# *The muon $g-2$: A theoretical challenge*

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# Generalities

In a world with $\mathcal{P}$ symmetry, a fermion, mass $m_f$, $q = p' - p$

$$\langle f, p' | J^\mu(0) | f, p \rangle = \overline{u}(p') \left[ F_1(q^2)\gamma^\mu + \frac{i}{2\, m_f}\, F_2(q^2)\, \sigma^{\mu\nu} q_\nu \right] u(p)$$

Charge: $F_1(0) = 1$

Anomalous magnetic moment: $F_2(0) = \frac{(g-2)_f}{2} = a_f$

Dimensional analysis:

- Only $f$ in loops $\Rightarrow a_f$ indep. of mass and universal $f = e, \mu, \tau$.
- Mass $m << m_f \Rightarrow a_f \sim \log \frac{m_f}{m}$
- Mass $M >> m_f \Rightarrow a_f \sim \frac{m_f^2}{M^2} \log \frac{M}{m_f}$

# $g-2$ for electron, muon and tau

- The measurement of $a_\tau$ :

$$a_\tau^{EXP} = -0.018(17) \qquad \text{(DELPHI '04)}$$

is not very constraining. Compare with

$$a_\tau^{TH} = 117721(5) \times 10^{-8} \qquad \text{(Eidelman et al.'07)}$$

- There is a very good measurement of $a_e$ :

$$a_e^{EXP} = 1159652180.73(28) \times 10^{-12} \quad [0.24\ ppb] \qquad \text{(Hanneke et al. '11)}$$

but we need it to <u>define</u> $\alpha$ , (Aoyama et al. '12):

$$\alpha^{-1}(a_e) = 137.0359991727\ \underbrace{(68)}_{\alpha^4}\underbrace{(46)}_{\alpha^5}\ \underbrace{(26)}_{QCD+EW}\ \underbrace{(331)}_{exp} \quad [0.25\ ppb]$$

to be able to make predictions.

(Notice that the error due to QCD begins to show up).

# $g-2$ for electron, muon and tau

- Only then does $a_\mu$ become calculable in the SM, :

$$a_\mu^{\rm SM} = 11\ 659\ 182.8\ (4.9) \times 10^{-10} \quad (\text{Hagiwara et al.}'11)$$

and with

$$a_\mu^{EXP} = 11\ 659\ 208.9\ \underline{(6.3) \times 10^{-10}}\ [0.54\ ppm] \quad (\text{Bennett et al.}'06)$$

one gets

$$a_\mu^{EXP} - a_\mu^{SM} = 26.1\ (8.0) \times 10^{-10} \quad \underline{[3.3\ \sigma]}$$

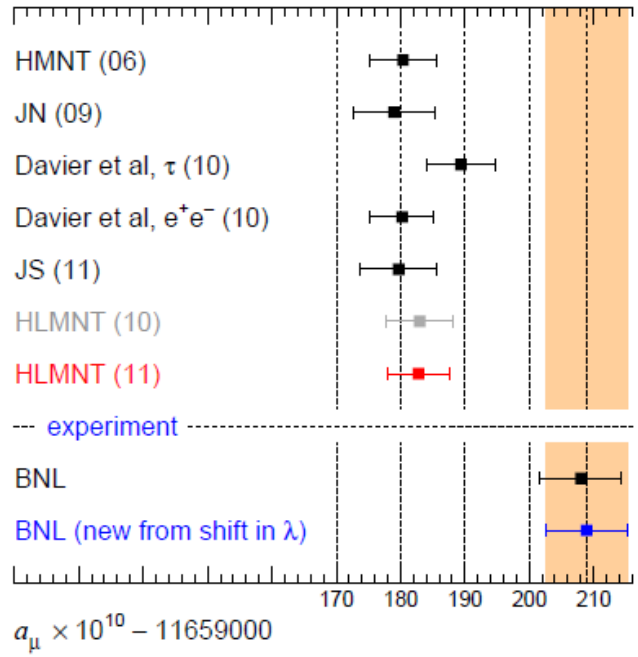
with the prospects of reducing the exp. error by a <u>factor of $\sim 4$</u> to $0.14$ ppm.

(FERMILAB E989, circa '17 ??). See also (J-PARC E34).

Since 2006, $\sim$ 250 papers on $(g-2)_\mu$ !

# $a_\mu$ Current Status

(K. Hagiwara et al. '11)




All SM determinations consistently below the exp. result.

(Precise discrepancy depends on details, though).

# *QED & EW & QCD*

# $a_\mu$: QED contributions

(Aoyama et al. '12 and many refs therein)

E.g. at order $\alpha^5$:

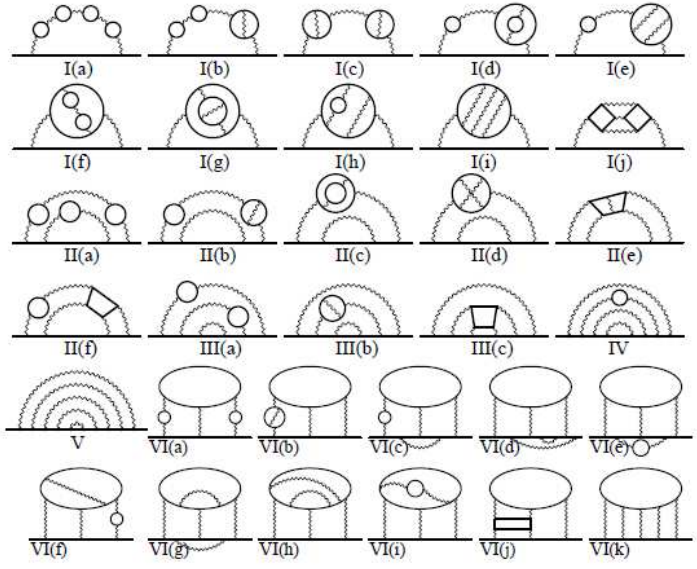



FIG. 2. Typical self-energy-like diagrams representing 32 gauge-invariant subsets contributing to the tenth-order lepton $g-2$. Solid lines represent lepton lines propagating in a weak magnetic field.

| | |
|---|---|
| $A^{(2)}$ | 0.5 |
| $A^{(4)}$ | 0.765857425(17) |
| $A^{(6)}$ | 24.05050996(32) |
| $A^{(8)}$ | 130.8796(63) |
| $A^{(10)}$ | 753.29(1.04) |

- $A^{(2n)}, n \leq 3$ are known analytically (errors are due to $e, \mu, \tau$ masses).
- $A^{(8)}, A^{(10)}$ known only numerically.

12672 diagrams later:

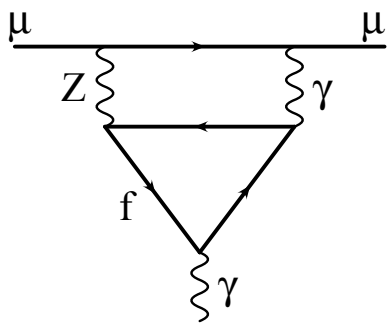
$$a_\mu^{QED} = A^{(2)}\left(\frac{\alpha}{\pi}\right) + A^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + A^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + A^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

$$= 116584718845(9)_{\ell\ \mathrm{mass}}(19)_{\alpha^4}(7)_{\alpha^5}(30)_{\alpha(a_e)} \times 10^{-14}$$

# $a_\mu$: EW contributions

(Czarnecki, Krause and Marciano '96)
( Knecht, SP, Perrottet and de Rafael '02)
(Czarnecki, Marciano and Vainshtein '03)
(Gnendiger, Stöckinger, Stöckinger-Kim. '13)

E.g. at two loops:




$$a_\mu^{EW} \sim G_F\, m_\mu^2 \left[1 + (\tfrac{\alpha}{\pi})\, \Phi(M_t, M_H, \textbf{chiral anomaly}, \ldots)\right]$$

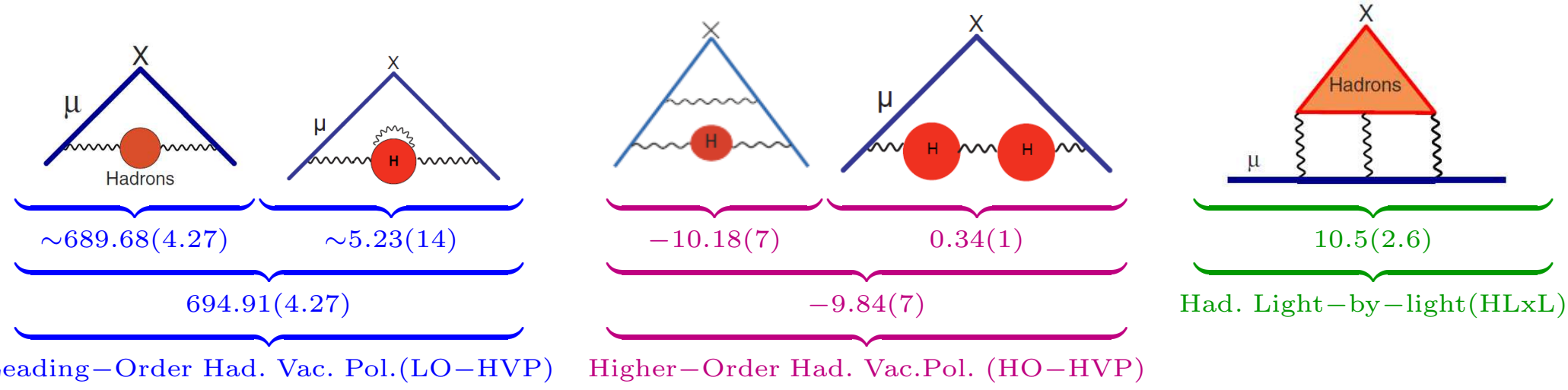
$$a_\mu^{EW} = 15.36(10) \times 10^{-10}$$

# $a_\mu$: Anatomy of QCD contributions

- Classification by Calmet et al. '77; Krause '97; see also Greynat and de Rafael '12.
- Numbers from, e.g., Hagiwara et al. '11. (see also Davier et al. '03)

(Recall $\Delta a_\mu^{EXP} = 6.3 \times 10^{-10}$.)

$a_\mu(\times 10^{-10})$:

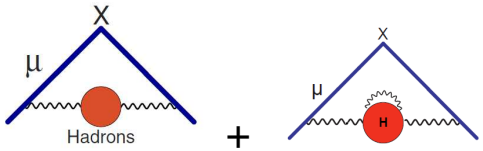



$$a_\mu^{\rm SM} = 11\ 659\ 182.8\ (4.9) \times 10^{-10}$$

- LO-HVP and HO-HVP are data based: $\sigma(e^+e^- \to had)$.
- Final error currently dominated by LO-HVP, but HLxL's is model based. Systematic error ?

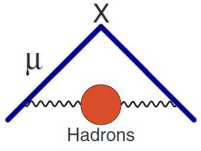
# *LO-HVP*

- Standard Method (Gourdin and de Rafael '69):




$$\sim \int_{m_\pi^2}^{\infty} \frac{dt}{t}\, K(t) \quad \overbrace{\mathrm{Im}\Pi(t)}^{e^+e^- \to \mathrm{had}(\gamma)} \quad , \quad K(t) \sim m_\mu^2/t, \quad t \to \infty$$

- Lattice method (Blum '03; Lautrup, Peterman, de Rafael '72):




$$\sim \quad \int_0^\infty dQ^2 \quad \underbrace{f(Q^2)}_{\mathrm{known}} \quad \Big[\underbrace{\Pi(Q^2) - \Pi(0)}_{\widehat{\Pi}(Q^2)}\Big]$$

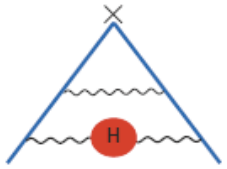
- Current error $\sim 5 - 10\%$. Goal for lattice: to compute this with less than $\sim 0.5\%$ error.

- Warning: This is currently not the same as what is obtained with the Standard Method: the "handbag" diagram is missing, no disconnected (i.e. OZI-violating) diagrams and $m_u = m_d$.

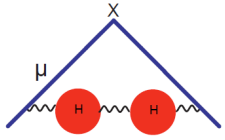
- Lots of recent lattice work devoted to this calculation...

# HO-HVP

- Standard Method (Hagiwara et al. '11):



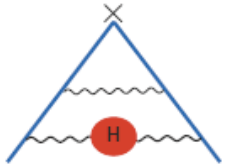

$$\sim \int_{4m_\pi^2}^{\infty} \frac{dt}{t}\, \widetilde{K}(t) \quad \overbrace{\mathrm{Im}\Pi(t)}^{e^+e^- \to \mathrm{had}(\gamma)} \quad , \quad \widetilde{K}(t) \sim m_\mu^2/t, \quad t \to \infty$$



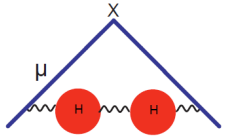

$$\sim \int_{4m_\pi^2}^{\infty} \frac{dt}{t}\, K(t) \left[\mathrm{Re}\widehat{\Pi}(t)\, \mathrm{Im}\Pi(t)\right] \quad , \quad K(t) \sim m_\mu^2/t, \quad t \to \infty$$

the latter is very small because $\int_{4m_\pi^2}^{\infty} \frac{dt}{t^2}\, \mathrm{Re}\widehat{\Pi}(t)\, \mathrm{Im}\Pi(t) = 0$ (Greynat, de Rafael '12).

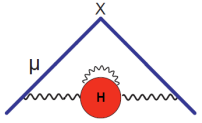
- Lattice Method :




$$\sim \int_0^{\infty} dQ^2\, \underbrace{\widetilde{f}(Q^2)}_{known}\, \widehat{\Pi}(Q^2)$$




$$\sim \int_0^{\infty} dQ^2\, \underbrace{f(Q^2)}_{known}\, \left[\widehat{\Pi}(Q^2)\right]^2$$




$$\sim \int_0^{\infty} dQ^2\, \underbrace{f(Q^2)}_{known}\, \widehat{\widetilde{\Pi}}(Q^2) \quad , \left[f(Q^2) \text{ is the same as LO-HVP.}\right]$$

No results yet.

# *Why* $\int_{4m_\pi^2}^{\infty} \frac{dt}{t^2}\, \mathrm{Re}\widehat{\Pi}(t)\, \mathrm{Im}\Pi(t) = 0$

Write (subtracted) disp. relation for $\widehat{\Pi}(Q^2)$:

$$\widehat{\Pi}(Q^2) = Q^2 \int_{4m_\pi^2}^{\infty} \frac{dt}{t(t+Q^2)}\, \frac{1}{\pi} \mathrm{Im}\Pi(t)\ , \approx a\, Q^2 + \mathcal{O}(Q^4) \qquad (1)$$

Again, now for $\left(\widehat{\Pi}(Q^2)\right)^2$ (it has same analytic structure):

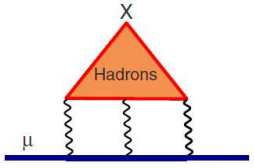
$$\left(\widehat{\Pi}(Q^2)\right)^2 = Q^2 \int_{4m_\pi^2}^{\infty} \frac{dt}{t(t+Q^2)}\, \frac{1}{\pi} \underbrace{\left(2\, \mathrm{Re}\widehat{\Pi}(t) \mathrm{Im}\Pi(t)\right)}_{\mathrm{Im}\widehat{\Pi}(t)^2} \approx b\, Q^2 + \mathcal{O}(Q^4)$$

Compare with square of (1) $\Rightarrow b = 0$ , $b = \int_{4m_\pi^2}^{\infty} \frac{dt}{t^2}\, \frac{1}{\pi} \left(2\, \mathrm{Re}\widehat{\Pi}(t) \mathrm{Im}\Pi(t)\right)$

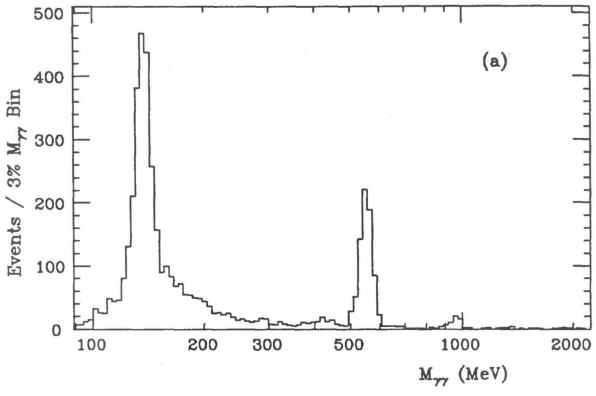
# $HL \times L$: The problem

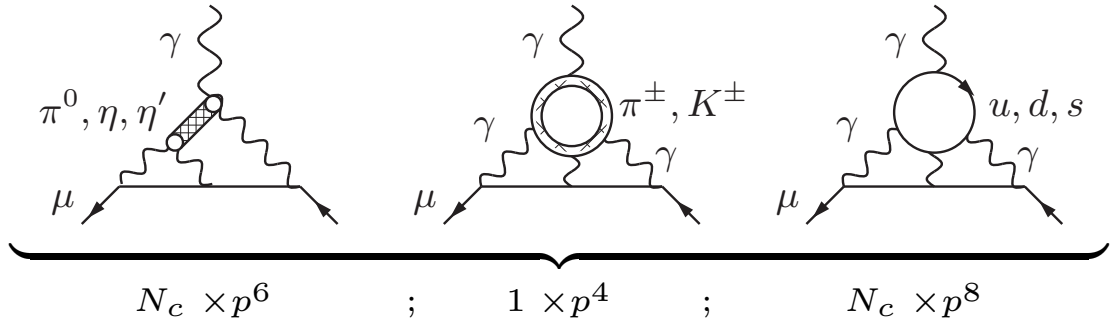
# $HL \times L$

- Standard Method (Bijnens,Pallante,Prades '95; Hayakawa, Kinoshita, Sanda '95; Knecht, Nyffeler '02; Prades, de Rafael, Vainshtein '09; Jegerlehner, Nyffeler '09):




$= 10.5(2.6) \times 10^{-10}$




Model Independent result for leading $\mathcal{O}(N_c \times p^6)$ term:

$$a_\mu^{\pi^0 - L\times L} = N_c \left(\frac{\alpha}{\pi}\right)^3 \frac{m_\mu^2 N_c}{48\pi^2 f_\pi^2} \left[\log^2 \frac{M_\rho}{m_\pi} + a \log \frac{M_\rho}{m_\pi} + b\right]$$

$\log^2$ dominant when $m_\pi \to 0$.

But in the real world $b$ regretfully unknown unless $\chi$ PT is "UV completed".
($a$ can be obtained from $\pi^0 \to e^+e^-$: $a \simeq -1$)

# $HL \times L$ (II)

- ”UV Completion" is based on Vector Meson Dominance (VMD) at different levels of sophistication/complexity.

$$VMD \equiv \text{Amplitude} \times \frac{M_V^2}{Q^2 + M_V^2}$$

Other resonances (e.g. axial-vector, scalar, pseudoscalar) mimic the VMD ansatz. Large-$N_c$ QCD requires this set to be *infinite* but only a *finite* number is used in practice.

Match onto the OPE at short distances insofar as possible. This **always** entails a compromise. (This is in fact related to the previous item.)

- Current estimates indeed lead to $a \simeq -1$ (as expected), but $b \sim 1$ while $b \sim 10$ would be required to match exp. result on $(g-2)_\mu$.

# $HL \times L$ (III)

Bijnens, Abyaneh '12

- **BPP[6]**: Bijnens,Pallante,Prades '95 (similar results by Hayakawa, Kinoshita, Sanda '95)
  **PdRV[7]**: Prades, de Rafael, Vainshtein '09 ("Glasgow Consensus")

**Table 2.** The different parts of the HLBL contribution.

| | BPP [6] | PdRV [7] |
|---|---|---|
| pseudo-scalar | $(8.5 \pm 1.3) \cdot 10^{-10}$ | $(11.4 \pm 1.3) \cdot 10^{-10}$ |
| axial-vector | $(0.25 \pm 0.1) \cdot 10^{-10}$ | $(1.5 \pm 1.0) \cdot 10^{-10}$ |
| quark-loop | $(2.1 \pm 0.3) \cdot 10^{-10}$ | — |
| scalar | $(-0.68 \pm 0.2) \cdot 10^{-10}$ | $(-0.7 \pm 0.7) \cdot 10^{-10}$ |
| $\pi K$-loop | $(-1.9 \pm 1.3) \cdot 10^{-10}$ | $(-1.9 \pm 1.9) \cdot 10^{-10}$ |
| errors | linearly | quadratically |
| sum | $(8.3 \pm 3.2) \cdot 10^{-10}$ | $(10.5 \pm 2.6) \cdot 10^{-10}$ |

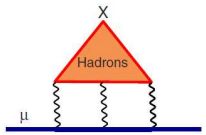
- But (Engel, Ramsey-Musolf '13):

| $\times 10^{-10}$ | Pointlike | Form Factor w/o $\chi$PT | Form Factor w. $\chi$PT |
|---|---|---|---|
| $\pi$ loop | $-4.1$ | $-1.6$ | $-4$ ( $-7$) !!! |

- There is recent proposal (Colangelo et al. '14) to express HL$\times$L in terms of exp. data ($\sim$ HVP). No results yet.

# $HL \times L \quad (IV)$

- <u>Lattice Method</u> (Blum, Chowdhury, Hayakawa and Izubuchi '14):

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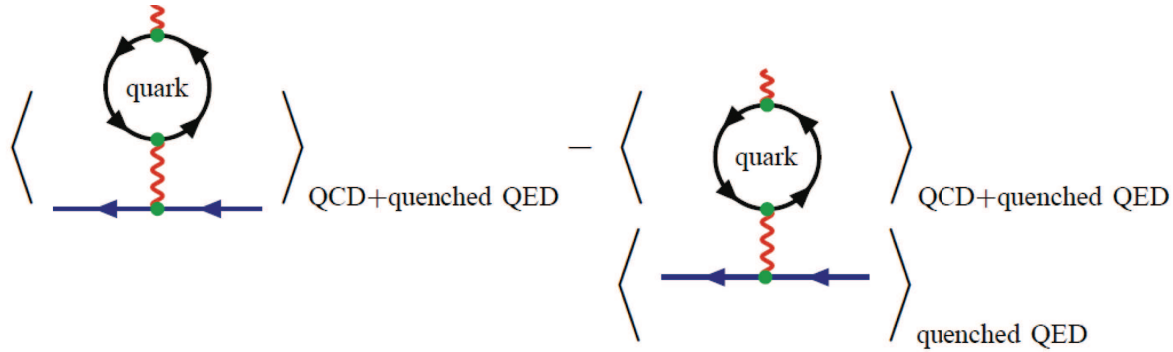


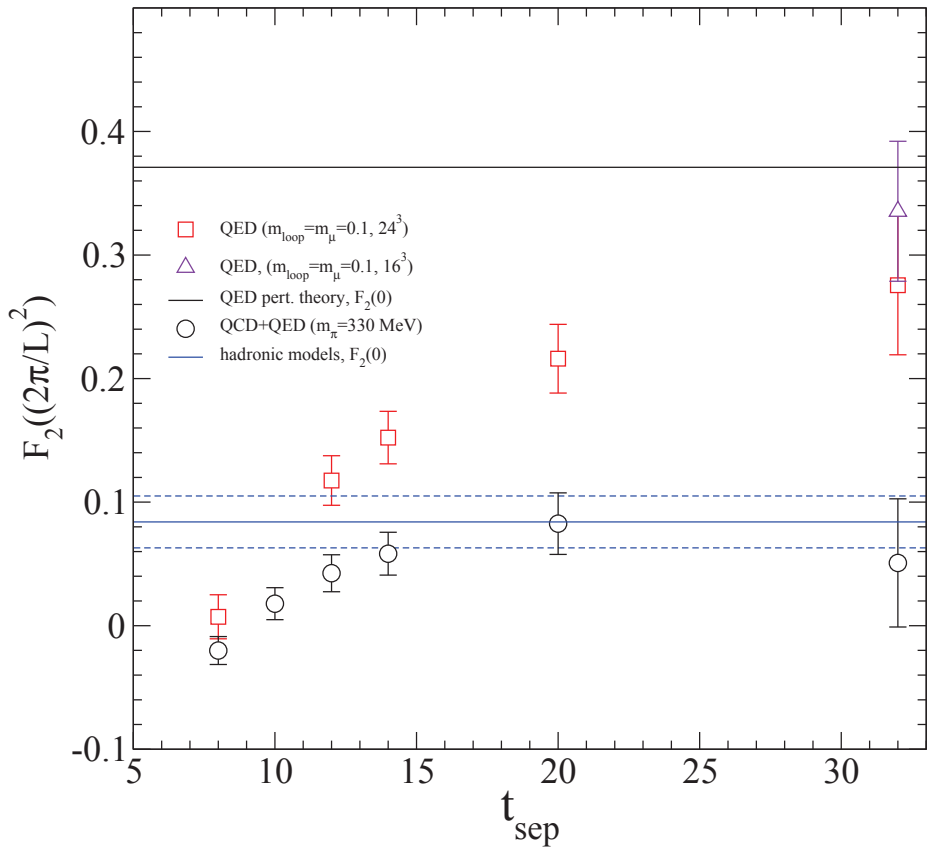

# *Conclusions*

# Conclusions

- Current results for HL$\times$L are "best possible estimate".

  ( Is “best possible” good enough ?)

- HL$\times$L has $\sim 30\%$ error, but model dependent.

  Discrepancy exp-th is $\sim 3 \cdot$ HL$\times$ L .

- Lattice is the only 1st principles calculation but requires QCD+QED. Encouraging results are being obtained.

  Current lattice estimates of HVP have too large errors. HL$\times$L is harder.

- (Giudice et al. ’12): Keep an eye on $(g-2)_e$.

  Currently $\Delta(g-2)_e \sim 8 \times 10^{-13}$.

  However, $\Delta(g-2)_e \sim \underbrace{\Delta(g-2)_\mu^{TH-EXP}}_{26\times 10^{-10}} \times (m_e/m_\mu)^2 \sim 0.6 \times 10^{-13}$

  Could this error on $(g-2)_e$ be attainable in the near future ?

The stakes are very high.


We have to get it right.

THANK YOU !