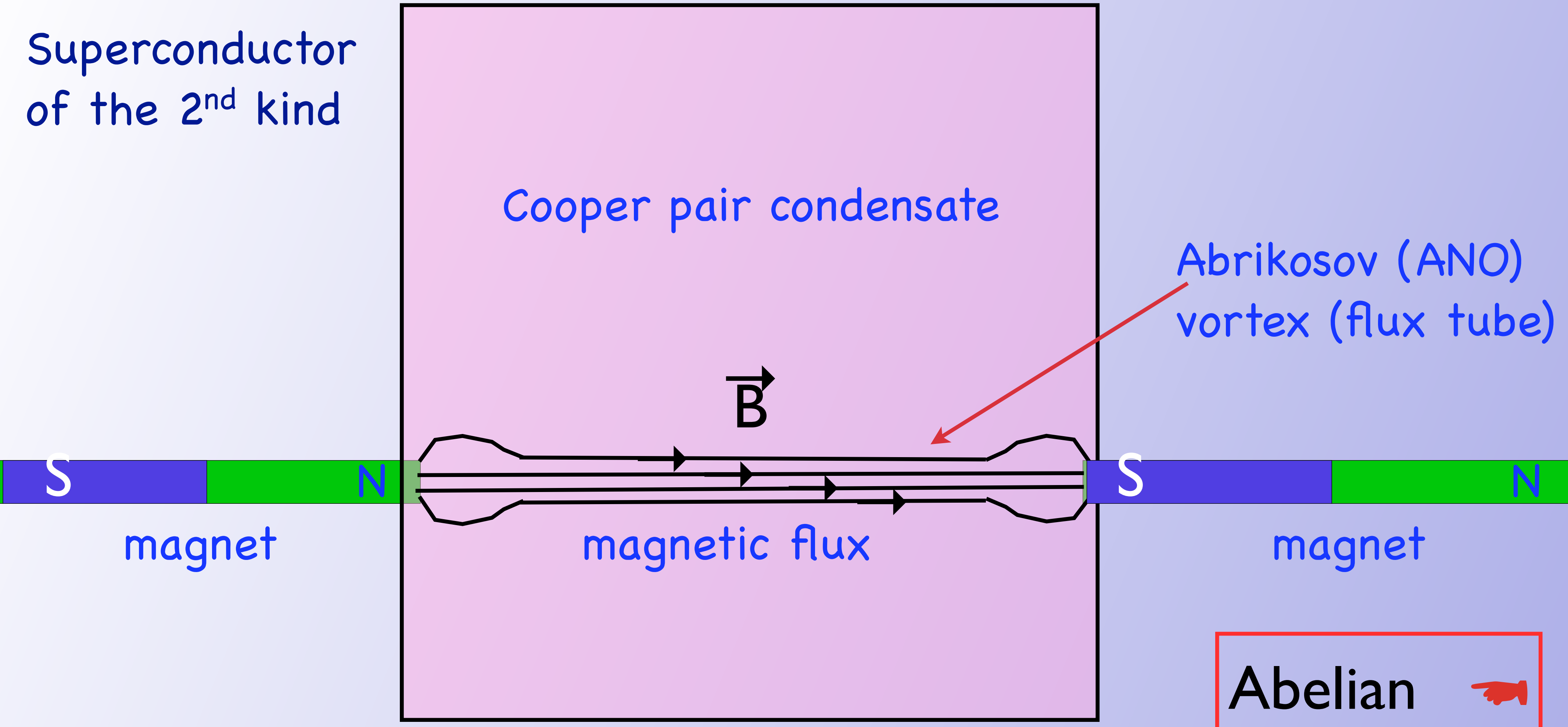
- Nitta-MS-Vinci (2013)
- MS-Yung (2013)
- Peterson-MS (2014)
- "Simple Models with Non-Abelian Moduli on Topological Defects," MS (2012)

SUSY

CM Applications

☞ The Meissner effect: 1930s, 1960s

Superconductor
of the 2nd kind



Cooper pair condensate

Abrikosov (ANO)
vortex (flux tube)

magnet

magnetic flux

magnet

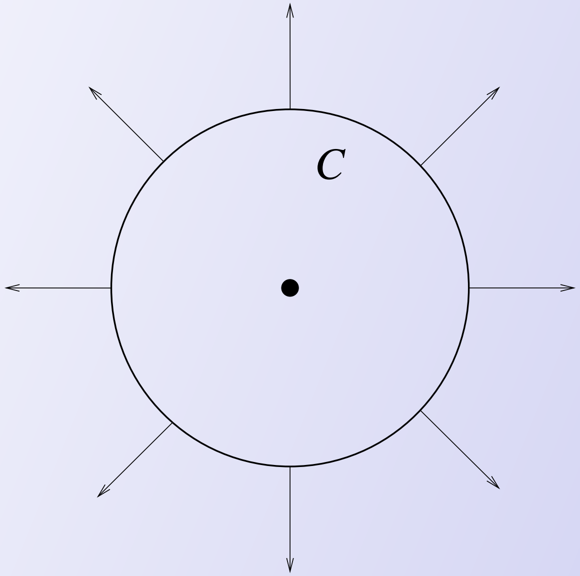
Abelian ☚

Abrikosov vortices; Type II superconductivity

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu}^2 + |\mathcal{D}^\mu \phi|^2 - U(\phi)$$

φ complex

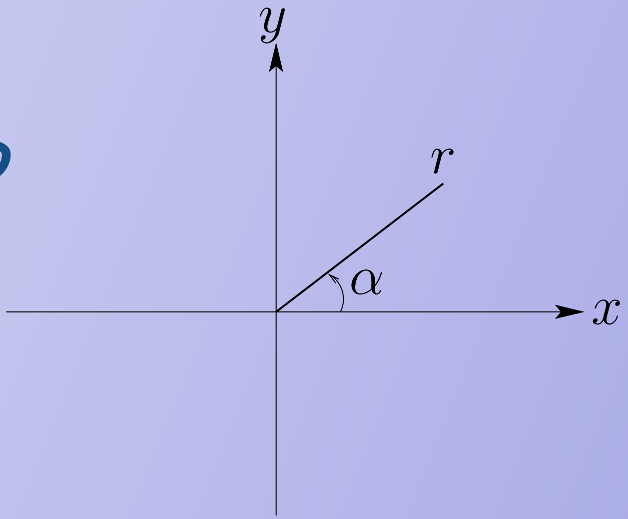
$$U(\phi) = \lambda \left(|\phi|^2 - v^2\right)^2$$



$\varphi = ve^{i\alpha}$ at $r \to \infty$

or

$\varphi = ve^{i(n)\alpha}$



$\int d^2x_\perp \ |\partial_\mu \ \varphi|^2 \to \infty$ logarithmically;

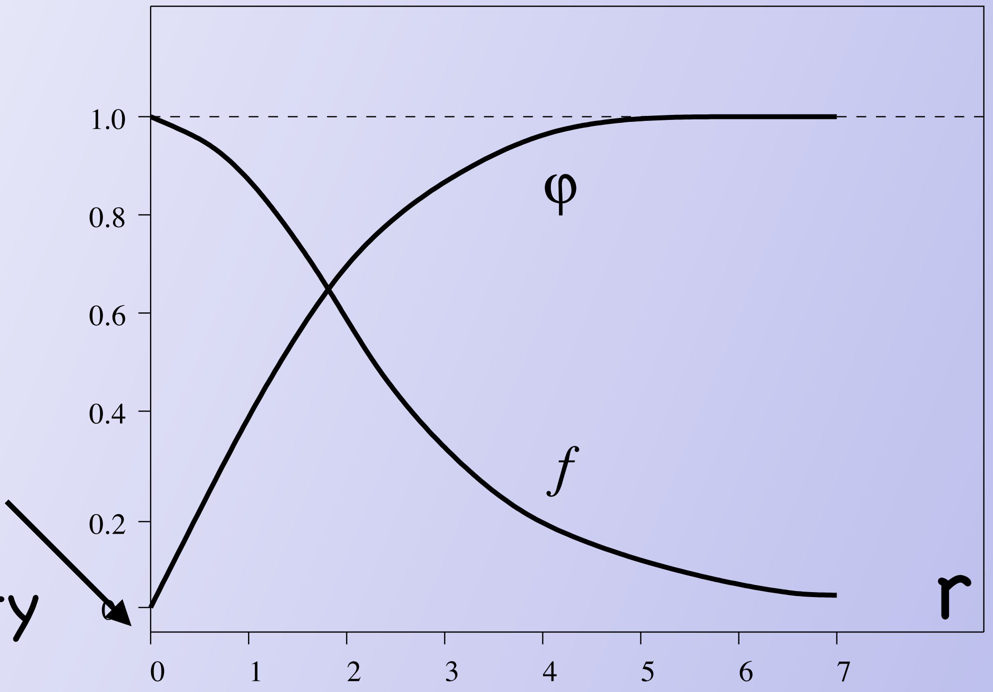
Hence,

$A_i = - \varepsilon_{ij} x_j / r^2 \to 0 \Leftarrow$ Pure Gauge

For arbitrary r

$$\phi(x) = v\,\varphi(r) e^{i\,\alpha}\,, \qquad A_i(x) = -\frac{1}{n_e}\varepsilon_{ij}\,\frac{x_j}{r^2}\,\left[1 - f(r)\right]$$

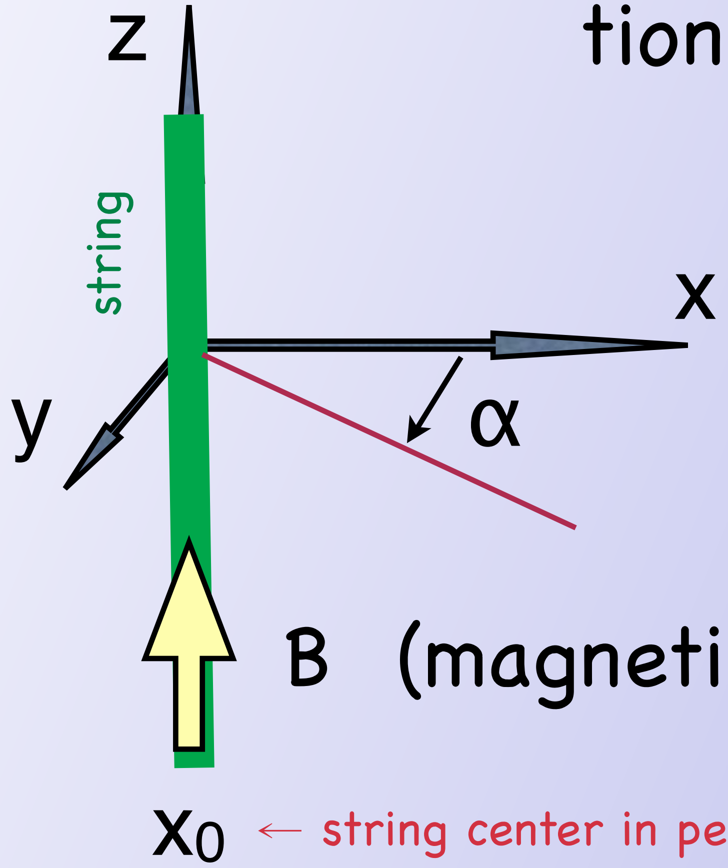
No superconductivity



No B

$\pi_1(U(1)) = Z$

massless (gapless) excitations correspond to oscilations in the perpendicular plane (translational zero modes)



B (magnetic field)

Flux quantized

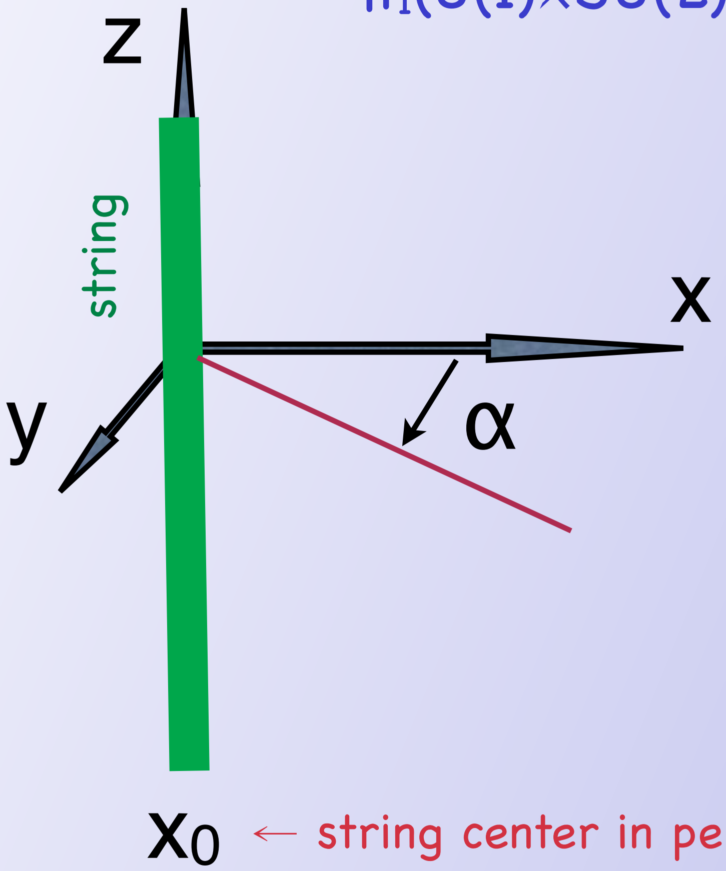
x_0 ← string center in perp. plane

- ★ ANO strings are there because of U(1)!
- ★ New strings:

$\pi_1(SU(2)\times U(1)) = Z_2$: **rotate by π around 3-d axis in SU(2)**

→ **-1; another -1 rotate by π in U(1)**

$\pi_1(U(1)\times SU(2))$ nontrivial due to Z_2 center of SU(2)



← string center in perp. plane

ANO $$\sqrt{\xi}\, e^{i\alpha} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

T=4πξ

Non-Abelian $$\sqrt{\xi} \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix}$$

$T_{U(1)} \pm T^3{}_{SU(2)}$

T=2πξ

SU(2)/U(1) ←orientational moduli; O(3) σ model

Prototype model

$$\begin{aligned} S \;&=\; \int \mathrm{d}^4x \left\{ \frac{1}{4g_2^2}\left(F^a_{\mu\nu}\right)^2 + \frac{1}{4g_1^2}\left(F_{\mu\nu}\right)^2 + \frac{1}{g_2^2}\left|D_\mu a^a\right|^2 \right. \\ &+\; \mathrm{Tr}\left(\nabla_\mu\Phi\right)^\dagger\left(\nabla^\mu\Phi\right) + \frac{g_2^2}{2}\left[\mathrm{Tr}\left(\Phi^\dagger T^a\Phi\right)\right]^2 + \frac{g_1^2}{8}\left[\mathrm{Tr}\left(\Phi^\dagger\Phi\right) - N\xi\right]^2 \\ &+\; \left. \boxed{\frac{1}{2}\mathrm{Tr}\left|a^aT^a\,\Phi + \Phi\,\sqrt{2}M\right|^2} + \frac{i\,\theta}{32\,\pi^2}\,F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu} \right\}, \end{aligned}$$

$$\Phi = \begin{pmatrix} \varphi^{11} & \varphi^{12} \\ \varphi^{21} & \varphi^{22} \end{pmatrix}$$

$$M = \begin{pmatrix} m & 0 \\ 0 & -m \end{pmatrix}$$

U(2) gauge group, 2 flavors of (scalar) quarks

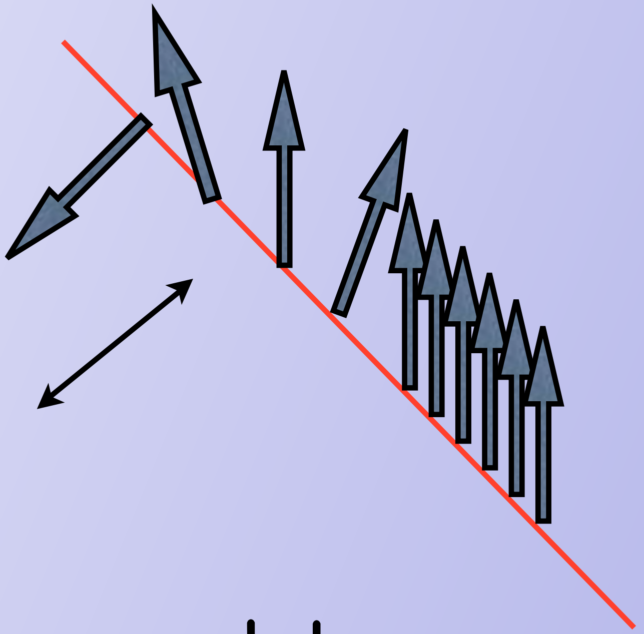
SU(2) Gluons A^a_μ + U(1) photon + gluinos+ photino

$$\Phi = \sqrt{\xi} \times I$$

Basic idea:

- Color-flavor locking in the bulk → Global symmetry G;
- G is broken down to H on the given string;
- G/H coset; G/H sigma model on the world sheet.

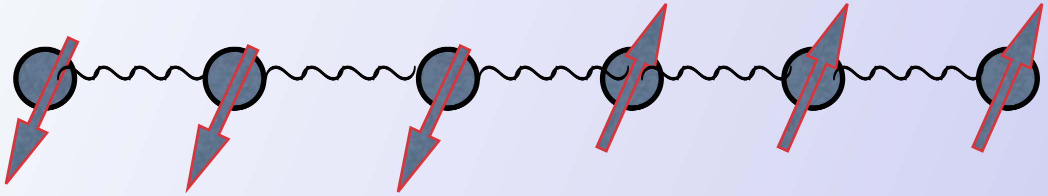
"Non-Abelian" string is formed if all non-Abelian degrees of freedom participate in dynamics at the scale of string formation

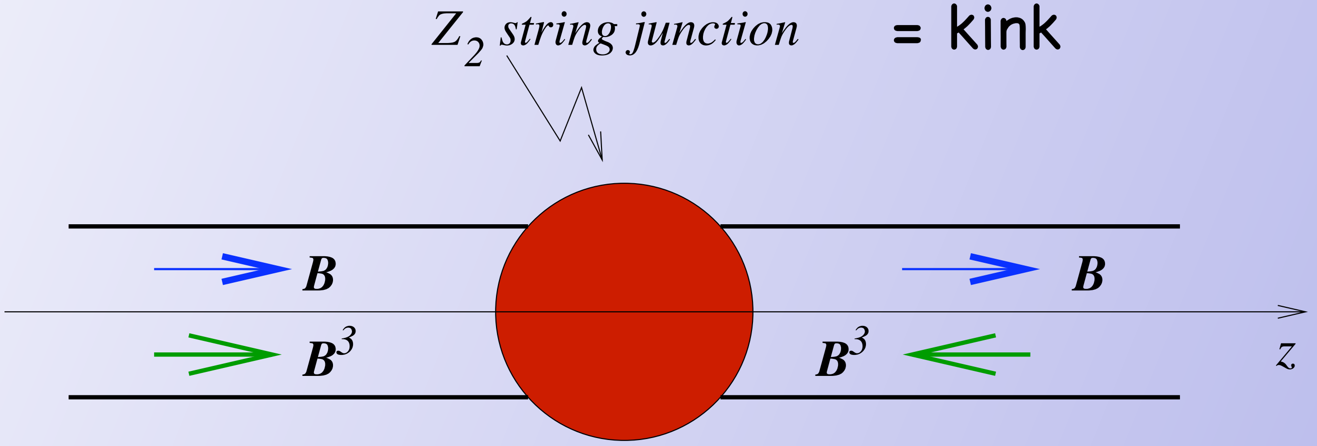


classically gapless excitation

SU(2)/U(1) = CP(1)~O(3) sigma model

$$S=\int d^2x\left\{\frac{2}{g^2}\,\frac{\partial_\mu\bar{\phi}\,\partial^\mu\phi-(\Delta m)^2\bar{\phi}\phi}{(1+\bar{\phi}\phi)^2}+fermions\right\}$$

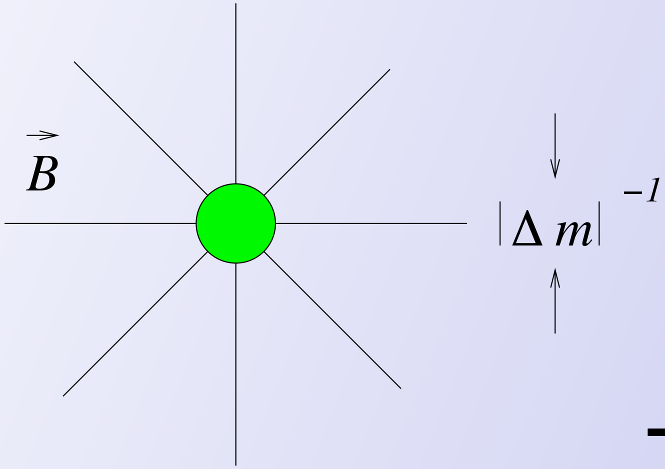




Evolution in dimensionless parameter m^2/ξ

$\xi = 0$

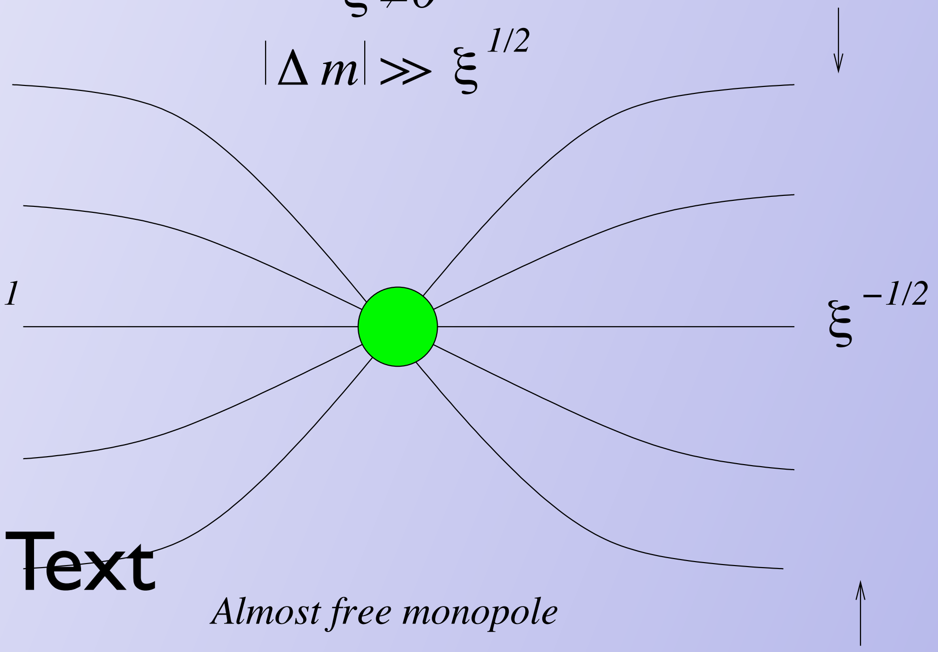
$\Delta m \neq 0$



The 't Hooft−Polyakov monopole

$\xi \neq 0$

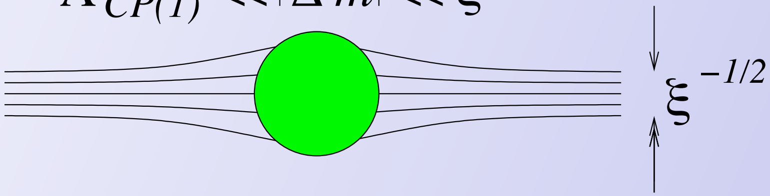
$|\Delta m| \gg \xi^{1/2}$



Text

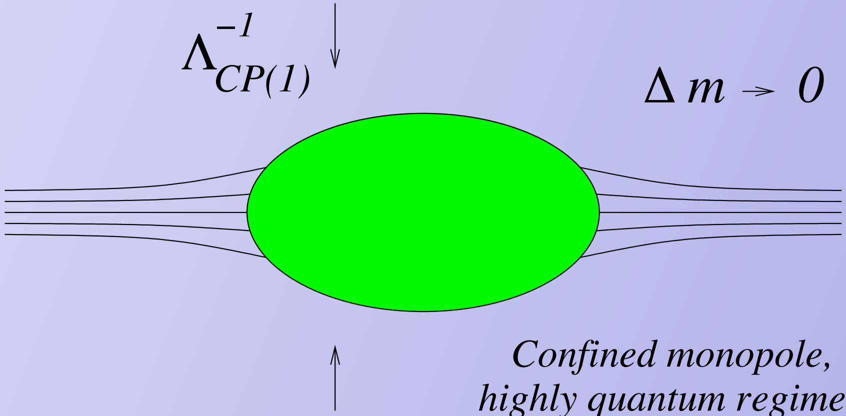
Almost free monopole

$\Lambda_{CP(1)} \ll |\Delta m| \ll \xi^{1/2}$



Confined monopole, quasiclassical regime

$\Delta m \to 0$



Confined monopole, highly quantum regime

- ✳ Kinks are confined in 4D (attached to strings).
- ✳ ✳ Kinks are confined in 2D:

Kink = Confined Monopole

4D ↔ 2D Correspondence

☛ World-sheet theory ↔ strongly coupled bulk theory inside

Dewar flask

World-sheet models on non-Abelian Strings

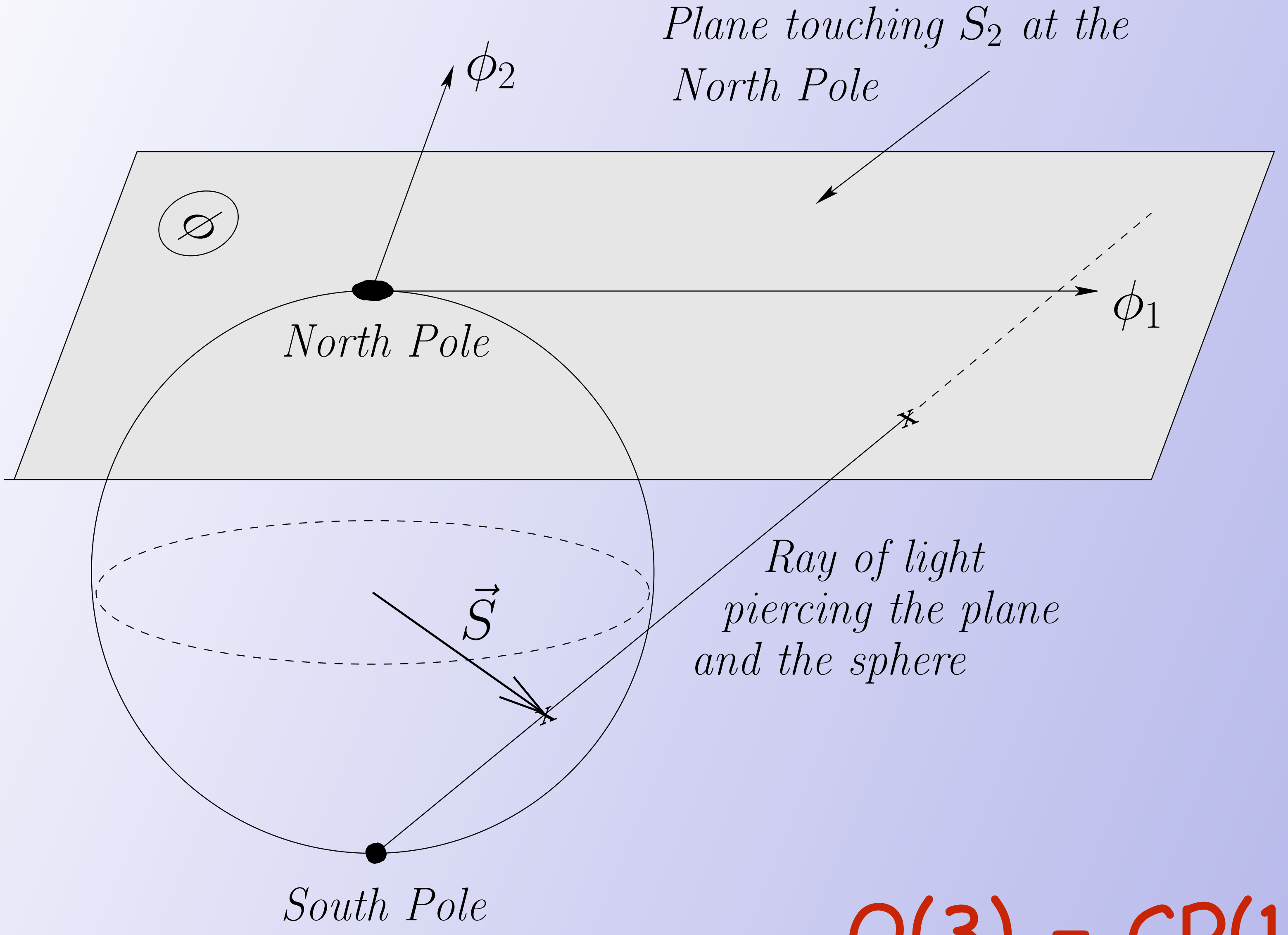
Bulk Theory parameters

↓

SU(N)×U(1), N_f, (S)quark masses

↑ ↑ ↑

Geometric vs Gauged descriptions



O(3) = CP(1)

1D complex plane (in general, N-1 dim)

Geometric/Sigma models on Kählerian target spaces

- ✷ $L = G_{\bar{A}B}\, \partial_\mu \bar{\Phi}^{\bar{A}}\, \partial^\mu \Phi^B$, $\mu = 0, z$ or $1,2$

$$G_{\bar{A}B} = (2/g^2)\, (\partial_{\bar{\Phi}}\, \partial_\Phi) \boxed{\log \Sigma_A\, (1 + \bar{\Phi}^{\bar{A}} \Phi^A)} + \ldots$$

$\underline{CP(N-1) \leftrightarrow SU(N)/SU(N-1)\times U(1)}$

$$R_{\bar{A}B} = G_{\bar{A}B} \times (Ng^2/2)$$

AF, IR strongly coupled

Kähler potential

Gauged/Sigma models on Kählerian target spaces

↓

n^i, $\bar{n}_i$, $i=1,2,\dots, N$ ← 2N rdof

$n^i\bar{n}_i = 1$, ← 2N - 1 rdof

U(1) gauging

2N - 2 rdof

$$\mathcal{L} = \frac{2}{g^2}\left|\mathcal{D}_\mu n^i\right|^2$$

$$\mathcal{D}_\mu n^i \equiv \left(\partial_\mu + iA_\mu\right) n^i$$

$$\mathcal{L}_\theta = \frac{\theta}{2\pi}\,\varepsilon^{\mu\nu}\partial_\mu A_\nu$$

Gauged/Ideal for large-N solution:

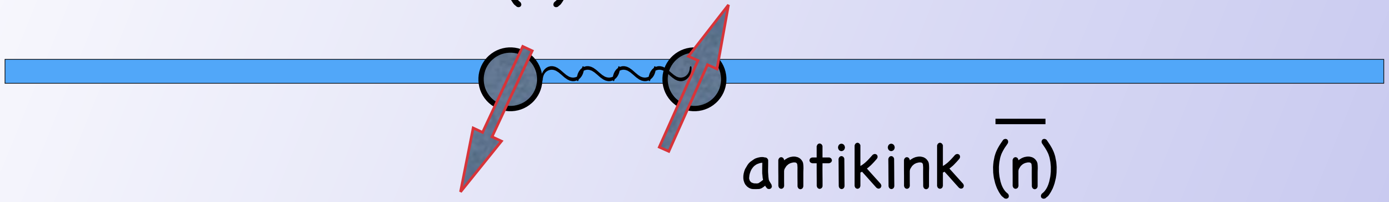
$$\mathcal{L} = | (\partial_\mu + i A_\mu) n^i |^2 + (\bar{n}_i n^i - 2\beta) D$$

$\beta=1/g^2$

quadratic in n

auxiliary

$$V(D) = (N/4\pi) D \log (D/e \Lambda^2) \longrightarrow D=\Lambda^2$$



kink (n)

antikink ($\bar{n}$)

2D confinement

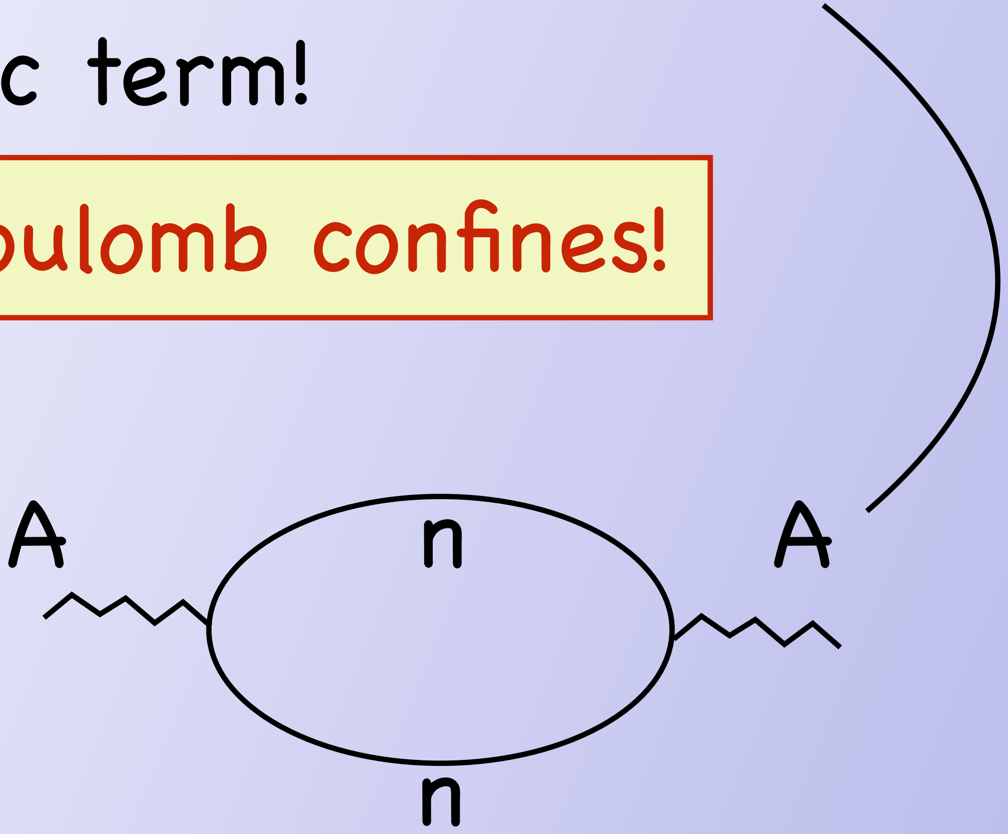
Impact:

$D=\Lambda^2 \longrightarrow$ mass generation

N-1 n fields $\longrightarrow$ N fields

Generation of $-(1/4e^2)\, F_{\mu\nu} F^{\mu\nu}$, i.e.
γ kinetic term!

In 2D Coulomb confines!



WCP(N,M)

$[N_f = 2N]$

$$\mathcal{L}_{\rm WCP}\text{[redacted]}\,(N,M)\text{[redacted]} = |\nabla_\mu n_i|^2 + |\tilde{\nabla}_\mu \rho_j|^2 \,.$$

$$\nabla_\mu n_i = (\partial_\mu - iA_\mu) n_i , \quad \tilde{\nabla}_\mu \rho_j = (\partial_\mu + iA_\mu) \rho_j$$

$$|n|^2 = 2\beta + |\rho|^2$$

$N = M \longrightarrow$ Conformal / Remember $N=M=2$

Ricci flat CY // 10D critical string

★ ★ ★ ★ $\mathcal{N}=2 \to \mathcal{N}=(2,2)$

$\mathcal{N}=1 \to \mathcal{N}=(2,0)$ nonminimal

(minimal)

No 2D confinement

$\mathcal{N}=0 \to \mathcal{N}=0$

bulk

2D world sheet

$\mathcal{N}$ =(2,2)

$$\mathcal{L}_{\text{ferm}} = G_{\bar{A}B}\, \bar{\Psi}^{\bar{A}}\, i \not{\nabla}\, \Psi^{B} + \text{Riemann}\ \bar{\Psi}\,\Psi\,\bar{\Psi}\,\Psi$$

$$\mathcal{L}_{\text{ferm}} = G_{\bar{A}B}\, \bar{\Psi}^{\bar{A}}\, i \not{\nabla}\, \Psi^{B}$$

minimal $\mathcal{N}$ =(0,2)

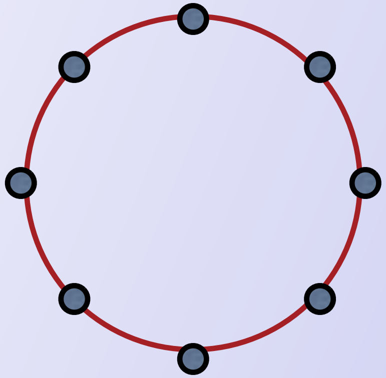
~~R… R~~

$$\Delta\, \mathcal{L}_{\text{ferm}} = \gamma R_{\bar{A}B}\, \bar{\Psi}^{\bar{A}}_{R}\ \ i\partial_{L}\varphi^{B} \boxed{\zeta_{R}}$$

$\mathcal{N}$ =(0,2) nonminimal

extra right-handed field

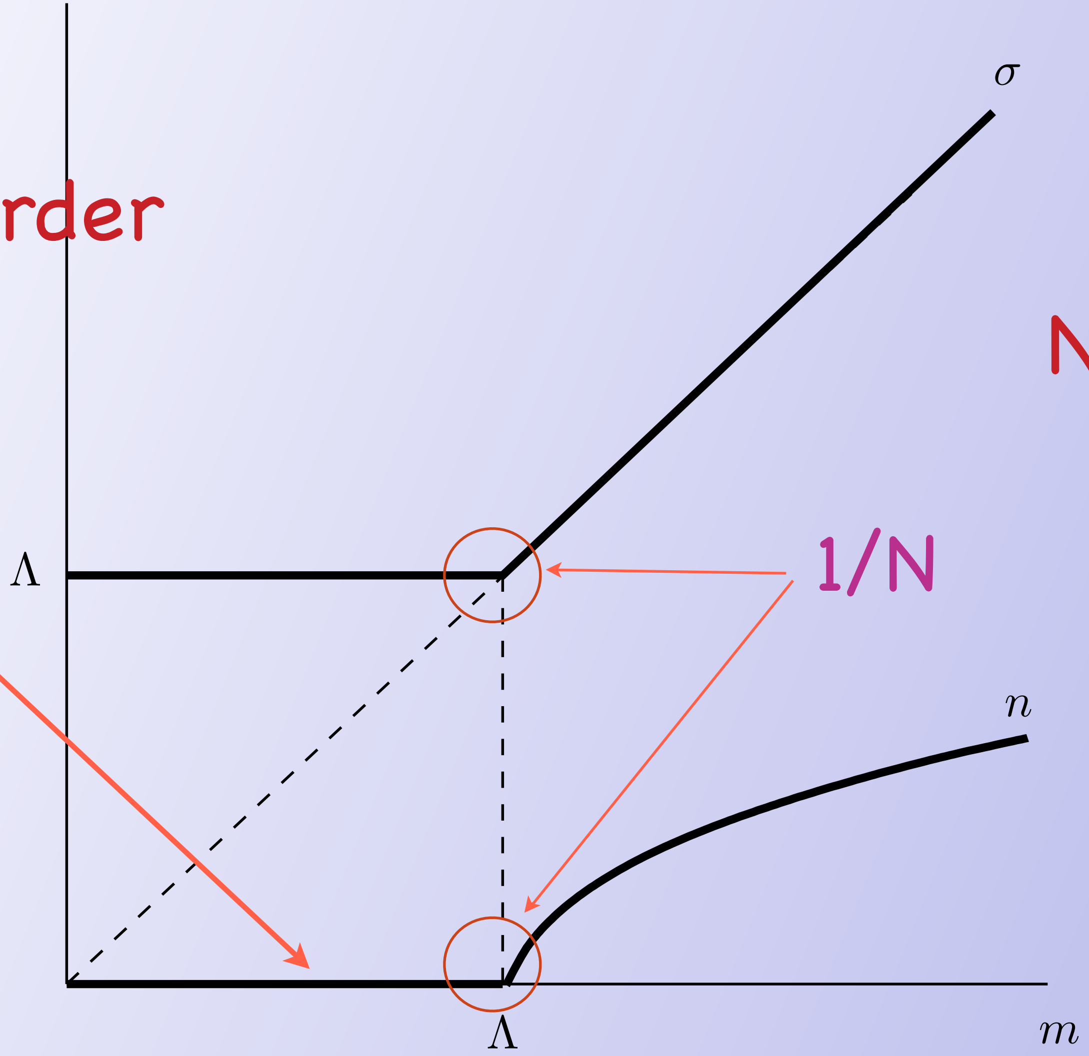
Twisted Masses breaking SU(N) but Z_N symmetry conserved



$$m = m_0 \left(e^{2\pi i/N},\ e^{4\pi i/N},\ \ldots,\ e^{2(N-1)\pi i/N},\ 1 \right)$$

m_0 real positive

Bifermion order parameter



No confinement

1/N

$E_{vac}=0$ always, SUSY unbroken,

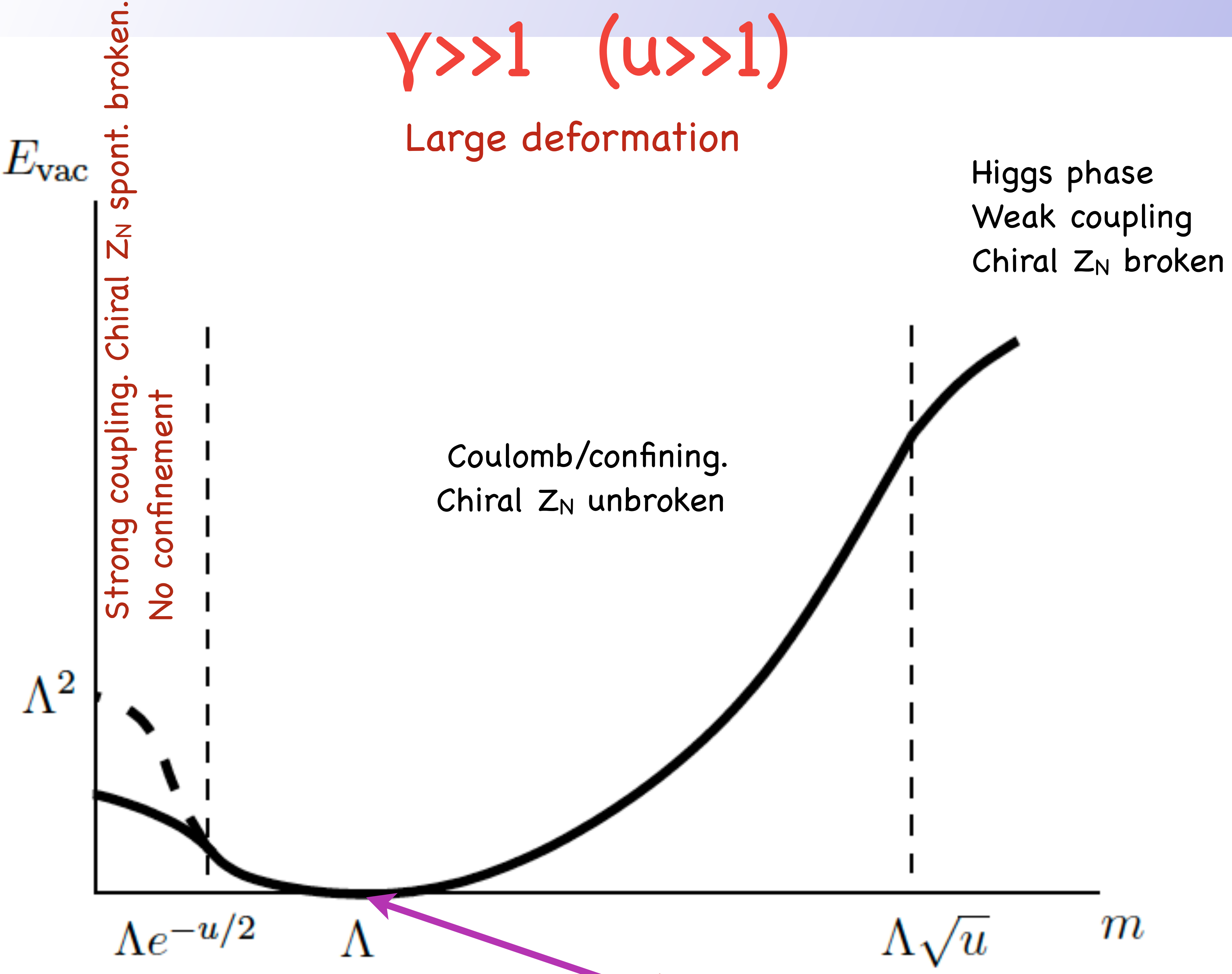
Z_N always broken, (N degenerate vacua)

Crossover instead of phase transition

Strong-coupling ↔ weak coupling Higgs regime

γ>>1 (u>>1)

Large deformation



Strong coupling. Chiral Z_N spont. broken.

No confinement

Coulomb/confining.
Chiral Z_N unbroken

Higgs phase
Weak coupling
Chiral Z_N broken

SUSY restored here

Conclusions

★Non-Abelian strings in SUSY bulk → CP(N-1) models (heterotic & nonheterotic) on string; a wealth of phase transitions;

★2D ↔ 4D Correspondence;

★A treasure trove of novel 2D models with intriguing dynamics.

The simplest model with non-Abelian moduli

Extra term to be added:

$$\mathcal{L}_\chi = \partial_\mu \chi^i \, \partial^\mu \chi^i - U(\chi, \phi)\,,$$

$$U = \gamma \left[\left(-\mu^2 + |\phi|^2 \right) \chi^i \chi^i + \beta \left(\chi^i \chi^i \right)^2 \right]$$

$$+ \; \lambda \left(|\phi|^2 - v^2 \right)^2$$

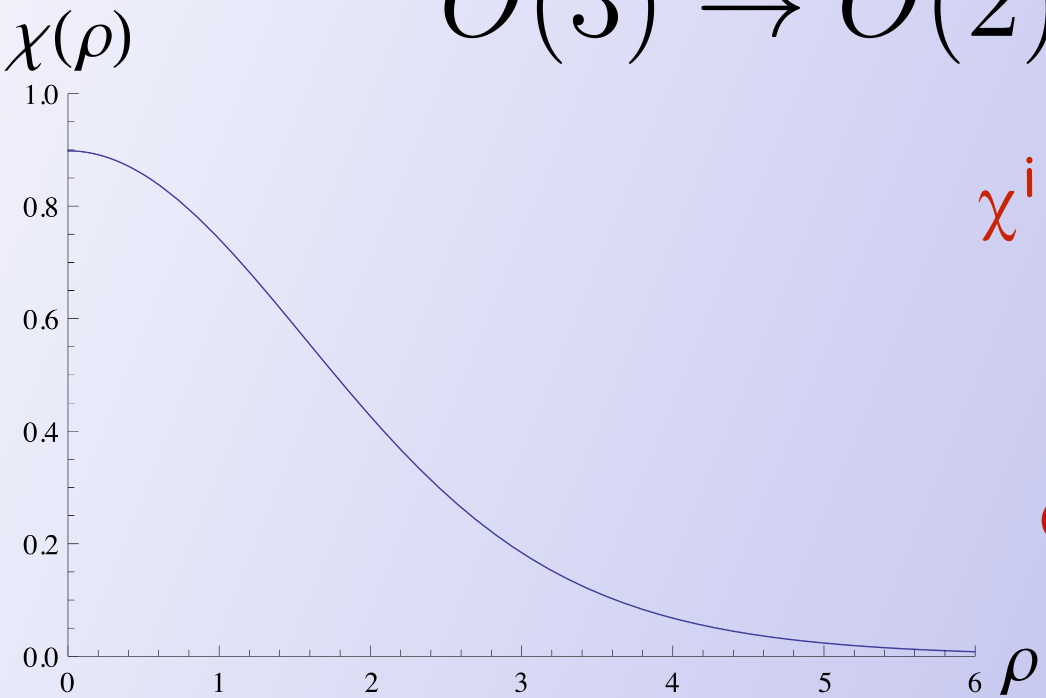
$$\boxed{\mu < v}$$

i=1,2,3 χ^i real

In the vacuum $\varphi = v$, $\chi = 0$

In the string core $\varphi = 0$, $\chi^2 = \mu^2/2\beta \neq 0$

$$O(3) \to O(2)$$



$\chi^i = (\mu^2/2\beta)^{1/2}\, S^i$

Classically 2 gapless modes

$$\mathcal{L}_{\rm sws} = \frac{1}{\beta}\left(\partial^a S^i\right)\left(\partial_a S^i\right), \qquad S^i S^i = 1\,, \qquad a = 0, 3\,.$$

$$S_{\text{world sheet}} = (\mu^2/2\beta)\int d^2x\, (\partial_\mu S^i)\, (\partial_\mu S^i)$$

$$\boxed{S^i S^i = 1}$$

Clasically two “rotational” zero modes (or 1?).

Q Mechanically may be lifted

☞ Goldstone modes (gapless excitations) on strings

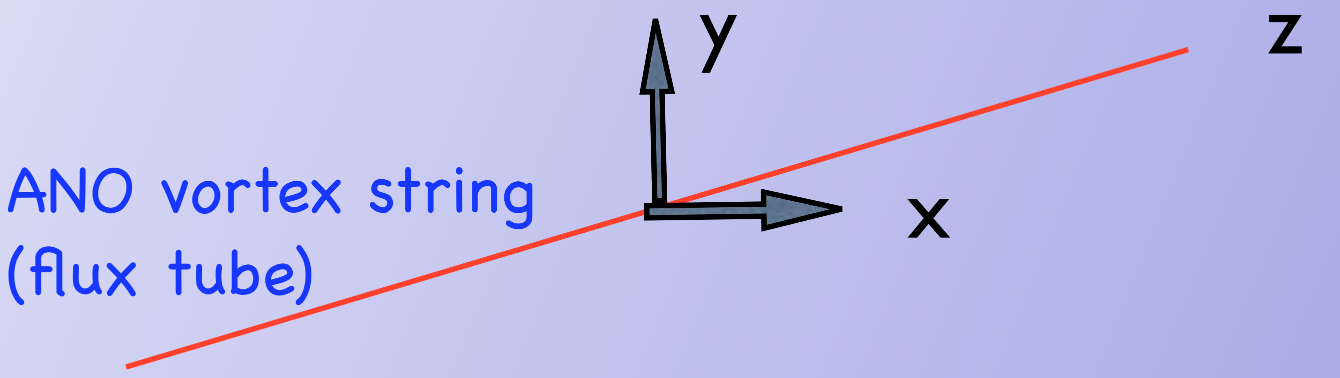
★ Internal symmetries $G \longrightarrow H$,

$$\nu_{\text{"rel"}} = \nu_G - \nu_H\,, \quad \nu_{\text{"non-rel"}} = (\nu_G - \nu_H)/2.$$

★ ★ This is not the case for geometric symmetries! (Ivanov-Ogievetsky, 1975; Low-Manohar, 2002)

★ ★ ★ E.g. (structureless) string breaks two translations and two rotations, but one should consider only translational zero modes!!!

M_{zx} & M_{zy} broken,
T_x & T_y broken .


ANO vortex string
(flux tube)

Time derivatives

$x\dot{x}$ nonrelativistic ☛ total derivative

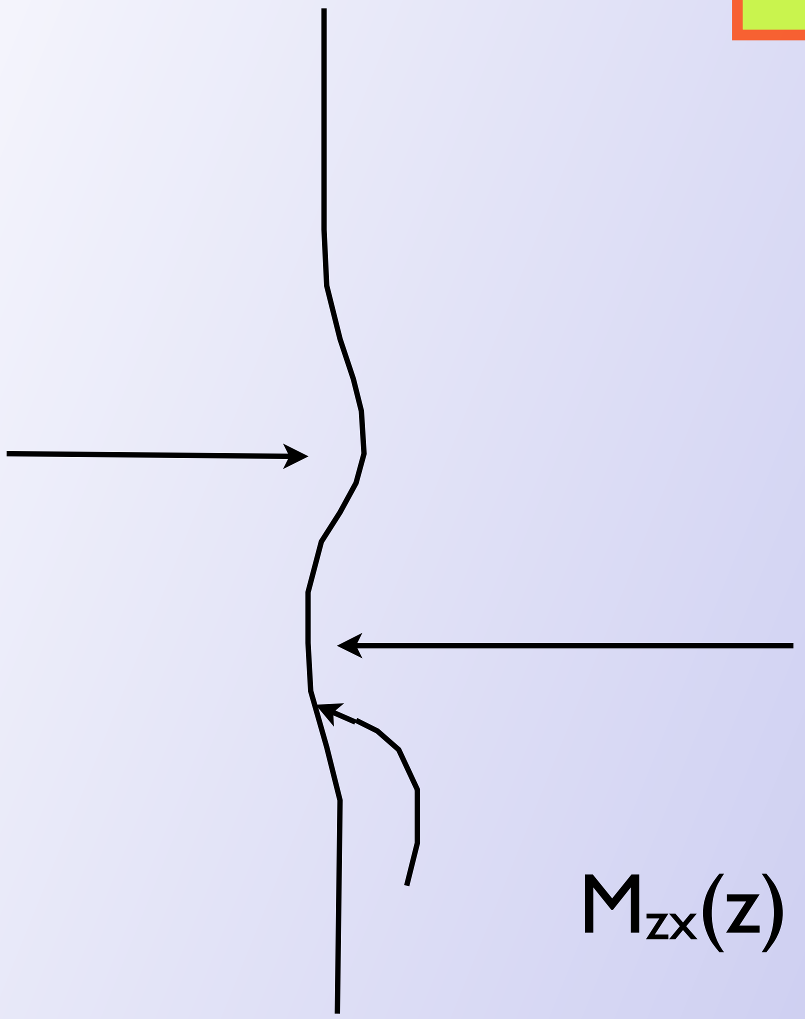
Hence, either

$\dot{x}\dot{x}$ relativistic or $y\dot{x}-x\dot{y}$

If x is canonic coordinate, y is canonic momentum,

$$[yx] = -i$$

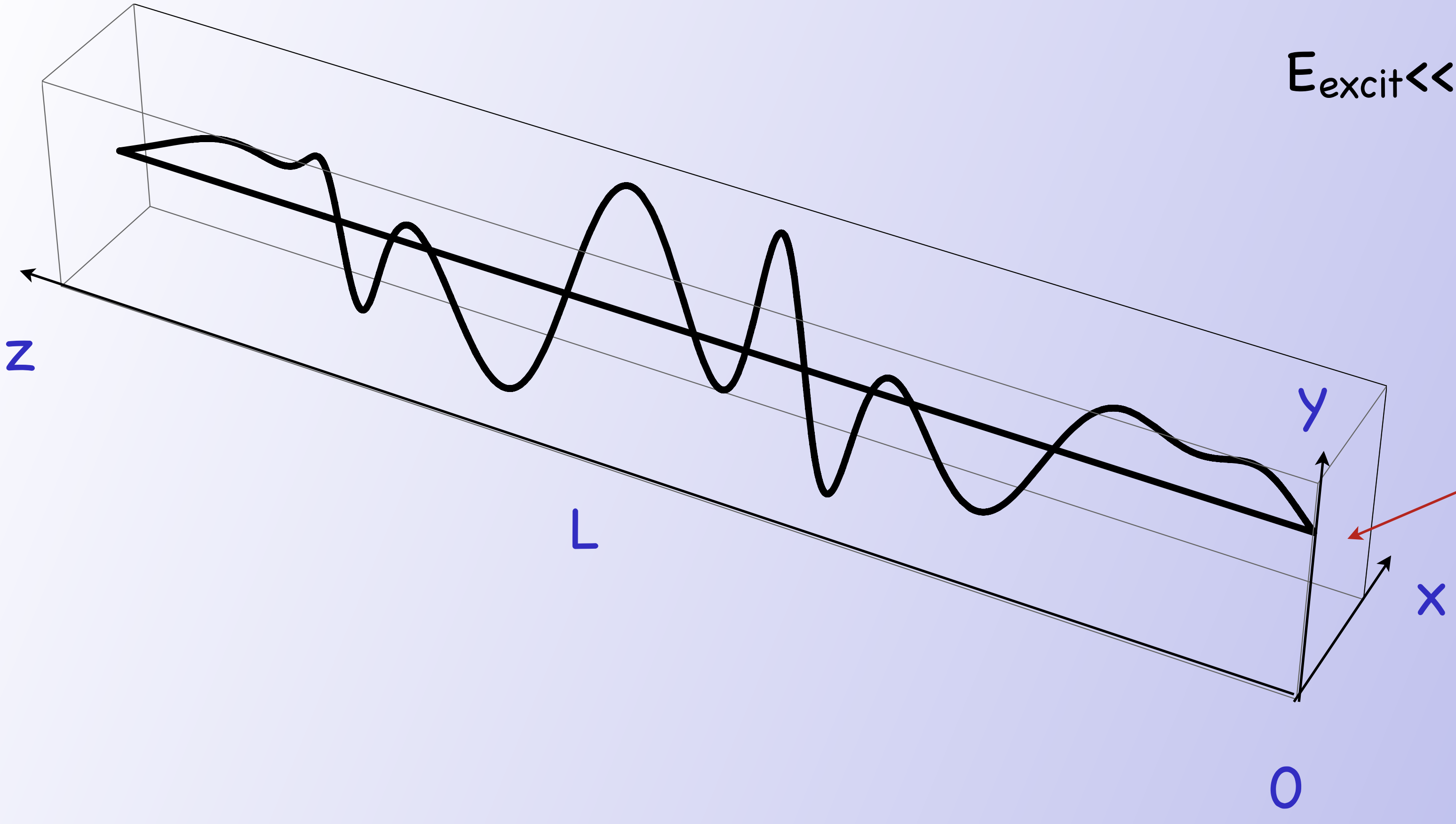
What if order parameter carries Lorentz indices?



$M_{zx}(z) \sim T_x$

Low-energy excitations (gapless modes) on vortices

$E_{\text{excit}} \ll m_\gamma \sim \text{ev}$



Perpendicular plane

◇◇ $\Delta H_{GL} = (T/2)(\vec{\partial}_z x_{\text{perp}}\, \vec{\partial}_z x_{\text{perp}}) + \text{h.d.}$ ➟ time derivatives can be linear.

or quadratic.

Nambu-Goto → String Theory

Kelvin modes or Kelvons

2 NG gapless modes in relat.

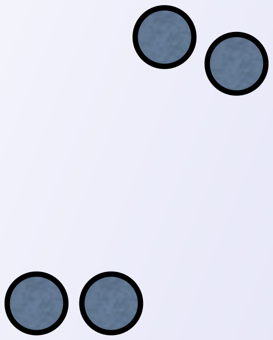
1 NG gapless mode in non-rel.

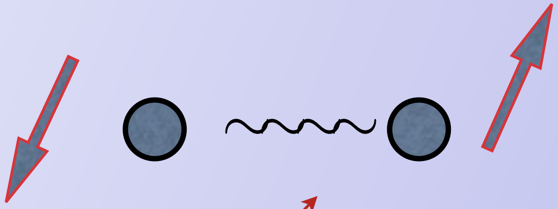
$E_{\text{str}} = TL + C/L$

Counts # of gapless modes !

3

^{3}He atoms





P-wave paring

Spin 1/2

L=1, S=1 ➡ Cooper pair order parameter $e_{\mu i}$ ← 3×3 matrix

Spin-orbit small, symmetry of H is $G = U(1)_p \times SO_S(3) \times SO_L(3)$

In the ground state $U_p(1) \times SO_S(3) \times SO_L(3) \to H_{\rm B} = SO(3)_{S+L}$

Hence, contrived NG modes in the bulk!

Spin-orbit interaction

With S. Monin

$$\Delta\,\mathcal{L}_\chi = -\,\varepsilon(\partial_i\chi^i)^2$$

$$\begin{aligned}
S &= \int dt\,dz\left(\mathcal{L}_{\mathrm{O}(3)} + \mathcal{L}_{x_\perp}\right),\\
\mathcal{L}_{\mathrm{O}(3)} &= \left\{\frac{1}{2g^2}\left[\left(\partial_k S^i\right)^2 - \varepsilon\left(\partial_z S^3\right)^2\right]\right\} - M^2\left(1-(S^3)^2\right)\\
\mathcal{L}_{x_\perp} &= \frac{T}{2}\,(\partial_k\vec{x}_\perp)^2 - M^2(S^3)^2\,(\partial_z\vec{x}_\perp)^2\\
&+\ 2M^2\left(S^3\right)\left(S^1\partial_z x_{1\perp} + S^2\partial_z x_{2\perp}\right),
\end{aligned}$$

Entanglement!

$M^2 \propto \varepsilon$

◇ Assume χ^i is spin field!

◇◇ Add $\Delta L = \varepsilon\, (\partial_i \chi^i)(\partial_k \chi^k)$

↓

✷ If $\varepsilon \to 0$, geometric symmetry is enhanced

Poincaré $\times$ O(3)

↓

✷✷ Two extra zero modes

What if $\varepsilon \neq 0$ but small?

$$\Delta_{CP(1)} S_{\text{world sheet}} = \varepsilon \int d^2x \, \{(\partial_z S^3)^2 - M^2[1-(S^3)^2]\}$$

$$\mathcal{L}_{x_\perp} = \frac{T}{2} (\partial_a \vec{x}_\perp)^2 - \tilde{M}^2 \left(S^3\right)^2 (\partial_z \vec{x}_\perp)^2 ,$$

★ EXTRA (quasi)gapless modes ★

★ ★ Translational (Kelvon) and orientational (spin) modes mix with each other ★ ★

Low energy dynamics of $U(1)$ vortices in systems with cholesteric vacuum structure 2014

With Peterson & Tallarita

$$\Delta\,\mathcal{L}_{\varepsilon} = -\eta\varepsilon_{ijk}\chi_i\partial_j\chi_k$$

P-odd, spin-orbit

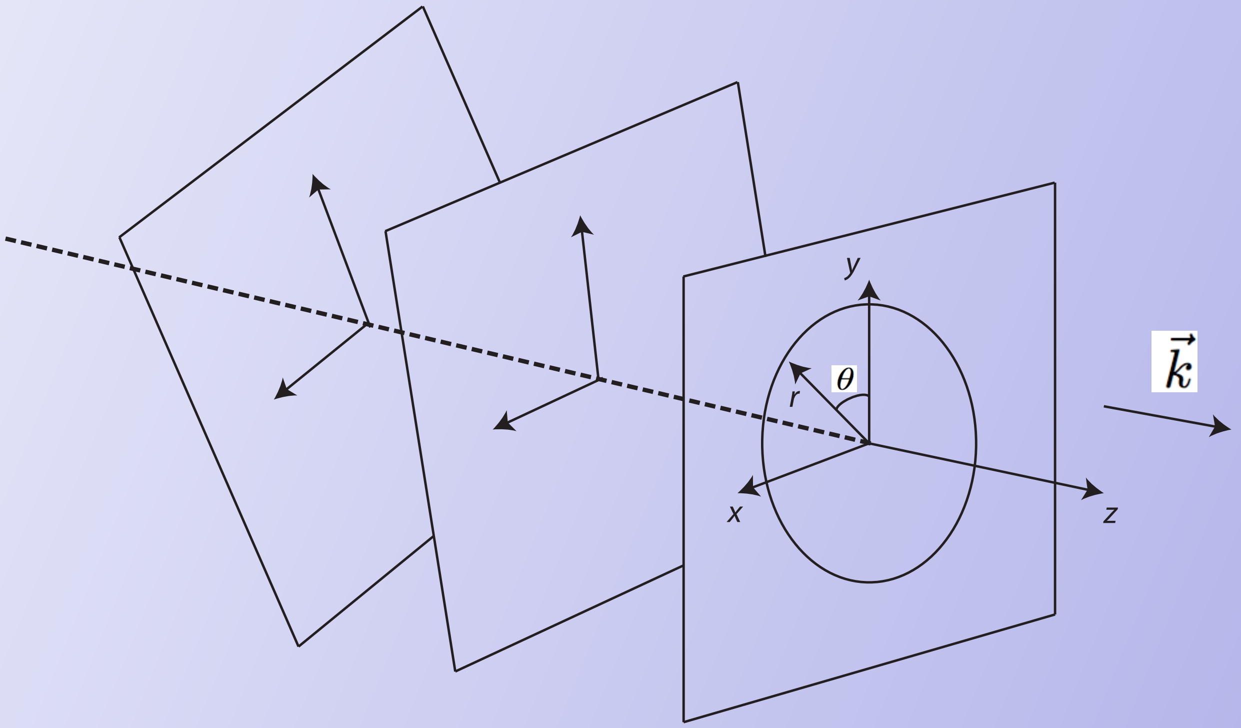
$$\chi_i \quad = \quad \chi_0\,\epsilon_i(z)\,, \qquad\qquad \chi_i = \frac{\mu}{\sqrt{2\beta}}\tilde{\chi}_0\,\epsilon_i(z)$$

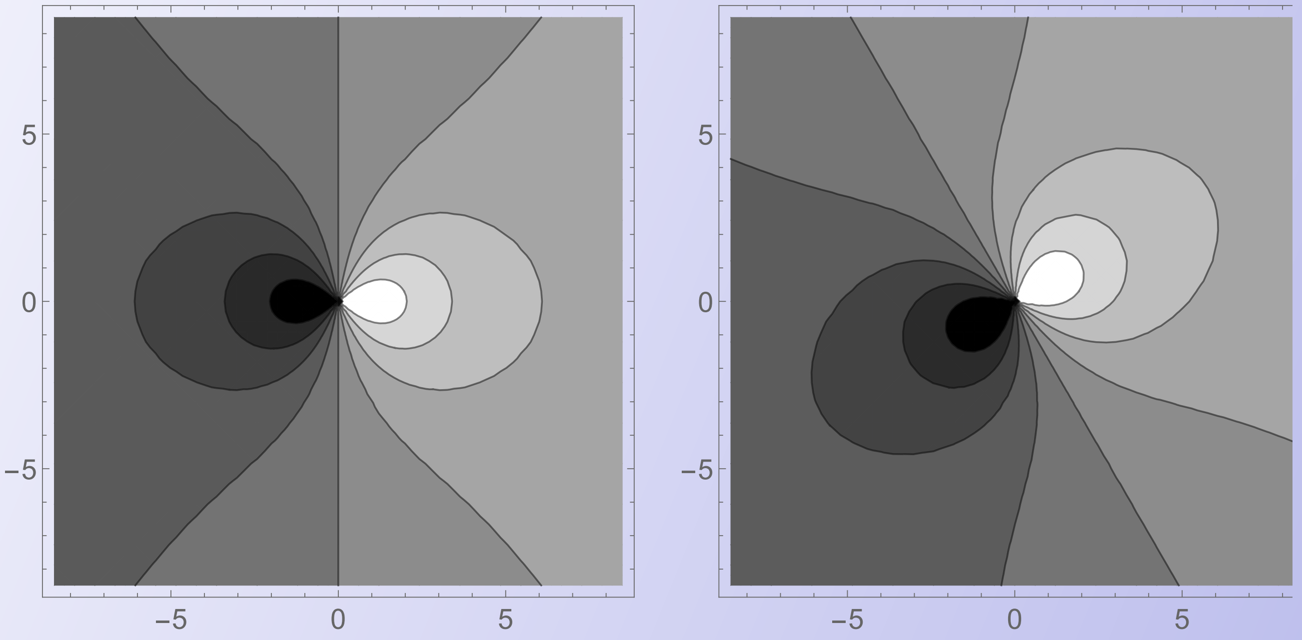
$$\vec{\epsilon}\,(z) \quad = \quad \Big\{\cos kz,\ \sin kz,\ 0\Big\}$$

Cholesteric vacuum
Liquid crystals

Goldstone mode associated to the broken translational symmetry in **z** direction. Namely if

$$\vec{\epsilon}(z) \to \vec{\epsilon}(x,y,z) = \left\{\, \cos\left(kz - k\xi(x,y,z)\right),\ \sin\left(kz - k\xi(x,y,z)\right),\ 0 \right\}$$





(a) χ^r, at $kz = 0$.

(b) χ^r at $kz = \pi/6$.