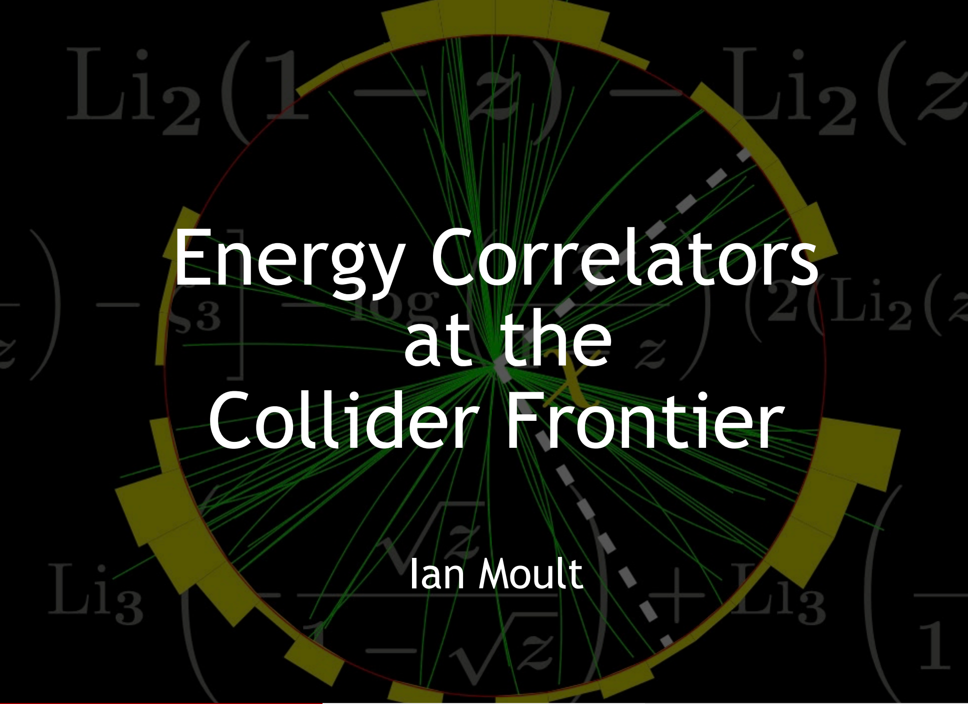
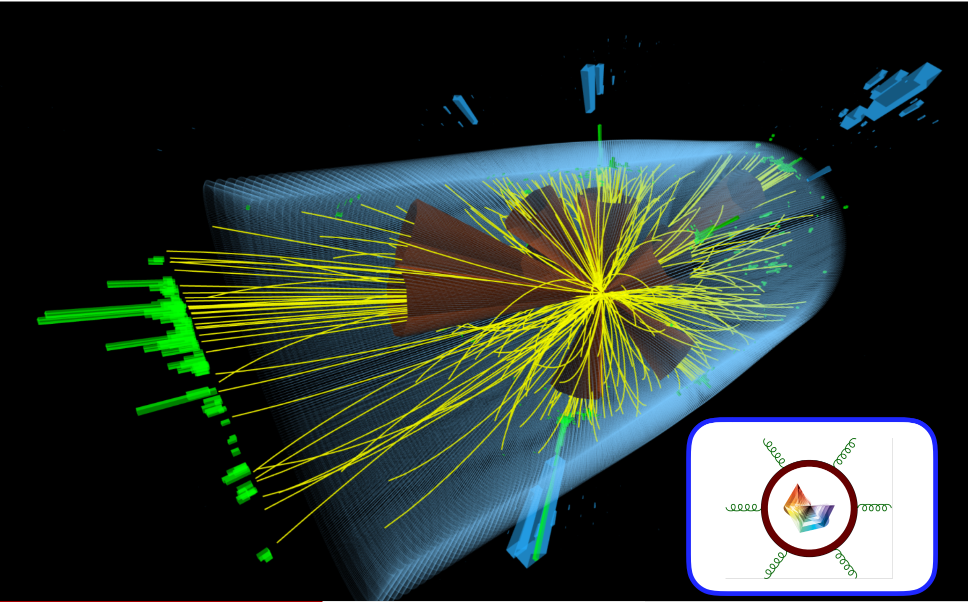


Energy Correlators at the Collider Frontier

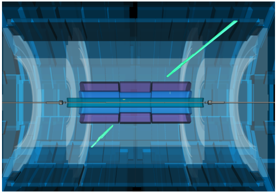
Ian Moult

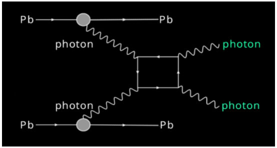
Jets!

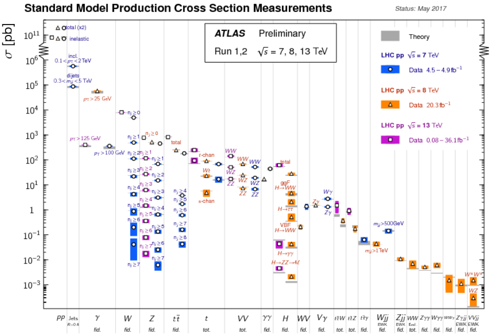


Exclusive Processes

- Tremendous progress in the understanding of exclusive scattering processes: analytic structure, multi-loop perturbative data, amplituhedron, S-matrix bootstrap,...



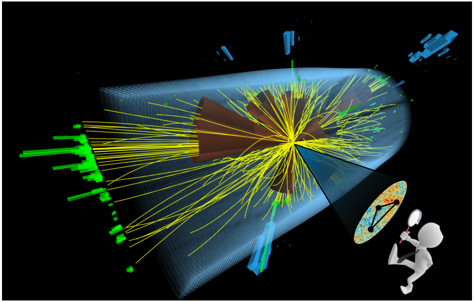


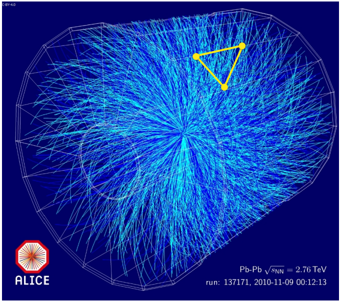


- Practical Outcome: Ability to accurately describe complicated SM scattering processes.

The High Multiplicity Regime

- A complementary regime: high multiplicity
  - Collisions with $E \gg m_{\text{gap}}$
  - Conformal Field Theories

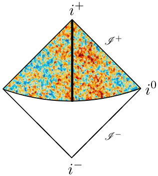




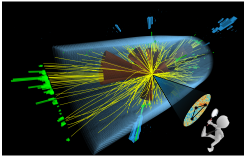
- Good observables are correlations in fluxes at (null) infinity.

Motivation

- How can we characterize a theory using asymptotic data?



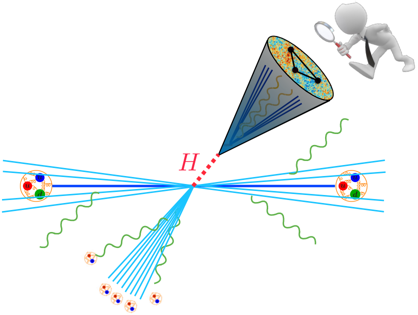
- Theoretical motivation:
  - What is the space of observables at null infinity?
  - How are they related to (C)FT data?
  - How do we constrain theories in the absence of S-matrix and/ or local ops (e.g. CFT coupled to gravity)
- Phenomenological motivation:
  - Can we relate asymptotic measurements to parameters of the underlying theory? (couplings, transport coefficients, ...)
  - Can we identify universal features that can be computed to high precision?

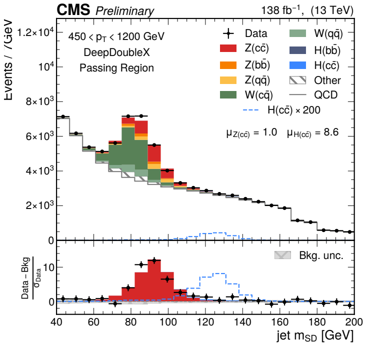


- Wealth of collider data provides a practical testing ground.

Jet Substructure: Searches

- Jet Substructure uses the internal structure of jets to provide qualitatively new ways to study physics at the LHC.

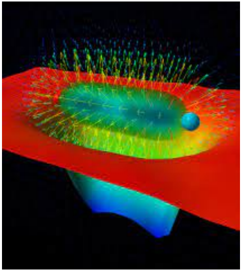


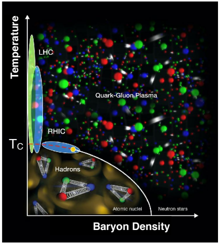


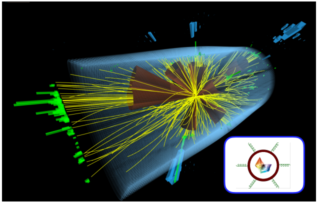
- Its introduction in 2008 by Butterworth, Davison, Rubin and Salam, along with anti-k_T by Cacciari, Soyez, Salam, and the starting of the LHC, reinvigorated the study of jets in QCD.

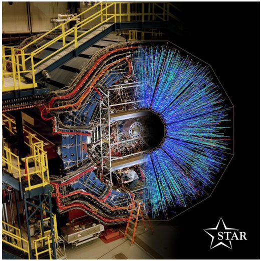
Jet Substructure: Quantum Field Theory

- Beyond searching for new physics, much more subtle questions about QCD are imprinted in collider energy flux:



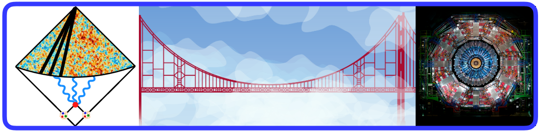


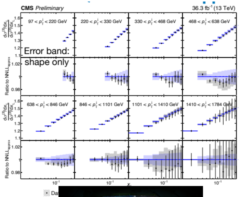


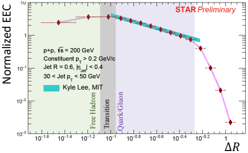


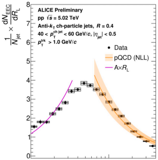
Energy Correlators in Data

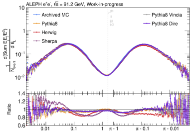
- Progress bridging theory and experiment across collider systems!

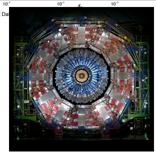


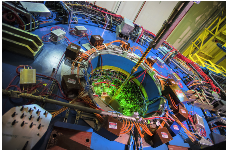


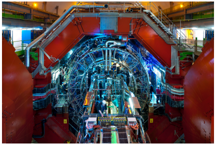


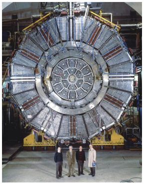






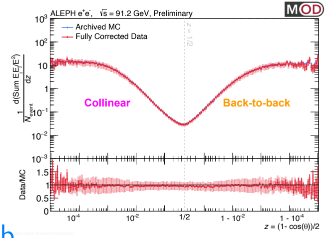




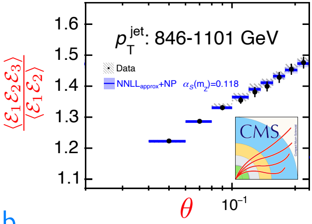


- Progress bridging theory and experiment across collider systems!

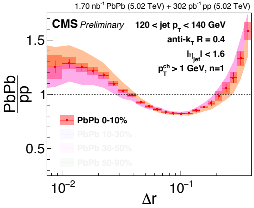
e^+e^-



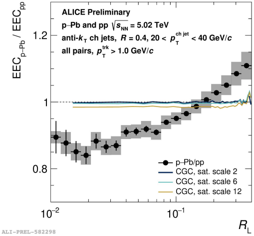
pp



Pb-Pb

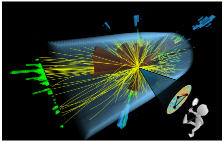


p-Pb

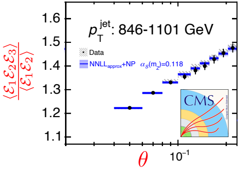


Outline

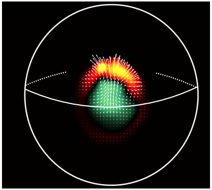
- Decoding Energy Flux



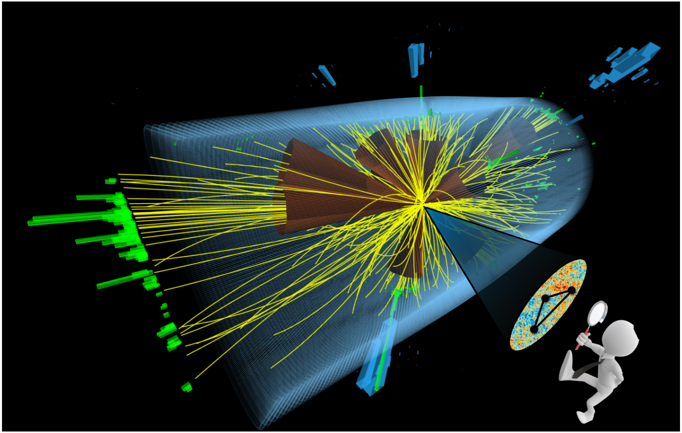
- Energy Correlators:
  Scaling and Multi-Point Correlators



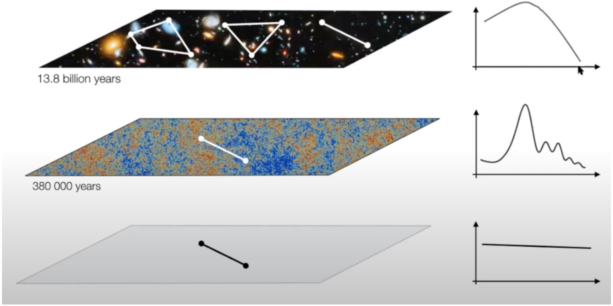
- Imaging Intrinsic and
  Emergent Scales of QCD

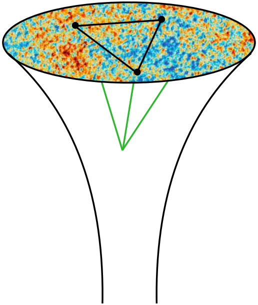


Decoding Energy Flux



- In condensed matter physics or cosmology we decode the underlying dynamics using correlation functions.

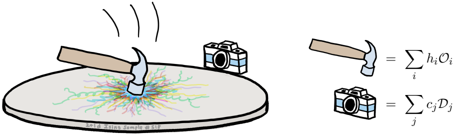




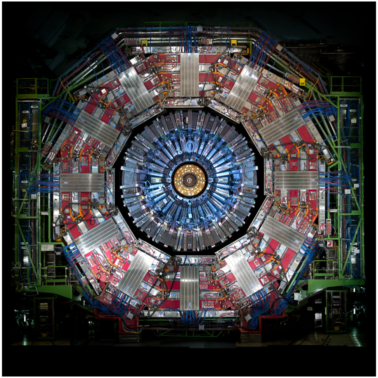
- Can we achieve a similarly coherent picture of collider physics?

Defining the Problem

- What is a detector?



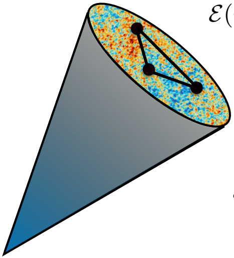
[Caron Huot, Kologlu, Kravchuk, Meltzer, Simmons Duffin]



- To be able to understand subtle signals in energy flux, we must understand what a detector is in Quantum Field Theory.

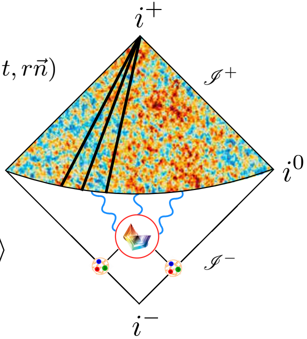
Calorimeter Cells in Field Theory

- Calorimeter cells can be given a field theoretic definition in terms of light-ray operators. [Hofman, Maldacena], [Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov] [Korchemsky, Sterman] [Ore, Sterman] [Basham, Brown, Ellis, Love]



$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\ n^i T_{0i}(t, r\vec{n})$$

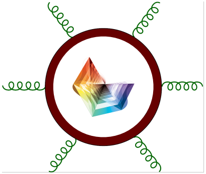
$$\langle \Psi | \mathcal{E}(\hat{n}_1) \cdots \mathcal{E}(\hat{n}_k) | \Psi \rangle$$

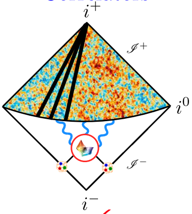


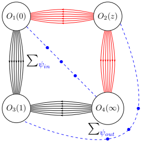
- From the perspective of QFT, jet substructure is the study of correlation functions of energy flow operators.

Energy Correlators

- Correlation functions $\langle 0|\mathcal{O}^\dagger \mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_k)\mathcal{O}|0\rangle$ take an interesting intermediate position between amplitudes and correlation functions.



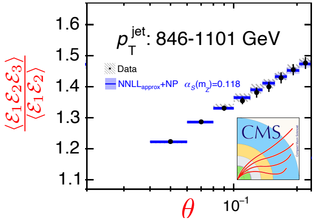


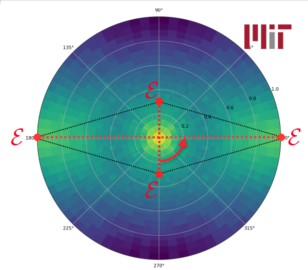


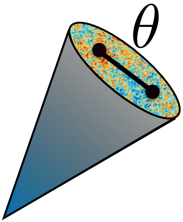
	Amplitudes	Energy Correlators	Correlation Functions
Boundary Observable	✓	✓	
IR Finite		✓	✓

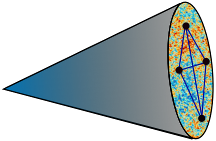
- Provide an interesting example of observables that are well defined at weak coupling, strong coupling, in a CFT, with gravity, ...

Energy Correlators:
Scaling and Multi-Point Correlators

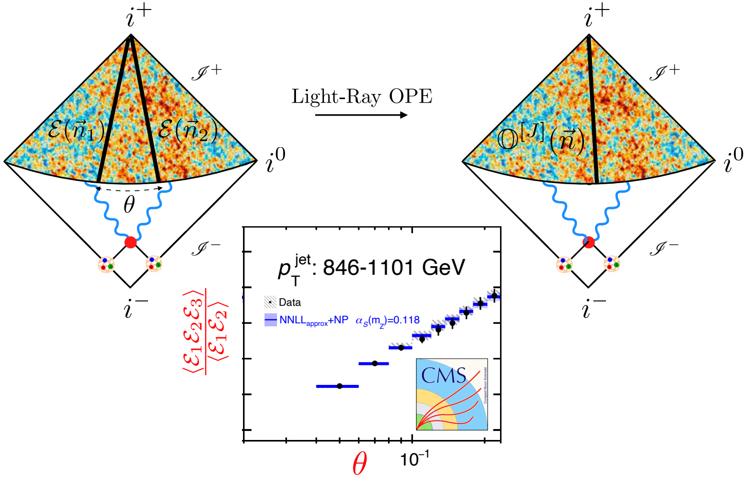








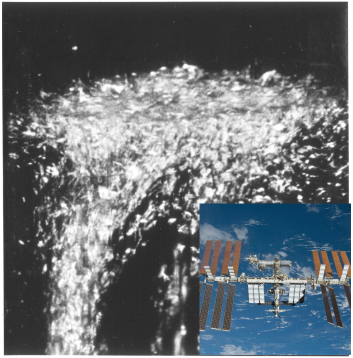
Scaling Behavior

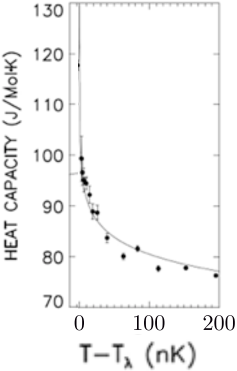


Scaling Behavior in QFT

- Scaling behavior in Euclidean regime well understood.

λ-point of Helium

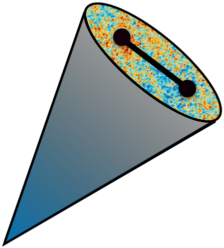


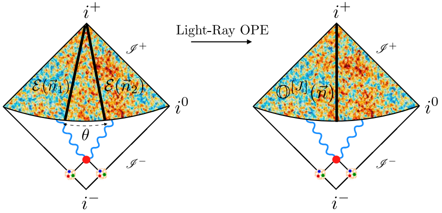


$$\mathcal{O}(x)\mathcal{O}(0) = \sum x^{\gamma_i} c_i \mathcal{O}_i$$

The OPE Limit of Lightray Operators

- Energy flow operators admit a Lorentzian OPE: "the lightray OPE"





$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i - 4} \mathbb{O}_i(\hat{n}_1)$$

[Hofman, Maldacena]
[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]
QCD: [Dixon, Moult, Zhu]

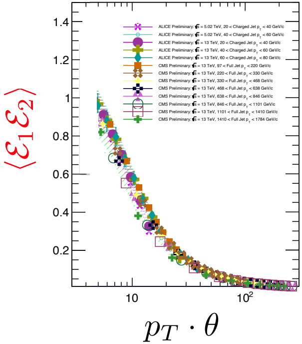
- Predicts universal scaling behavior in correlations of energy flux at energies $E \gg \Lambda_{\text{QCD}}$.

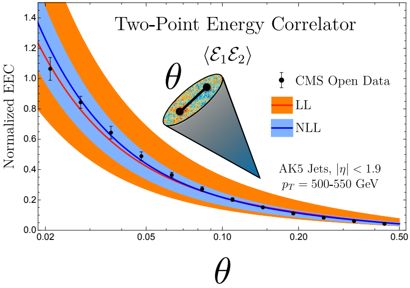
See early work by [Konishi, Ukawa, Veneziano]

Scaling Behavior in Jets

- Scaling measured inside jets by STAR, ALICE and CMS from 15 GeV to 1784 GeV:

  An experimental realization of the detector OPE!

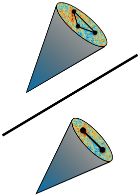




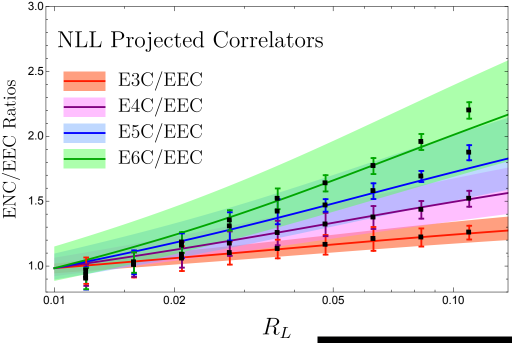
- Can we accurately extract anomalous exponents of different detectors?

The Spectrum of a Jet

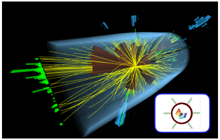
- The light-ray OPE predicts that the N-point correlators develop an anomalous scaling that depends on N.



$$\frac{\langle \mathcal{E}_1\mathcal{E}_2\cdots\mathcal{E}_{J-1}\rangle}{\langle \mathcal{E}_1\mathcal{E}_2\rangle} \sim \frac{\langle \mathbb{O}^{[J]}\rangle}{\langle \mathbb{O}^{[3]}\rangle}$$



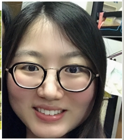
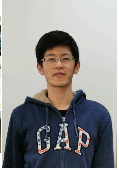
- Directly probes the spectrum of (twist-2) lightray operators from asymptotic energy flux.



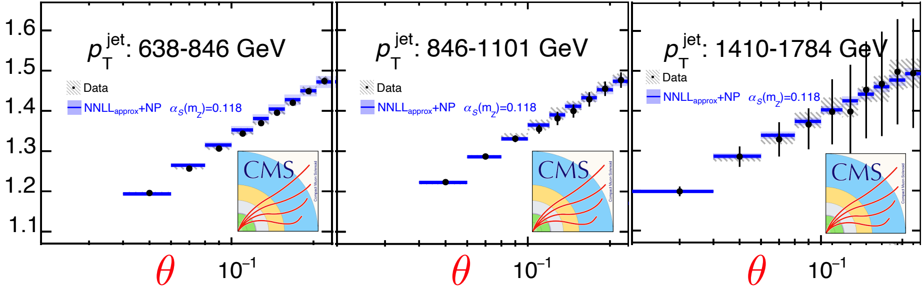
Anomalous Scaling

- Universal quantity in complicated hadronic environment.

$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \mathcal{E}_3 \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim \frac{\langle \mathbb{O}^{[4]} \rangle}{\langle \mathbb{O}^{[3]} \rangle} \sim \theta^{\gamma(4)-\gamma(3)}$$

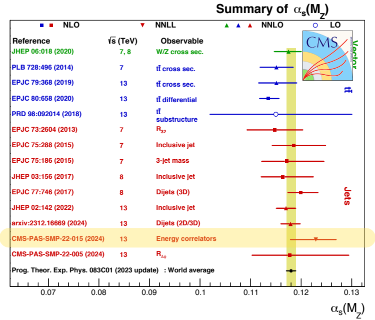
XiaoYuan Zhang **Yibei Li** **Hao Chen**



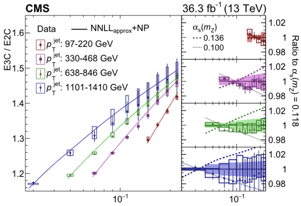
- Uses scaling anomalous dimensions at three-loop order.
- Beautiful quantitative test of QFT!

The Strong Coupling

- Use scaling to extract value of the strong coupling constant α_s at 4% accuracy.



$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \mathcal{E}_3 \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle}$$



θ

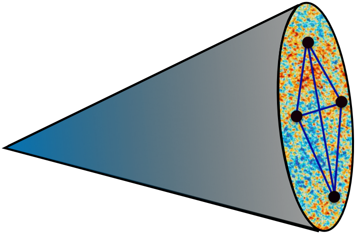
This yielded the worlds most precise α_S measurement from jet substructure: $\alpha_S = 0.1229^{+0.0040}_{-0.0050}$.

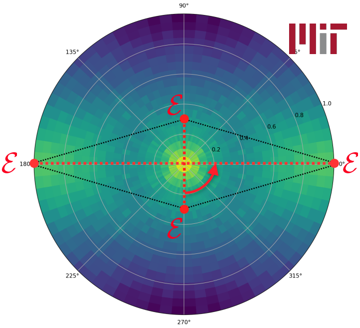
$$\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050}$$

$$= 0.1229^{+0.0014(stat.)+0.0030(theo.)+0.0023(exp.)}_{-0.0012(stat.)-0.0033(theo.)-0.0036(exp.)}$$

- Very clear target for improved perturbative calculations. e.g. NNLO $2 \to 3$ hard functions, NP corrections, ... not yet included.

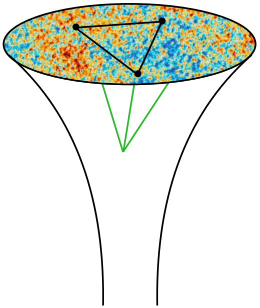
Higher Point Functions in Energy Flux

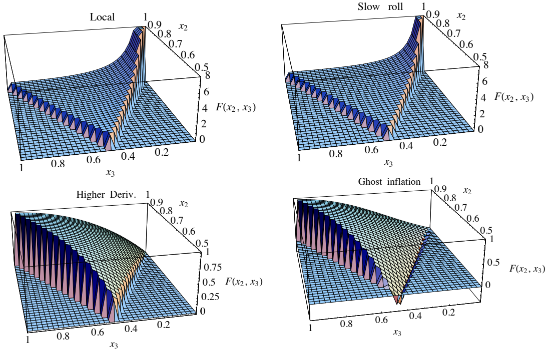




Multipoint Correlators

- Higher-point correlators probe detailed aspects of the underlying microscopic interactions. e.g. CMB three-point functions allow to distinguish models of inflation.





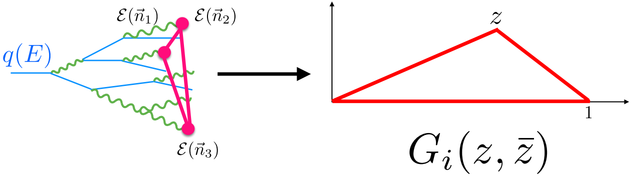
- What is the structure of higher-point functions of energy flux?

Multipoint Correlators

- The only explicit results for correlators with $N > 2$ were the remarkable strong coupling results of Hofman and Maldacena:

$$\langle \mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_n)\rangle = \left(\frac{q}{4\pi}\right)^n \Bigg[1 + \sum_{i<j}\frac{6\pi^2}{\lambda}[(\vec{n}_i.\vec{n}_j)^2 - \frac{1}{3}] + + \frac{\beta}{\lambda^{3/2}}[\sum_{i<j<k}(\vec{n}_i.\vec{n}_j)(\vec{n}_j.\vec{n}_k)(\vec{n}_i.\vec{n}_k) + \cdots] + o(\lambda^{-2})\Bigg]$$

- The wealth of techniques developed to compute perturbative scattering amplitudes can be applied to multi-point correlators at weak coupling.



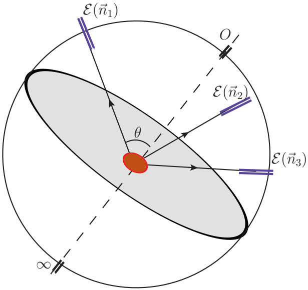
Correlators in Perturbation Theory

- Two approaches to calculate energy correlators:
  1. Light transforming N-point functions of stress tensors:

$$\langle 0|\mathcal{O}^\dagger T \cdots T \mathcal{O}|0\rangle \to \langle 0|\mathcal{O}^\dagger \mathcal{E}(\vec{n}_1) \cdots \mathcal{E}(\vec{n}_k)\mathcal{O}|0\rangle$$

Two Point NLO in $\mathcal{N}=4$: [Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov]
Two Point NNLO in $\mathcal{N}=4$: [Henn, Sokatchev, Yan, Zhiboedov]
LO Charge-Charge Correlator in QCD: [Chicherin, Henn, Sokatchev, Yan]

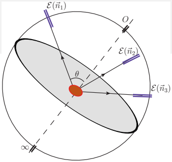
  2. Perturbative phase space integrals using (squared) form factors:



$$\frac{\langle \Psi|\mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_k)|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \sum_{i_1,\ldots,i_k} \int d\sigma \prod_{j=1}^{k} E_{i_j}\delta(\vec{n}_j - \vec{p}_{i_j}/p^0_{i_j})$$

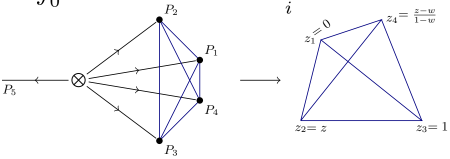
Two Point LO in QCD: [Basham, Ellis, Brown, Love]
Two Point NLO in QCD: [Dixon, Luo, Shtabovenko, Yang, Zhu]
Three Point Collinear LO in QCD: [Chen, Luo, Moult, Yang, Zhang, Zhu]
Three Point General Angle LO in $\mathcal{N}=4$: [Yan, Zhang]
Three Point General Angle LO in QCD: [Yang, Zhang]
Four Point Collinear LO in $\mathcal{N}=4$: [Chicherin, Moult, Sokatchev, Yan, Zhu]

Correlators in Perturbation Theory

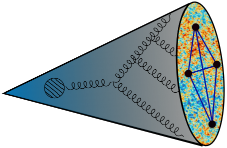


- For generic angles, the correlator depends on the cross ratios $\zeta_{ij} = \frac{1-\cos\theta_{ij}}{2}$, and the source.
- In the collinear (OPE) limit, $\zeta_{ij} \to 0$, it becomes a function of $2(N-2)$ variables that is independent of the source.
- The LO contribution to the N-point function is given by a *finite* integral in $(N-1)$ dimensional projective space of the *universal splitting functions*:

$$\mathrm{E^NC} \overset{\text{coll.}}{=} \int_0^1 dx_1 \cdots dx_N \, \delta(1 - \sum_i x_i) \, (x_1 \cdots x_N)^2 \, \mathcal{P}^{(0)}_{1\to N}$$

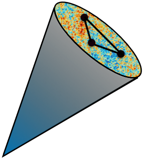


- This limit can be physically measured inside high energy jets at the LHC.



Three-Point Correlator at Weak Coupling

- First non-trivial correlator: tree level three-point correlator in the collinear limit $G(z, \bar{z})$. [Chen, Luo, Moult, Yang, Zhang, Zhu]
- Turns out to have an elegant perturbative structure. e.g. in $\mathcal{N} = 4$



$$G_{\mathcal{N}=4}(z) = \frac{1+u+v}{2uv}(1+\zeta_2) - \frac{1+v}{2uv}\log(u) - \frac{1+u}{2uv}\log(v)$$
$$-(1+u+v)(\partial_u + \partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-\bar{z})^2(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z)$$
$$+ \frac{(u-1)(u+1)}{2uv^2}D_2^+(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^+(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^+\left(\frac{z}{z-1}\right)$$

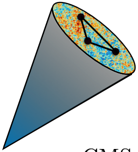
- where Φ and D_2^+ are

$$\Phi(z) = \frac{2}{z-\bar{z}}\left(\mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z}) + \frac{1}{2}\left(\log(1-z) - \log(1-\bar{z})\right)\log(z\bar{z})\right)$$
$$D_2^+(z) = \mathrm{Li}_2(1-|z|^2) + \frac{1}{2}\log(|1-z|^2)\log(|z|^2)$$

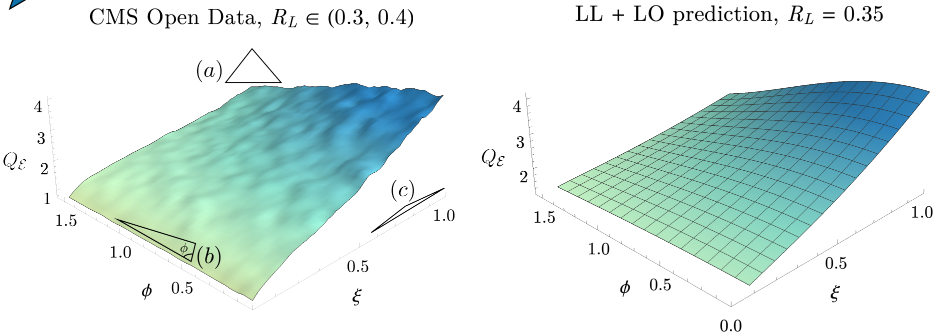
- Provides important perturbative data for the development of the lightray OPE.

Shape Dependence of Non-Gaussianities

- Multipoint correlators can be directly measured in high energy jets: Simple analytic functions for the *actual measured observable!*



- Non-Gaussianities inside high energy jets at the LHC:

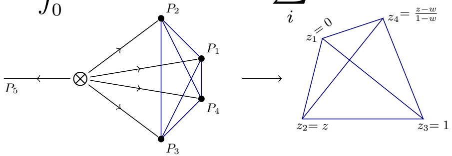


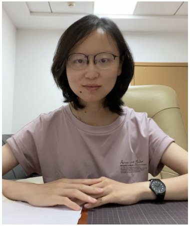
Four Point Correlator

[Chicherin, Moult, Sokatchev, Yan, Zhu]

- Simple structure makes energy correlators a nice playground for exploration of *physical observables* in perturbation theory.
- Four point correlator computed in $\mathcal{N}=4$ SYM by direct integration in parameter space, using simple form of $1\to 4$ splitting function.

$$\mathrm{E^NC} \overset{\text{coll.}}{=} \int_0^1 dx_1 \cdots dx_N \, \delta(1-\sum_i x_i)\,(x_1\cdots x_N)^2\, \mathcal{P}^{(0)}_{1\to N}$$



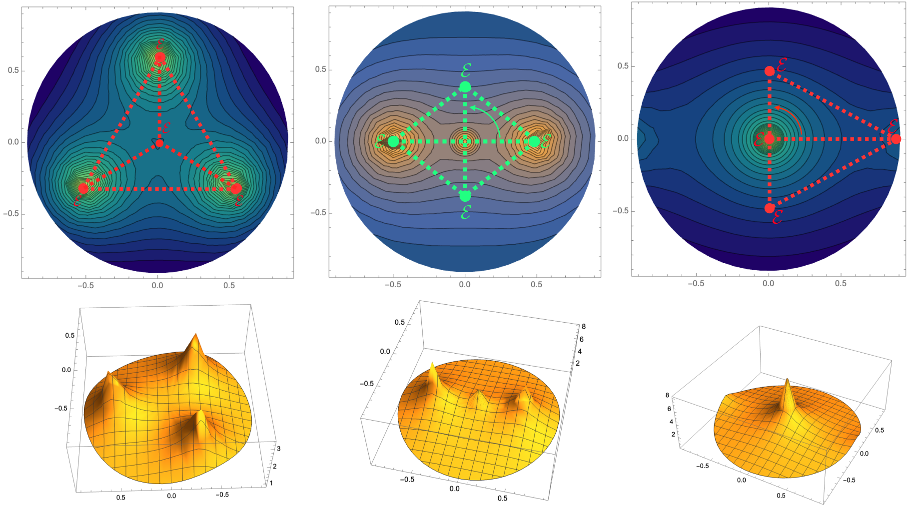


Kai Yan

- Compact result expressed in terms of weight three polylogarithms: much structure still to be explored.
- Would be interesting to extend to QCD using known $1\to 4$ splitting functions. [Del Duca, Duhr, Haindl, Lazopoulos, Michel]
- Can one push to higher points or make general statements?.

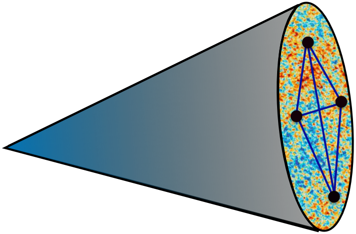
The Four Point Correlator

- Intricate view of correlations of energy flow. Access to OPE limits, spinning operators, ...



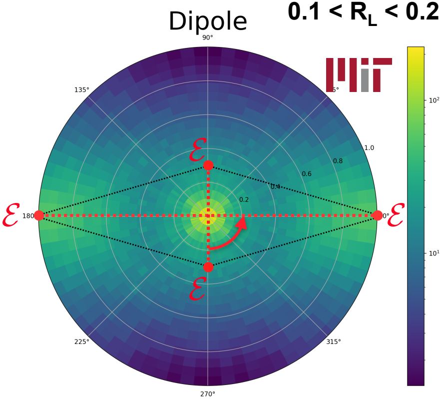
The Four Point Correlator

Experimental tour de force to enable precision measurements of higher point correlators.



$p_T > 100$ GeV

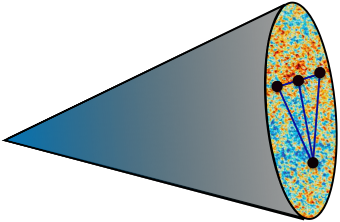
$0.1 < R_L < 0.2$

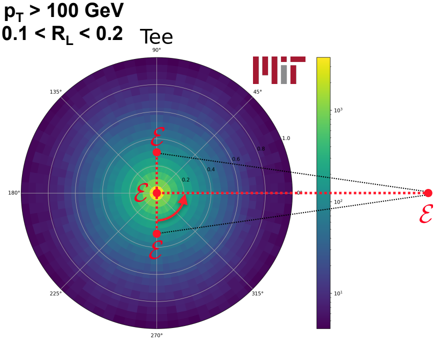


Thanks to Simon Rothman and Phil Harris + Kyle Lee

The Four Point Correlator

Experimental tour de force to enable precision measurements of higher point correlators.

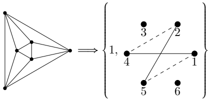


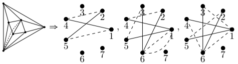


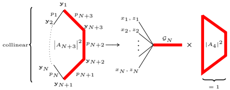
Thanks to Simon Rothman and Phil Harris + Kyle Lee

Multi-point Correlators at Weak Coupling

- Has motivated the theoretical exploration of higher point correlators.
- Integrand up to 11 points in $\mathcal{N} = 4$ super Yang-Mills.
- Hints of elliptic and Calabi-Yau structures in integrals for 5 points and beyond. [He, Jiang, Yang, Zhang]





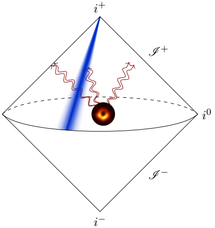


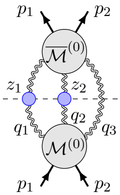
- Nice interplay between theory and experiment.

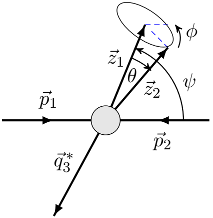
Structure in Perturbative Gravity

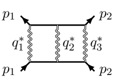
[Herrmann, Kologlu, Moult]

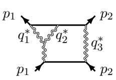
- Also have an interesting structure in perturbative quantum gravity.

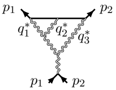


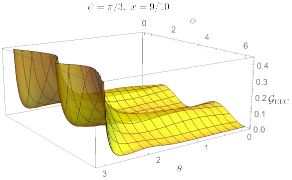






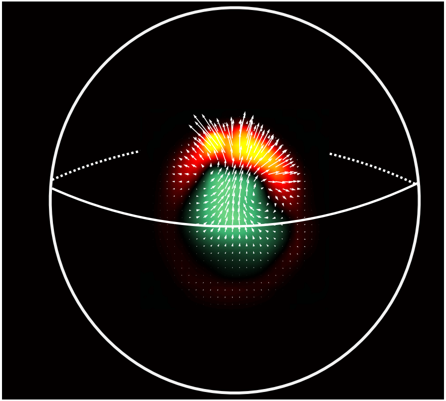






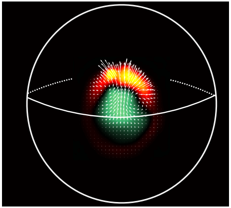
- Currently investigating higher point correlators...

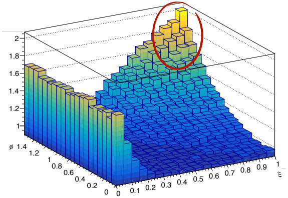
Imaging Intrinsic and Emergent Scales of QCD



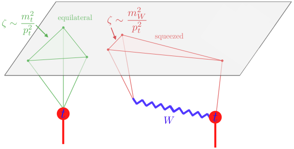
LHC Targets:

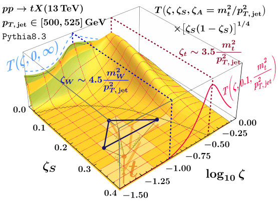
- Measurements on more complicated states:
  - Imaging the Quark Gluon Plasma



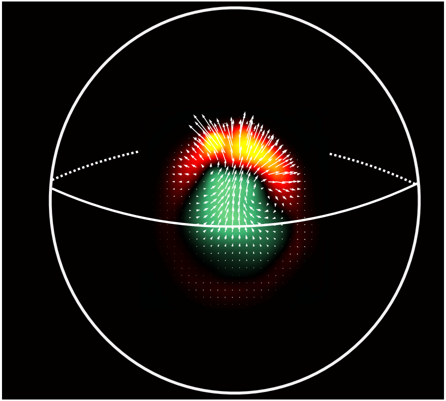


  - Weighing the Top Quark





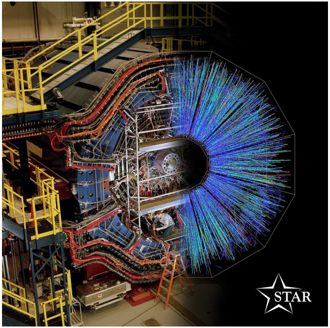
Resolving the Scales of the QGP

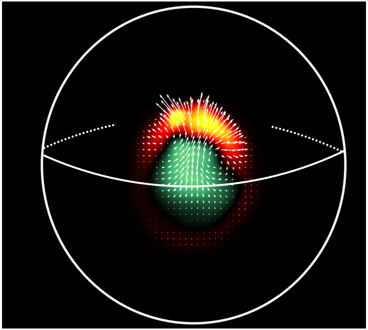


[Andres, Dominguez, Holguin, Kunnawalkam Elayavalli, Marquet, Moult]

Quark Gluon Plasma

- Heavy ion collisions provide an example of an extremely complicated asymptotic state, where we do not understand the microscopic dynamics that created it.

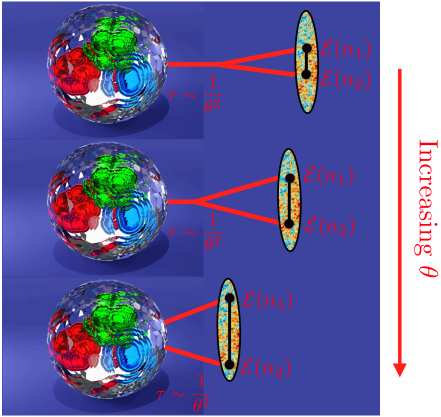


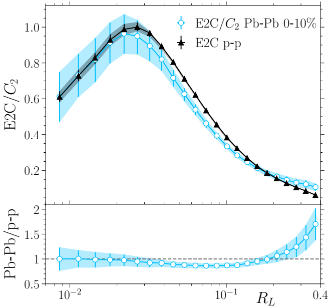


- Nice interplay between pp and heavy ion jet substructure communities.

Correlators in the QGP

- QGP scales cleanly imprinted in two-point correlation.



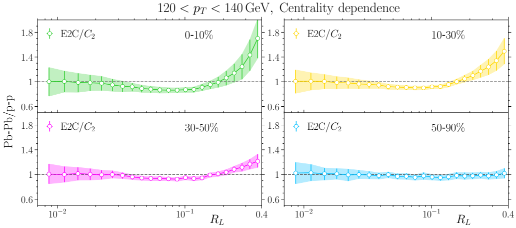


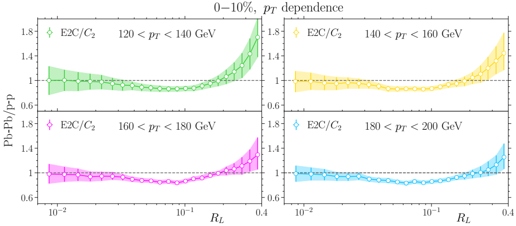
[CMS-PAS-HIN-23-004]

[Andres, Dominguez, Holguin, Kunnawalkam Elayavalli, Marquet, Moult]

Correlators in the QGP

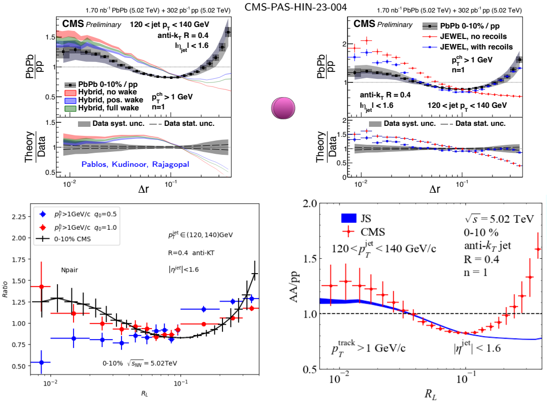
- Large angle enhancement visible in ratio.





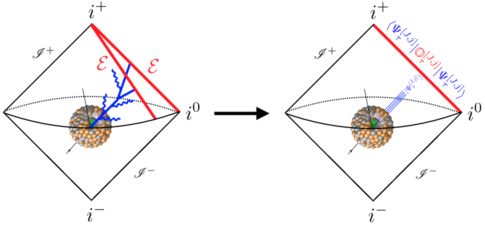
Correlators in the QGP

- Theory calculations in heavy ion extremely complicated, and require large computing power.



- Can we identify robust scalings?

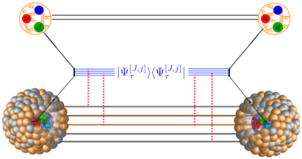
- Lightray OPE allows us to reduce the transverse structure of energy correlator to well defined scaling laws, with coefficients given by matrix elements of lightray operators in nuclear states:



$$\mathcal{E}(n_1)\mathcal{E}(n_2) = \frac{1}{\theta^2}\mathbb{O}^{[j=3]}_{\tau=2} + \mathbb{O}^{[j=3]}_{\tau=4} + \cdots$$

Correlators in the QGP

- An old argument of Sterman and Qiu shows that matrix elements of higher twist operators are enhanced in nuclear medium.
- Predicts that the result in Pb-Pb and p-Pb should in fact give a simple scaling law.

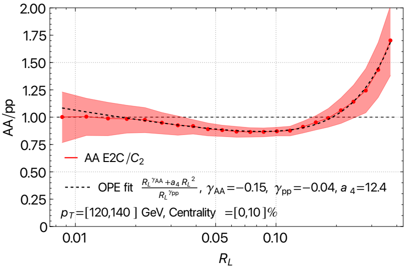


$$\frac{\langle\Psi_{QGP}|\mathcal{E}(n_1)\mathcal{E}(n_2)|\Psi_{QGP}\rangle}{\langle\Psi_{pp}|\mathcal{E}(n_1)\mathcal{E}(n_2)|\Psi_{pp}\rangle} = \frac{\langle\Psi_{QGP}|\frac{1}{\theta^2}\mathbb{O}_{\tau=2}^{[j=3]}+\mathbb{O}_{\tau=4}^{[j=3]}+\cdots|\Psi_{QGP}\rangle}{}$$

$$\sim \frac{\langle\Psi_{QGP}|\mathbb{O}_{\tau=2}^{[j=3]}|\Psi_{QGP}\rangle}{\langle\Psi_{pp}|\mathbb{O}_{\tau=2}^{[j=3]}|\Psi_{pp}\rangle} + \theta^2\frac{\langle\Psi_{QGP}|\mathbb{O}_{\tau=4}^{[j=3]}|\Psi_{QGP}\rangle}{\langle\Psi_{pp}|\mathbb{O}_{\tau=2}^{[j=3]}|\Psi_{pp}\rangle}$$

Correlators in the QGP

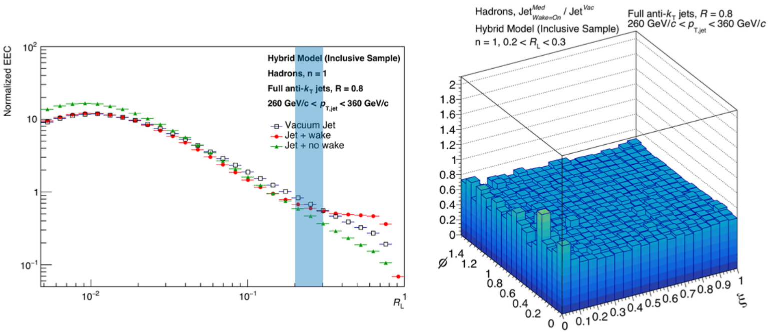
- Simple OPE picture provides excellent description of CMS data.



- Allows us to use jet substructure to study matrix elements of higher twist operators, and their evolution.

Resolving the Scales of the QGP

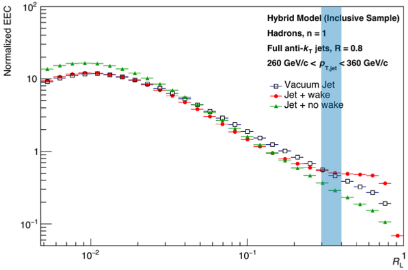
- Higher point correlators allow us to probe the "shape" of medium modifications as a function of scale:

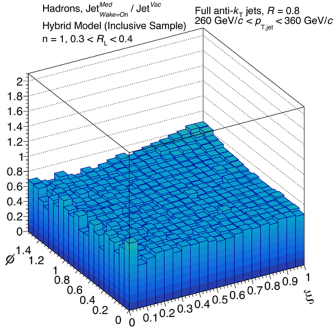


[Bossi, He, Kudinoor, Moult, Pablos, Rai, Rajagopal]

Resolving the Scales of the QGP

- Higher point correlators allow us to probe the "shape" of medium modifications as a function of scale:

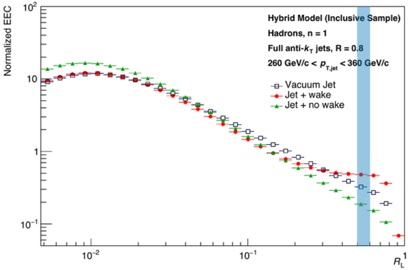


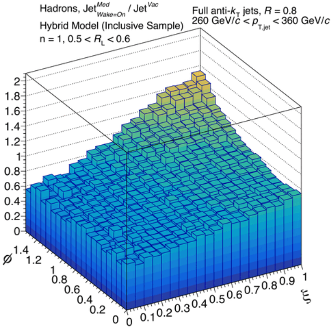


[Bossi, He, Kudinoor, Moult, Pablos, Rai, Rajagopal]

Resolving the Scales of the QGP

- Higher point correlators allow us to probe the "shape" of medium modifications as a function of scale:

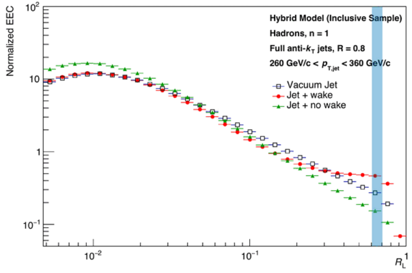


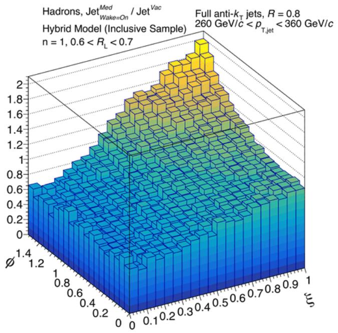


[Bossi, He, Kudinoor, Moult, Pablos, Rai, Rajagopal]

Resolving the Scales of the QGP

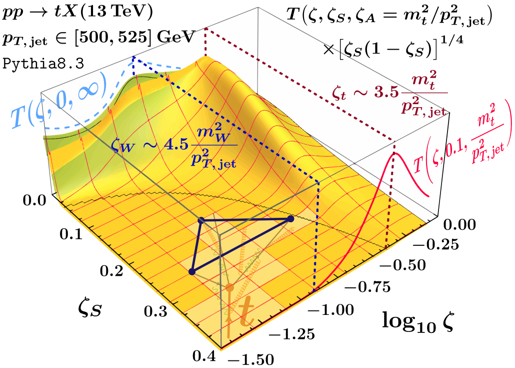
- Higher point correlators allow us to probe the "shape" of medium modifications as a function of scale:





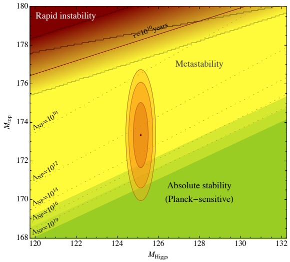
[Bossi, He, Kudinoor, Moult, Pablos, Rai, Rajagopal]

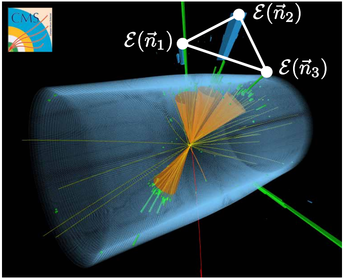
Weighing the Top Quark

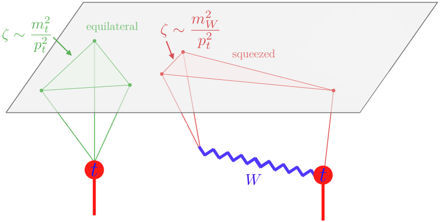


Weighing the Top Quark

- The top quark mass is one of the most important parameters of the SM. e.g. electroweak vacuum stability/criticality, electroweak fits, etc.
- Need simple observables with top mass sensitivity that can be computed from first principles field theory.

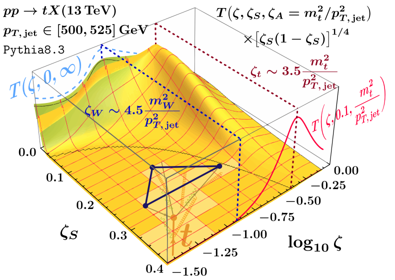


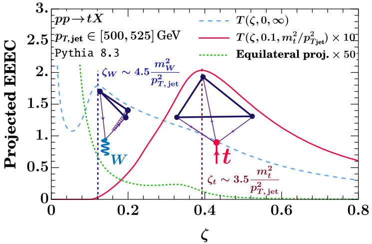


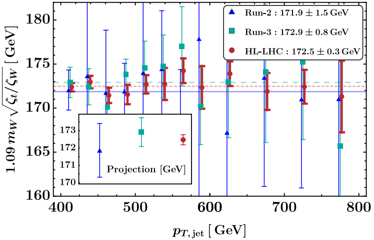


Weighing the Top Quark

- Extract the mass ratio between the W and top quark from the shape of the three-point correlator.







[Holguin, Moult, Pathak, Procura, Schofbeck, Schwarz]

See also: [Xiao, Ye, Zhu]

- Motivates precision calculations of correlators on top decays.

Weighing the Top Quark

- Initial investigations illustrate has minor sensitivity to experimental systematics, and global event: successfully isolates dynamics of top decay.

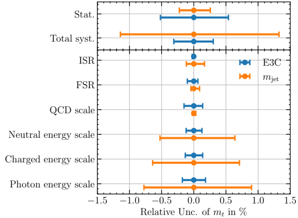
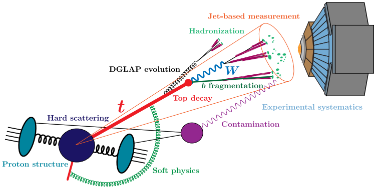
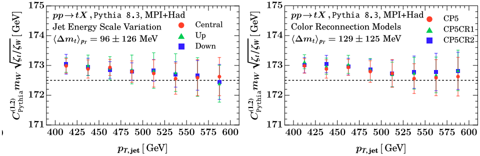


Figure 2. The expected uncertainties of m_t (in % of m_t = 171 GeV) using E3C and $m_{\rm jet}$ distributions, at $\mathcal{L} = 36\,\text{fb}^{-1}$. The statistical uncertainties and a breakdown of the systematic uncertainties are shown.

[Xiao, Ye, Zhu]

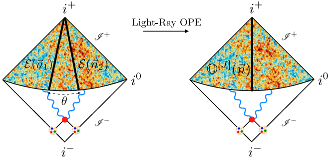




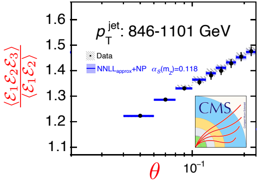
- Motivates precision calculations of correlators on top decays, and further experimental investigation.

Summary

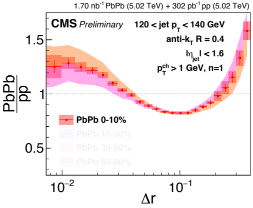
- Significant recent progress in the theoretical characterization of asymptotic energy flux.

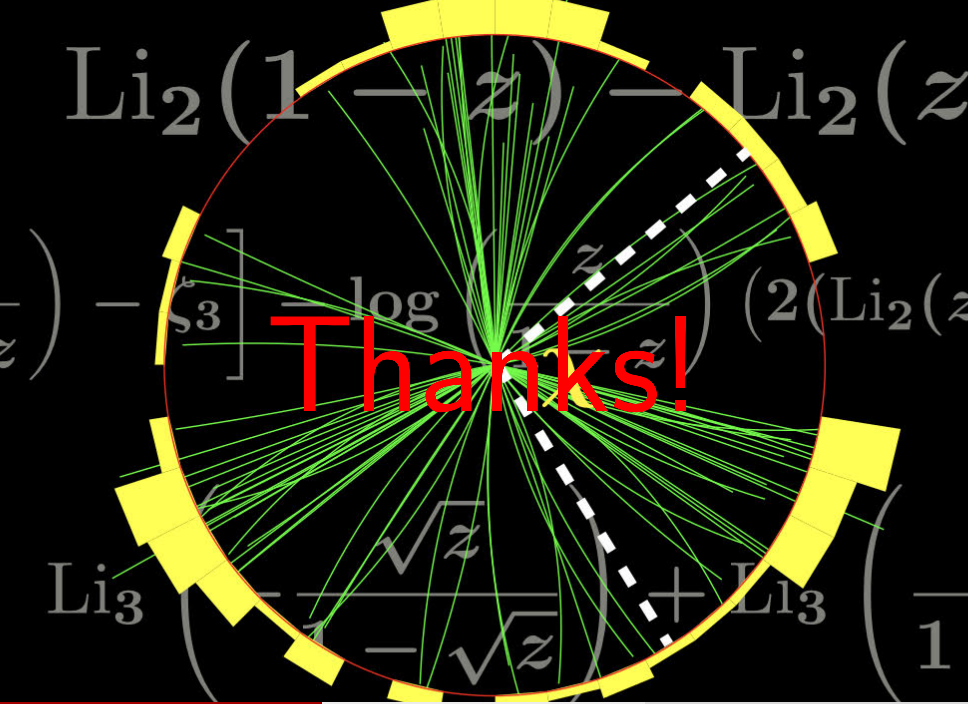


- Scaling and shape dependence of multi-point energy correlators can be directly measured at the LHC: How can we best use them?



- Provides the opportunity to use theoretically beautiful objects to learn about the real world.





Thanks!