BERNHARD MISTLBERGER

EXPLORING COLLINEAR LIMITS OF SCATTERING CROSS SECTIONS

- With Gherardo Vita

MOTIVATION FOR PRECISION

The LHC will deliver measurements at the percent level.

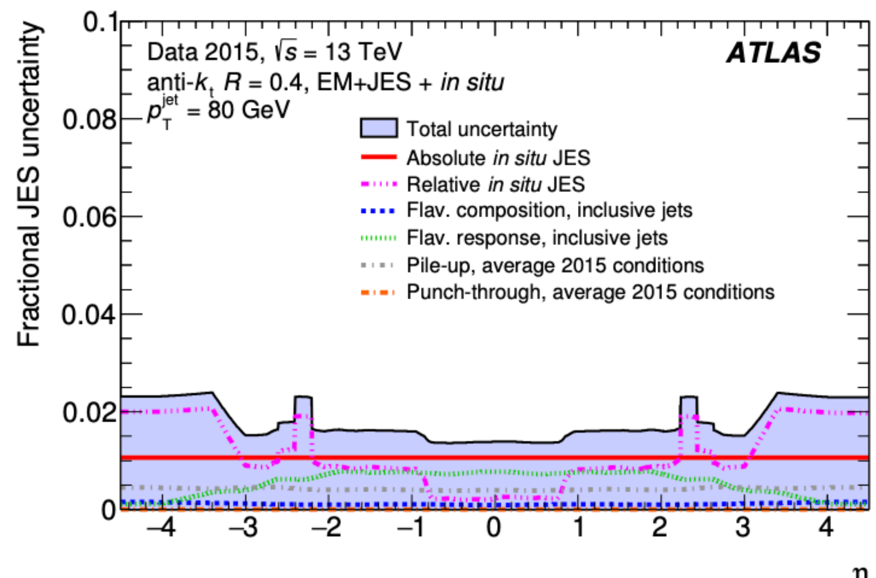
Fundamental Limitations:

[CMS, 2104.01927]

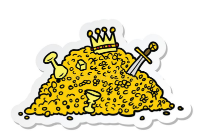
- Luminosity
  (any absolute measurement)

Source	2015 [%]	2016 [%]	Corr
Total normalization uncertainty	1.3	1.0	—
Total integration uncertainty	1.0	0.7	—
Total uncertainty	1.6	1.2	—

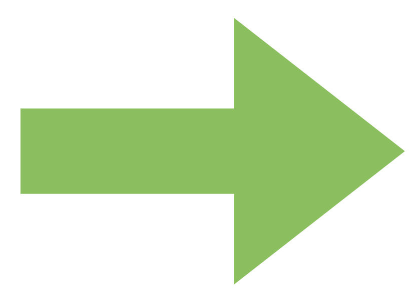
- Jet-Energy Scale



- Statistics



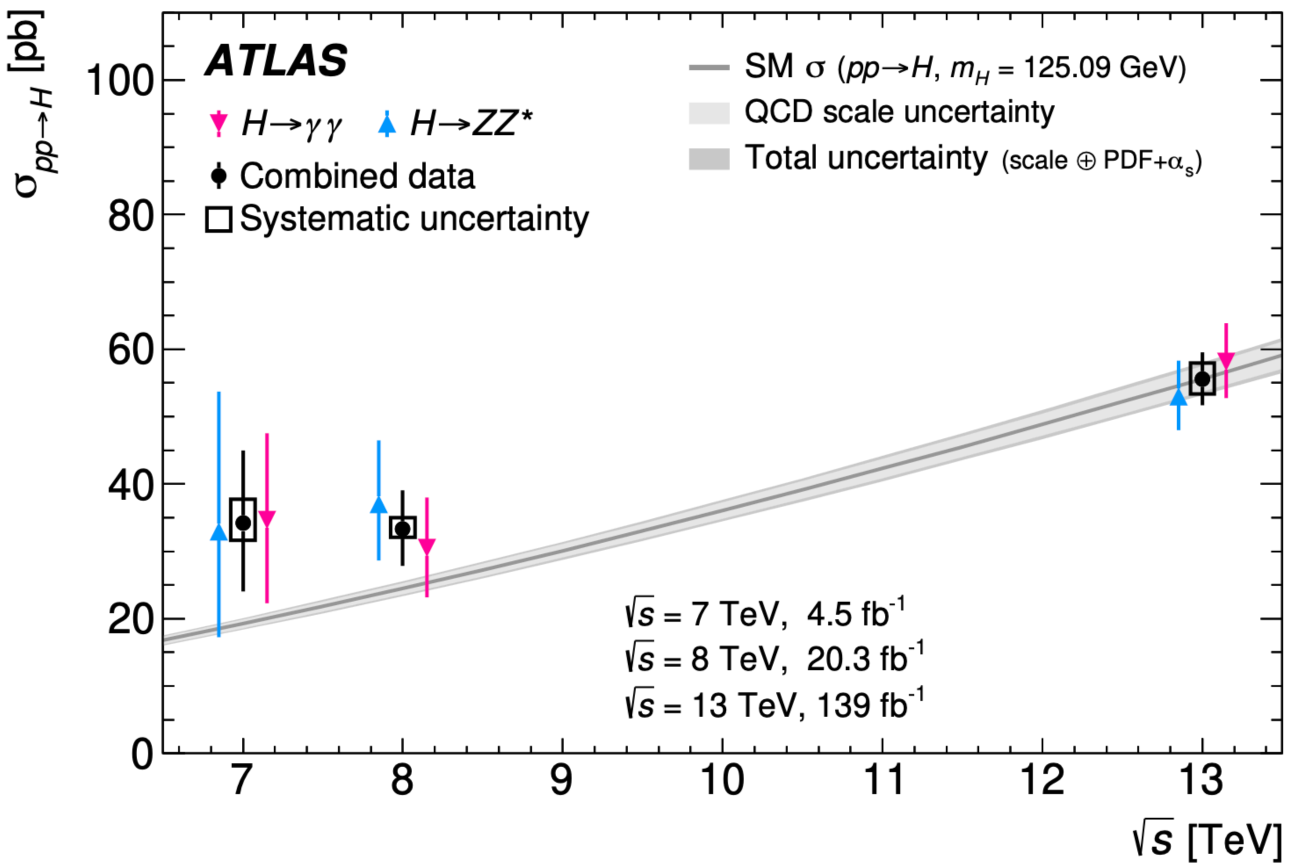
140 fb^-1



3000 fb^-1

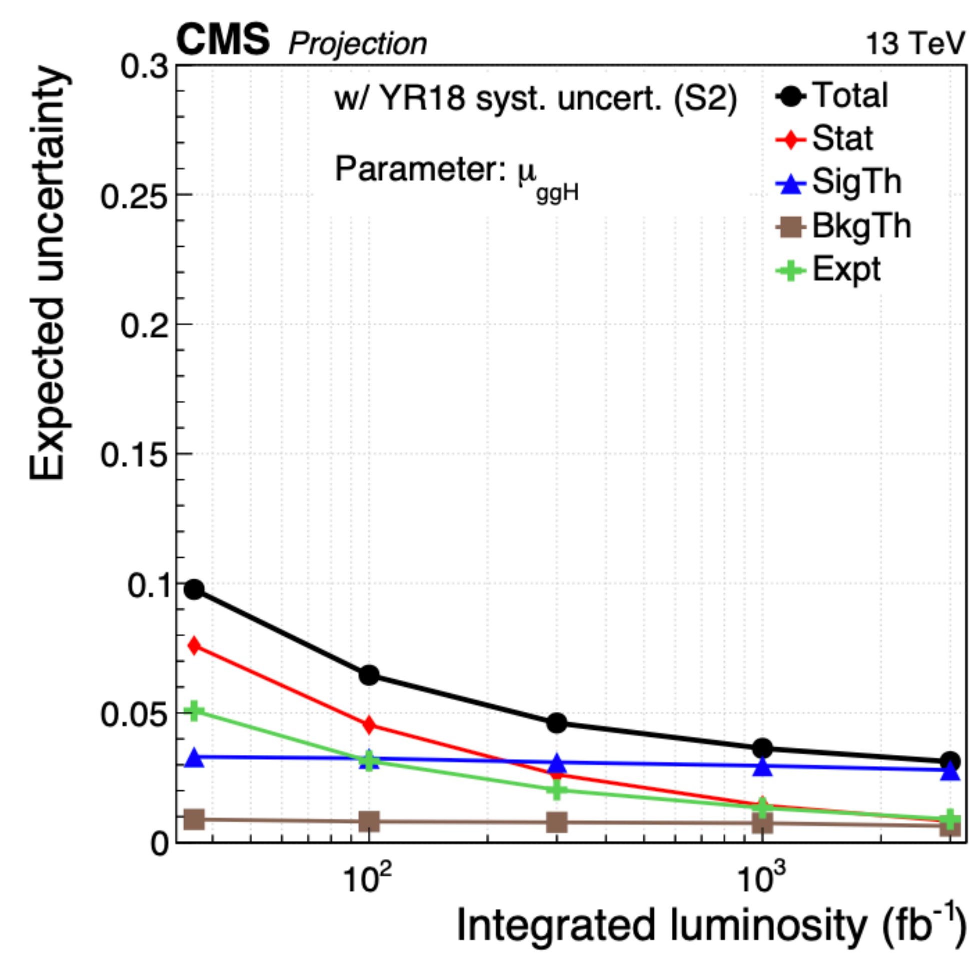
PRECISION OBSERVABLES

How many Higgs bosons are produced at the LHC?



$$\mu = \frac{\sigma_{\text{obs.}}}{\sigma_{\text{SM}}}$$

- Agreement of EXP and TH at ~ 7%!
- TH Uncertainty ~ Exp Uncertainty
- Incredible future precision envisioned! ~ 2.5 %



PRECISION OBSERVABLES

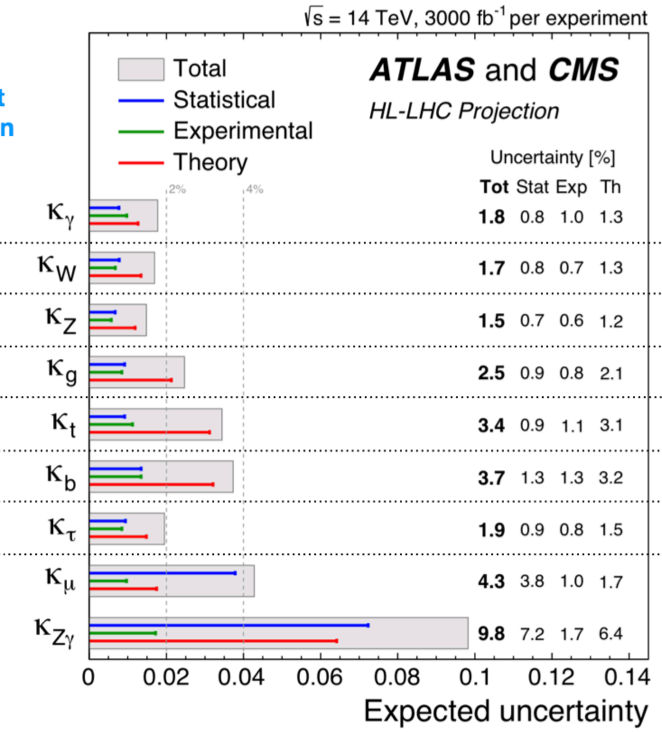
The LHC will deliver measurements at the percent level.

Phenomenology:

- Higgs Properties:

	ATLAS - CMS Run 1 combination	ATLAS Run 2	CMS Run 2	Current precision
κ_γ	13%	1.04 ± 0.06	$1.01^{+0.09}_{-0.14}$	6%
κ_W	11%	1.06 ± 0.06	$-1.11^{+0.14}_{-0.09}$	6%
κ_Z	11%	0.99 ± 0.06	0.96 ± 0.07	6%
κ_g	14%	$0.92^{+0.07}_{-0.06}$	$1.16^{+0.12}_{-0.11}$	7%
κ_t	30%	0.92 ± 0.10	1.01 ± 0.11	11%
κ_b	26%	0.87 ± 0.11	$1.18^{+0.19}_{-0.27}$	11%
κ_τ	15%	0.92 ± 0.07	0.94 ± 0.12	8%
	JHEP 08 (2016) 045	ATLAS-CONF-2021-53	CMS-PAS-HIG-19-005	

Measurements here assume no BSM in Higgs width



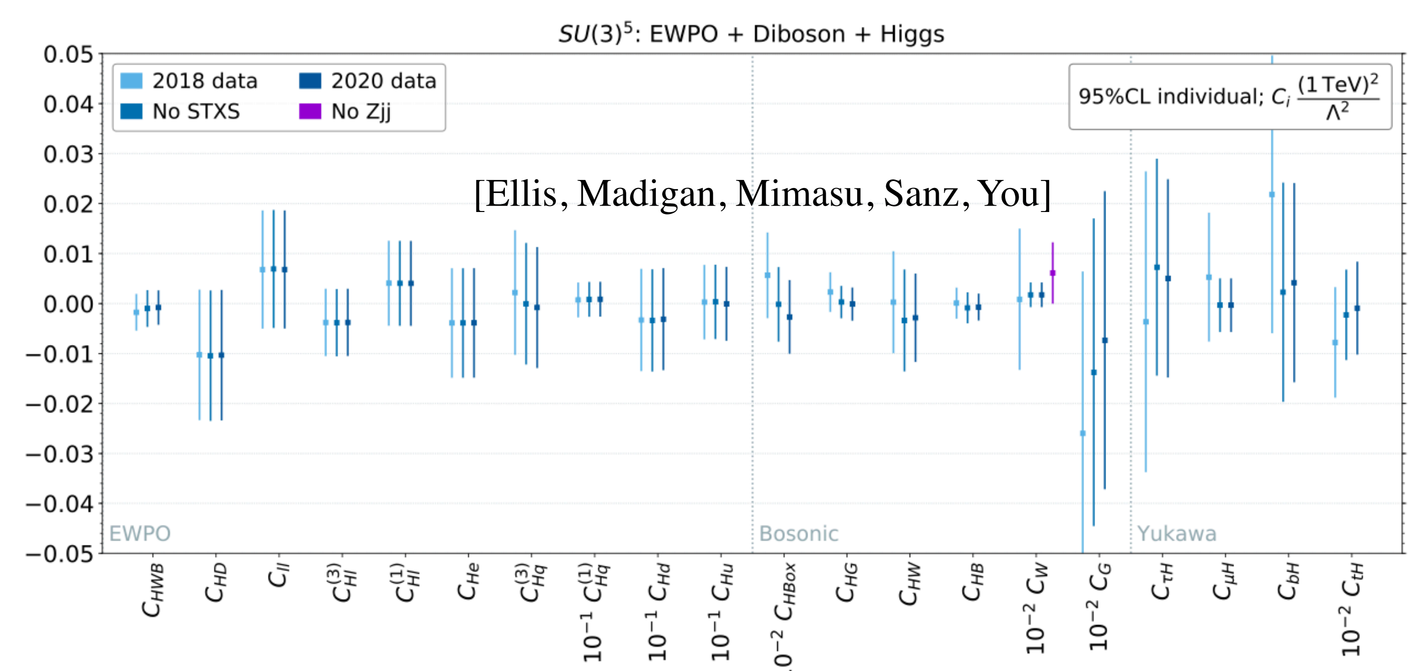
Still 25 times more data and reduction of a factor of 3 uncertainty!

TH uncertainty dominant!

- Establish the SM at the 1 % level

Z **W** **top** **jets**

$$m_W^2\left(1-\frac{m_W^2}{m_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_\mu}.$$



- Search for deviations

PRECISION OBSERVABLES

The Method: Predict & Compare.

$$\mu = \frac{\sigma_{\rm obs.}}{\sigma_{\rm SM}}$$

$$\sigma_{\rm obs.} - \sigma_{\rm SM} =\sim \frac{1}{\Lambda^2} \sum_i C_i \sigma^i_{\rm EFT}$$

- **Percent level comparison of a LARGE range of observables is about to be a reality**

PRECISION OBSERVABLES

The Method: Predict & Compare.

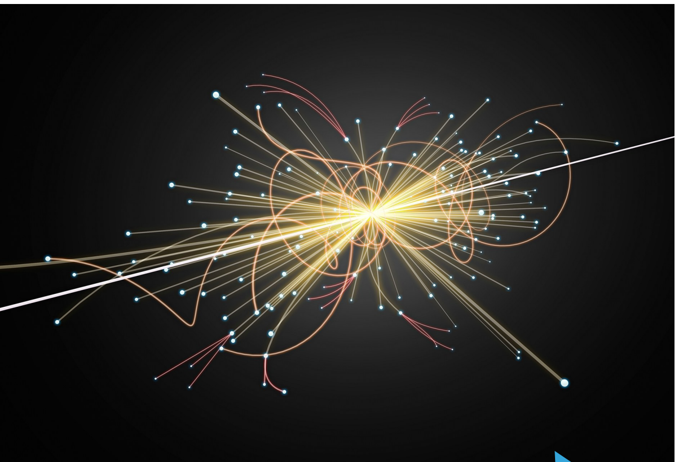
$$\mu = \frac{\sigma_{\text{obs.}}}{\sigma_{\text{SM}}}$$

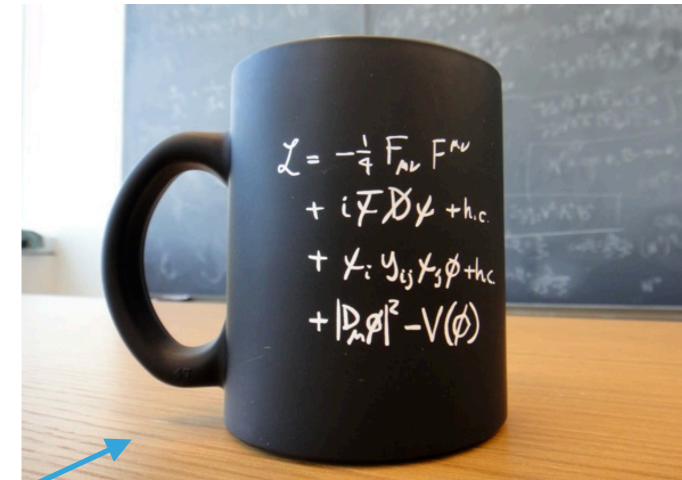
$$\sigma_{\text{obs.}} - \sigma_{\text{SM}} = \sim \frac{1}{\Lambda^2} \sum_i C_i \sigma^i_{\text{EFT}}$$

- **Percent level comparison of a LARGE range of observables is about to be a reality**

Precision Theory

THE WAY TO PRECISION LHC PREDICTIONS



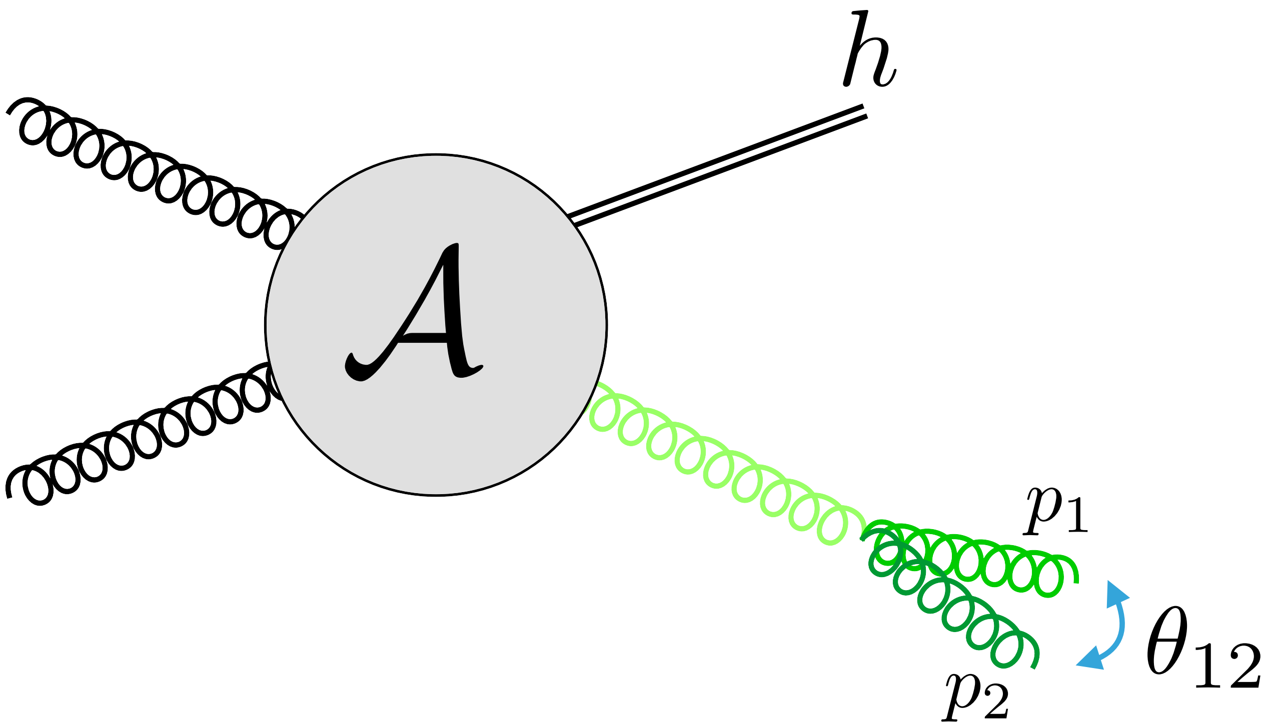


$$\sigma \sim \int dx dy f(x) f(y) \hat{\sigma} + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$

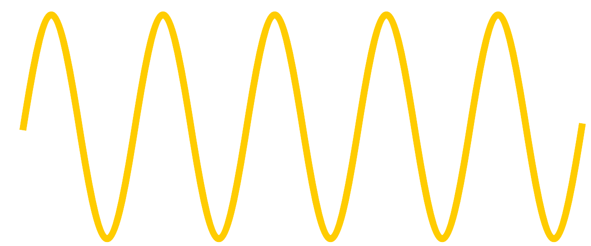
- Perturbative partonic cross sections
- Making progress is hard but desired on many fronts!
- The path we want to follow here:

Explore one **universal** problem with perturbative computations and use our improvements for **new phenomenology.**

COLLINEAR LIMITS OF CROSS SECTIONS



Collinear Particles



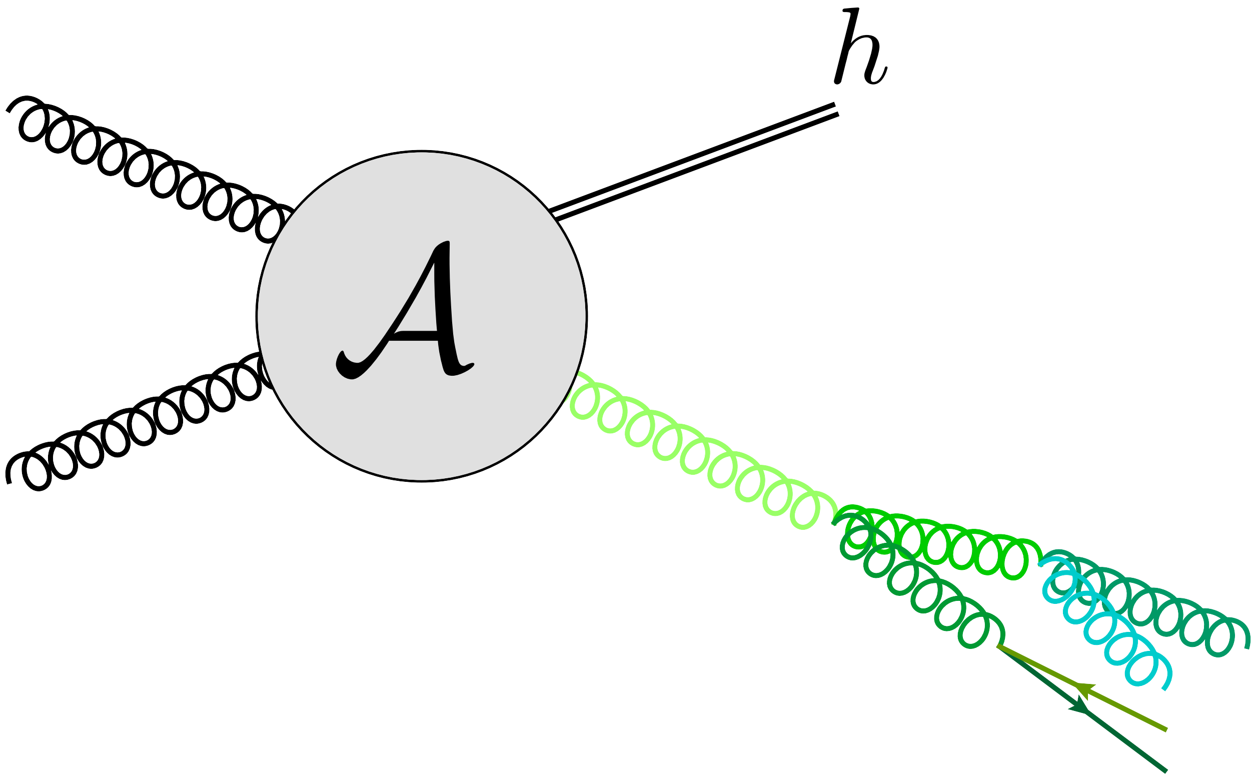
Consider angular separation smaller than resolution of our detectors.

Two particles look like one!

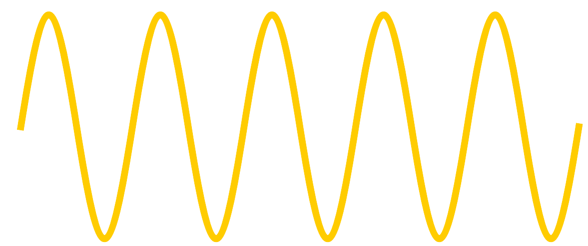
Sum of quantum numbers of many particles = quantum number of one mother particle.

$$\lim_{p_1 || p_2} \mathcal{A}_{ggggh} \sim \mathcal{A}_{gggh} \times g_S \frac{1}{p_1 . p_2} \sim \frac{1}{1 - \cos(\theta_{12})}$$

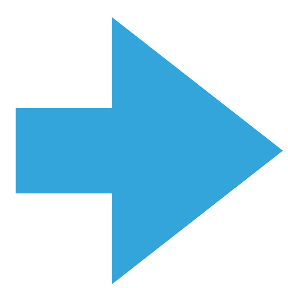
COLLINEAR LIMITS OF CROSS SECTIONS



Collinear Particles



Consider angular separation smaller than resolution of our detectors.

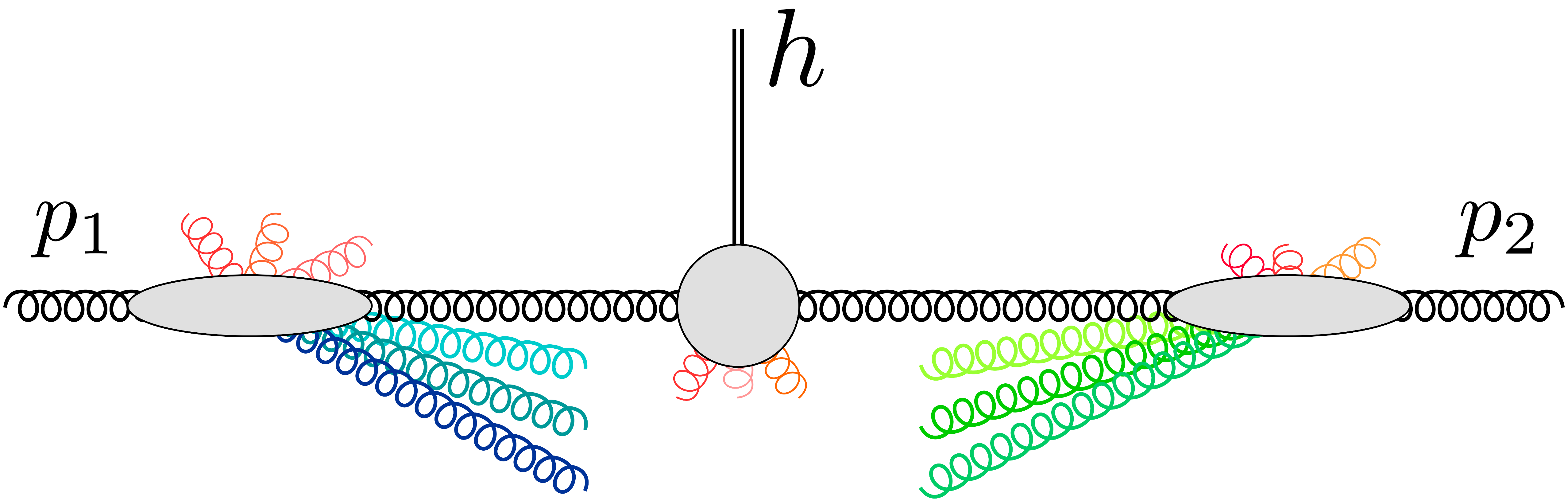


Many particles look like one!

- Breakdown of perturbation theory in limit of small particle angles.
- If we probe this limit with observation - resolve it - we need to include (infinitely) many orders in perturbation theory.

COLLINEAR LIMITS OF CROSS SECTIONS

Higgs Boson: $g\,g \to h$

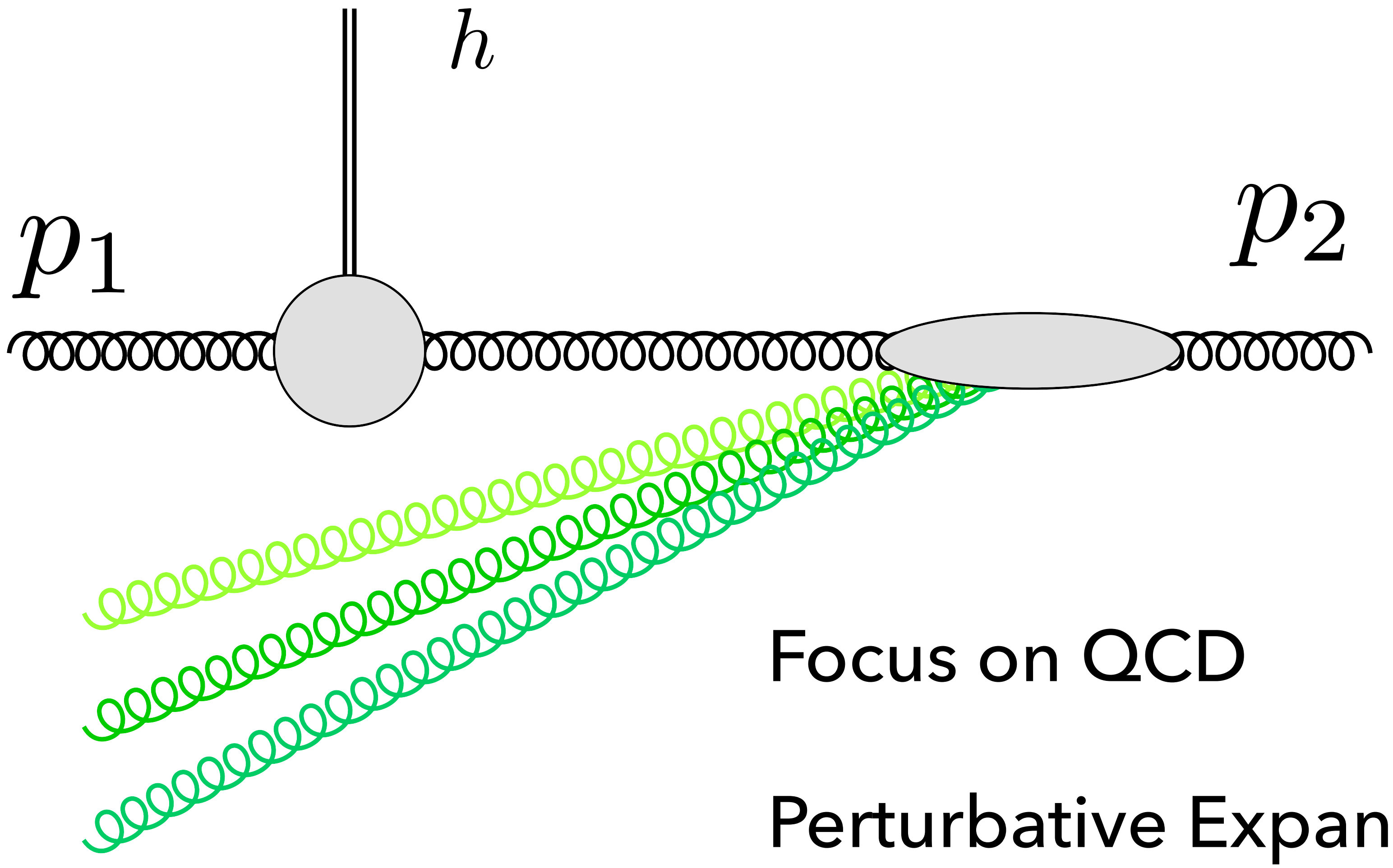


Interesting Radiation:

- Collinear to p1
- Collinear to p2
- Soft

COLLINEAR LIMIT

Partonic Scattering Process:



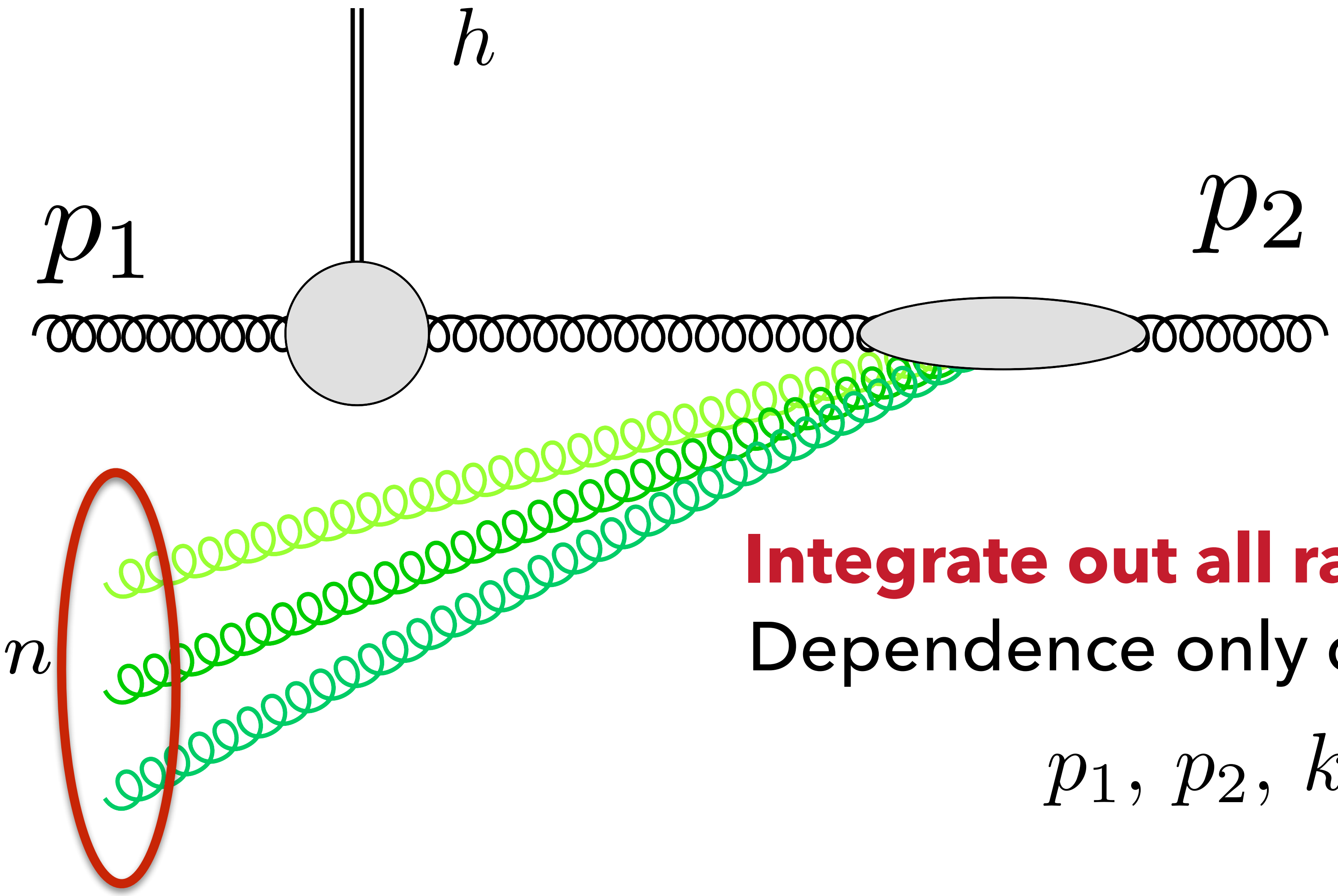
Focus on QCD

Perturbative Expansion:

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S^1 \hat{\sigma}^{(1)} + \alpha_S^2 \hat{\sigma}^{(2)} + \alpha_S^3 \hat{\sigma}^{(3)} \ldots$$

COLLINEAR LIMIT

Partonic Scattering Process:



$$\int d\Phi_n$$

$$k = \sum_i p_i$$

Integrate out all radiation

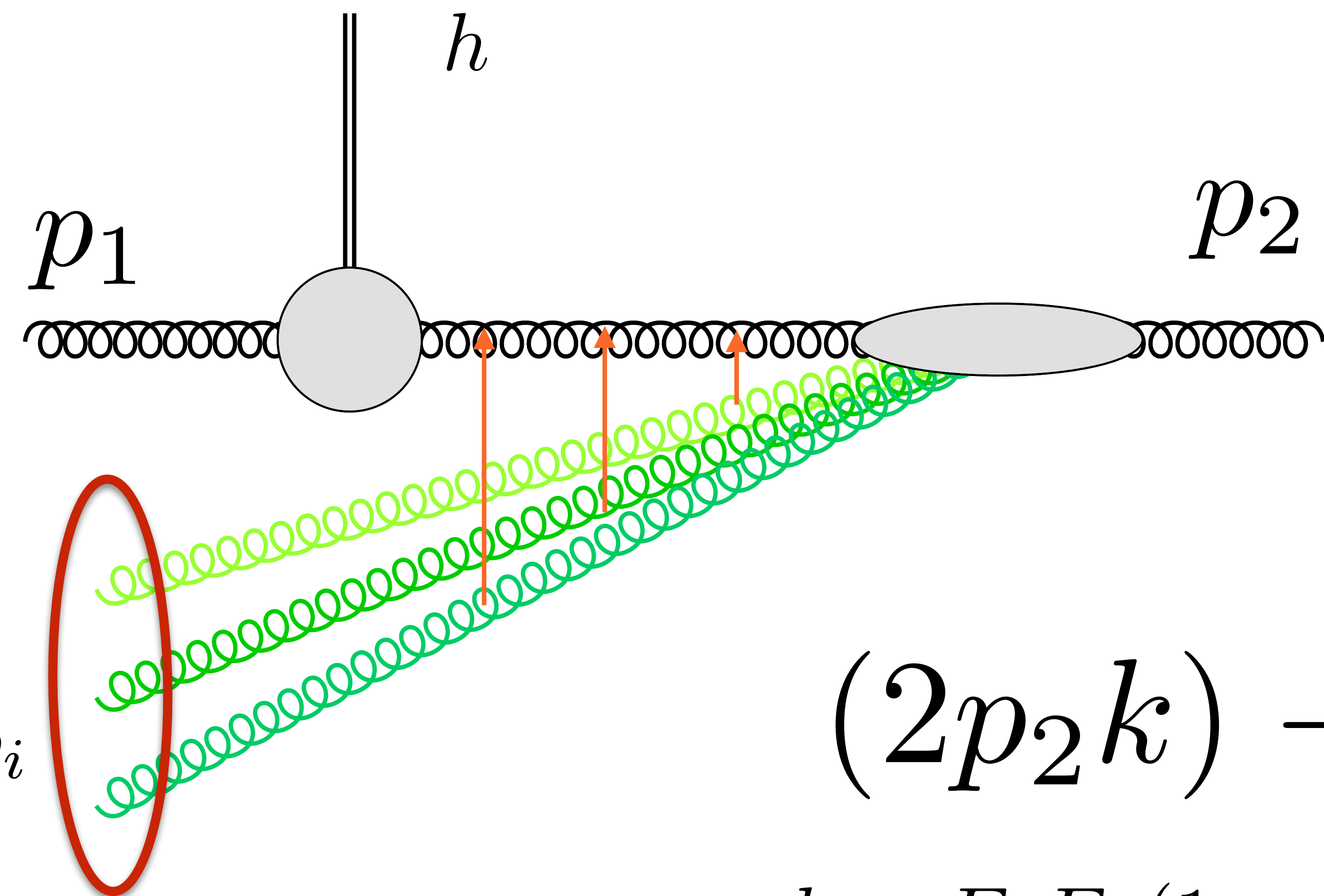
Dependence only on momenta

$$p_1,\ p_2,\ k,\ p_h$$

Variables: $\{s, p_\perp^2, Y, m_h^2\}$

COLLINEAR LIMIT

Approaching the limit:



$$k = \sum_i p_i$$

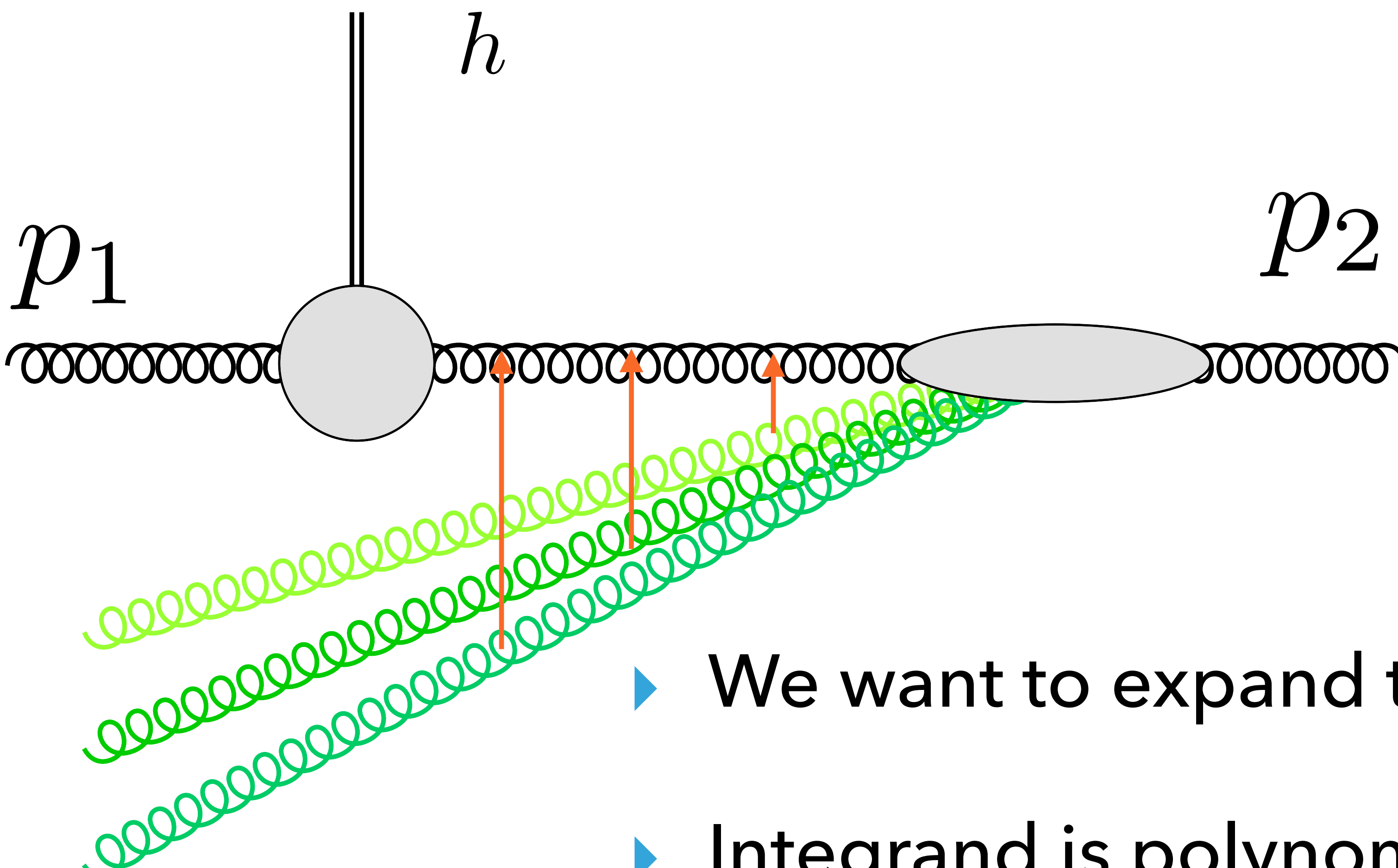
$$(2p_2 k) \to 0$$

$$p_2 k \sim E_2 E_k (1 - \cos\theta_{2k})$$

EXPANDING AROUND COLLINEAR LIMITS

Goal:

Formulate a systematic expansion around the collinear limit of colorless final state LHC cross sections.

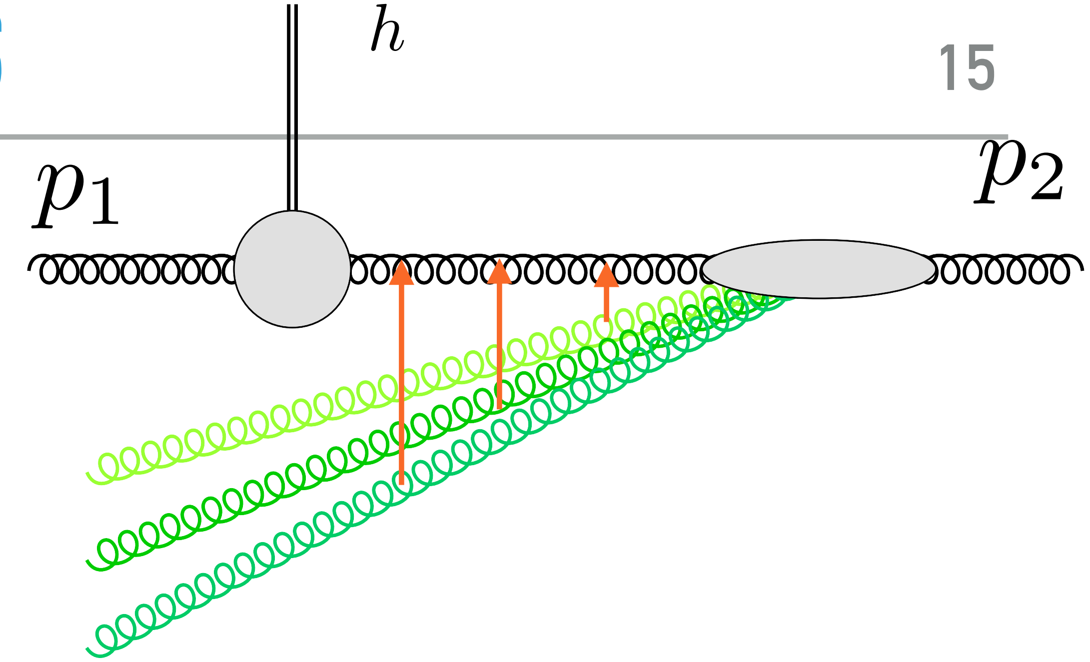


Expansion parameter:

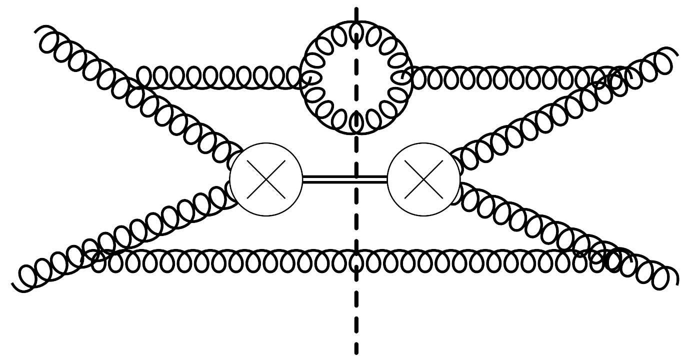
$$\sim 2p_2k$$

- We want to expand the Feynman integrand!
- Integrand is polynomial in momenta: easy to expand!
- Work in DimReg with $d = 4 - 2\epsilon$

EXPANDING AROUND COLLINEAR LIMITS



1 Write down all your Feynman diagrams!
(Drell-Yan + Higgs production)



2 Expand all integrands around the limit of all final state momenta collinear to p2

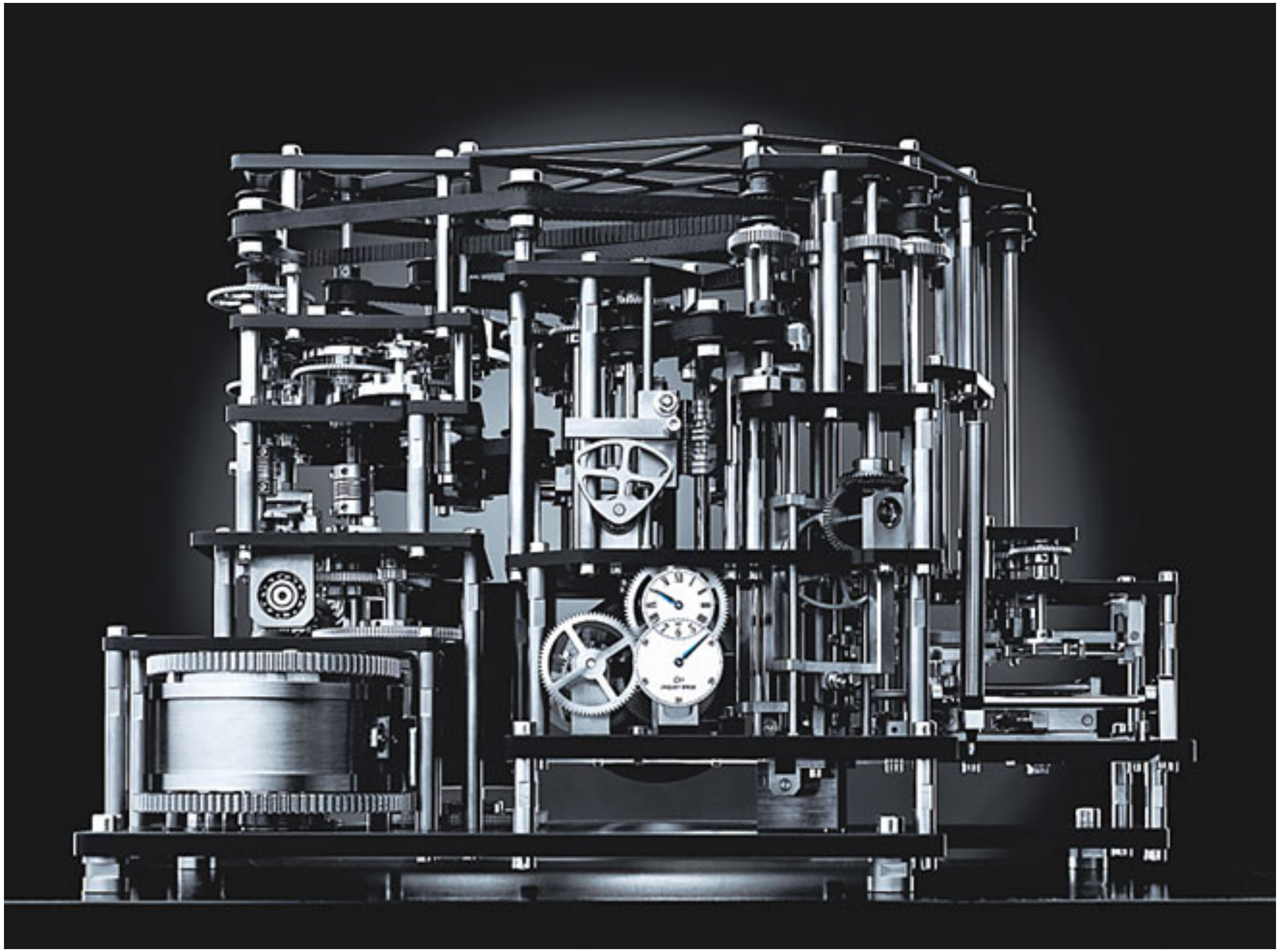
$$p_i \to \alpha p_2 + \delta^2 \beta p_1 + \delta p_{i,\perp} \qquad\qquad \delta \to 0$$

(Similar to SCET / CSS / Catani+Grazzini, apply method of regions!)

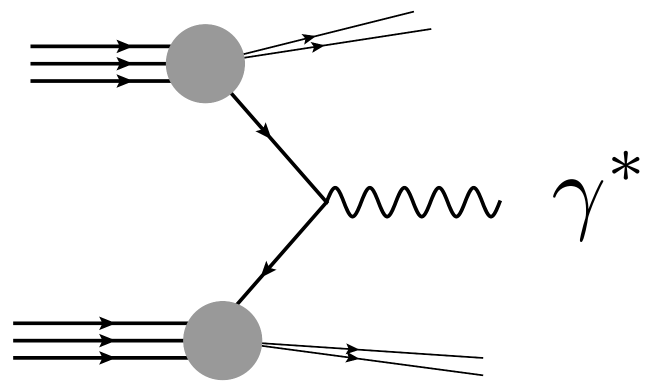
3 Carry out all our loop and phase-space integrals!

[Ebert,BM,Vita 2006.03055]

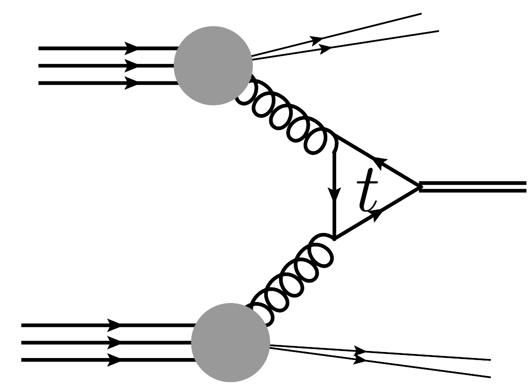
THE CROSS SECTION CALCULATION MACHINE



Drell - Yan



Higgs

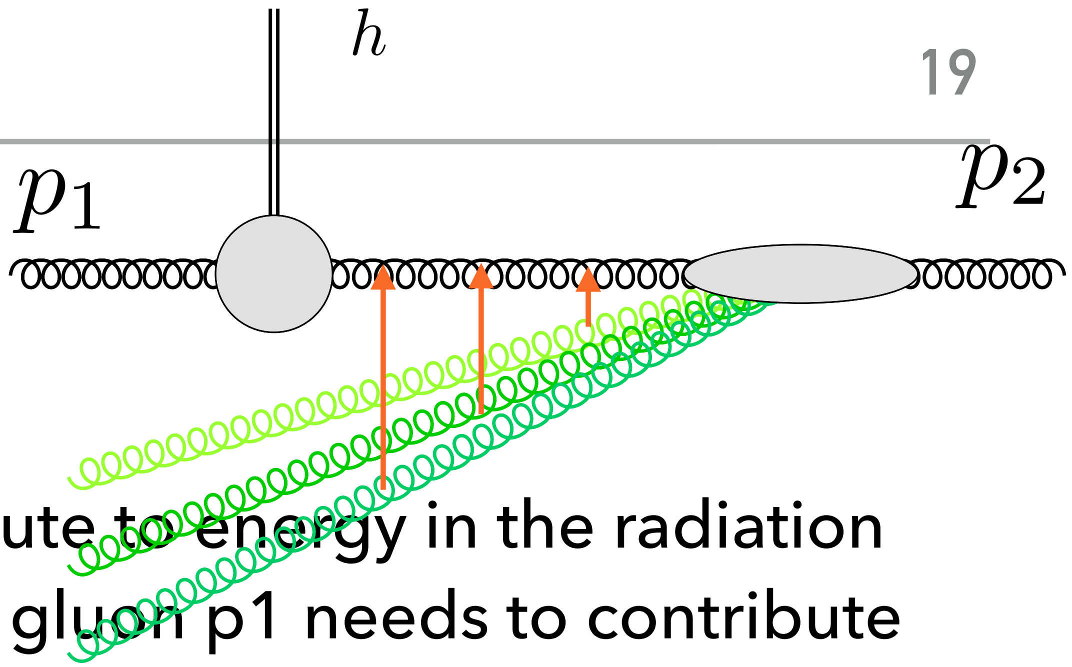


- **Expanded partonic cross section up to N3LO in QCD to first order in collinear limit.**

- Feynman diagrams: $\mathcal{O}(10^6)$
- 492 new master integrals using differential equations.
- Fully analytic results for
  the collinear limit of the partonic cross section.

APPLICATIONS
APPROXIMATIONS FOR CROSS SECTIONS

APPROXIMATIONS OF CROSS SECTIONS



- We found a systematic expansion around collinear limit.

- k || p2: Gluon with p1 does not have to contribute to energy in the radiation -> all its energy can go to the Higgs. The more gluon p1 needs to contribute to radiation, the smaller phase space gets -> collinear enhancement.

- Born level observable: Y - Rapidity distribution.

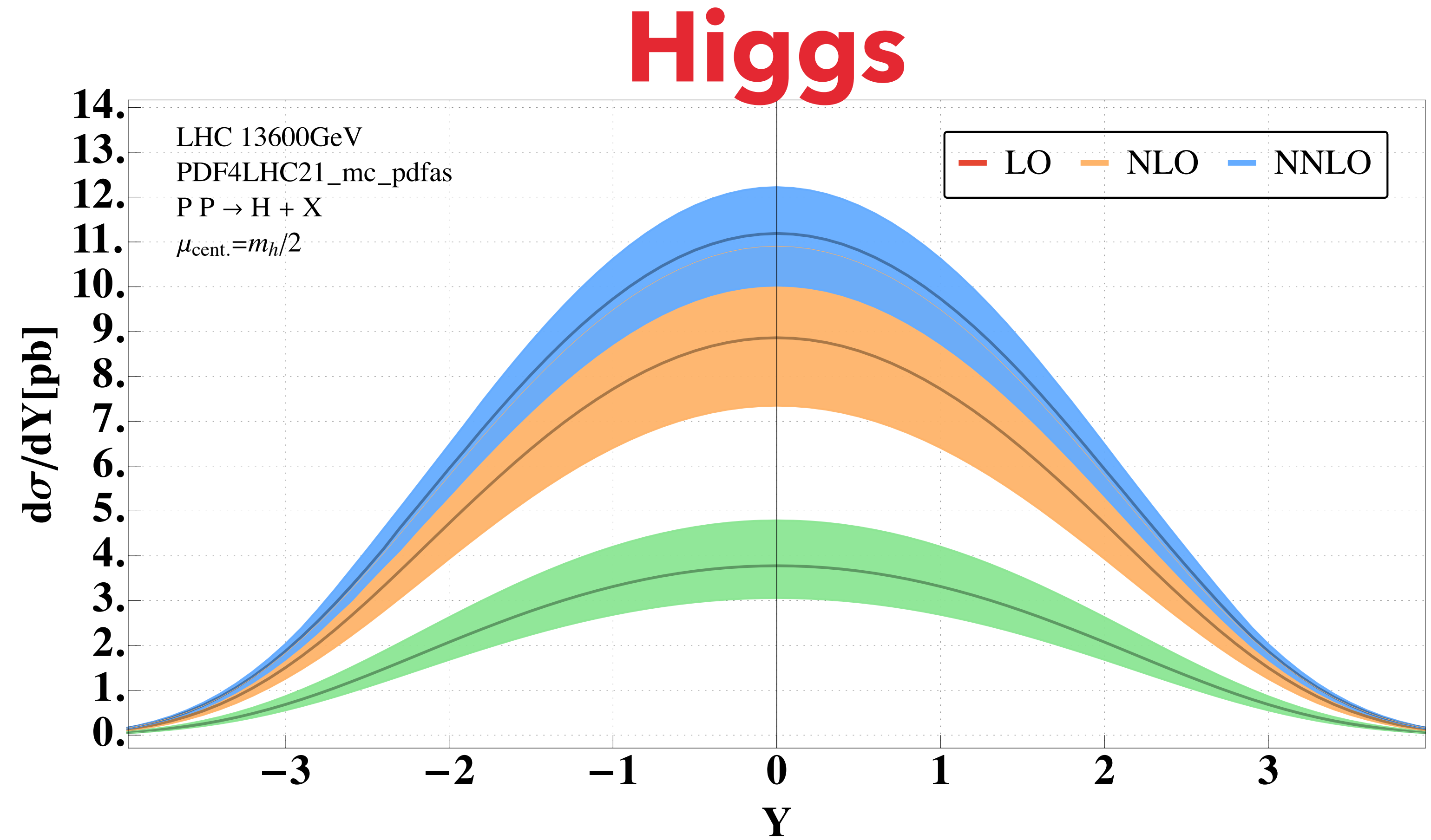
- Create approximation:

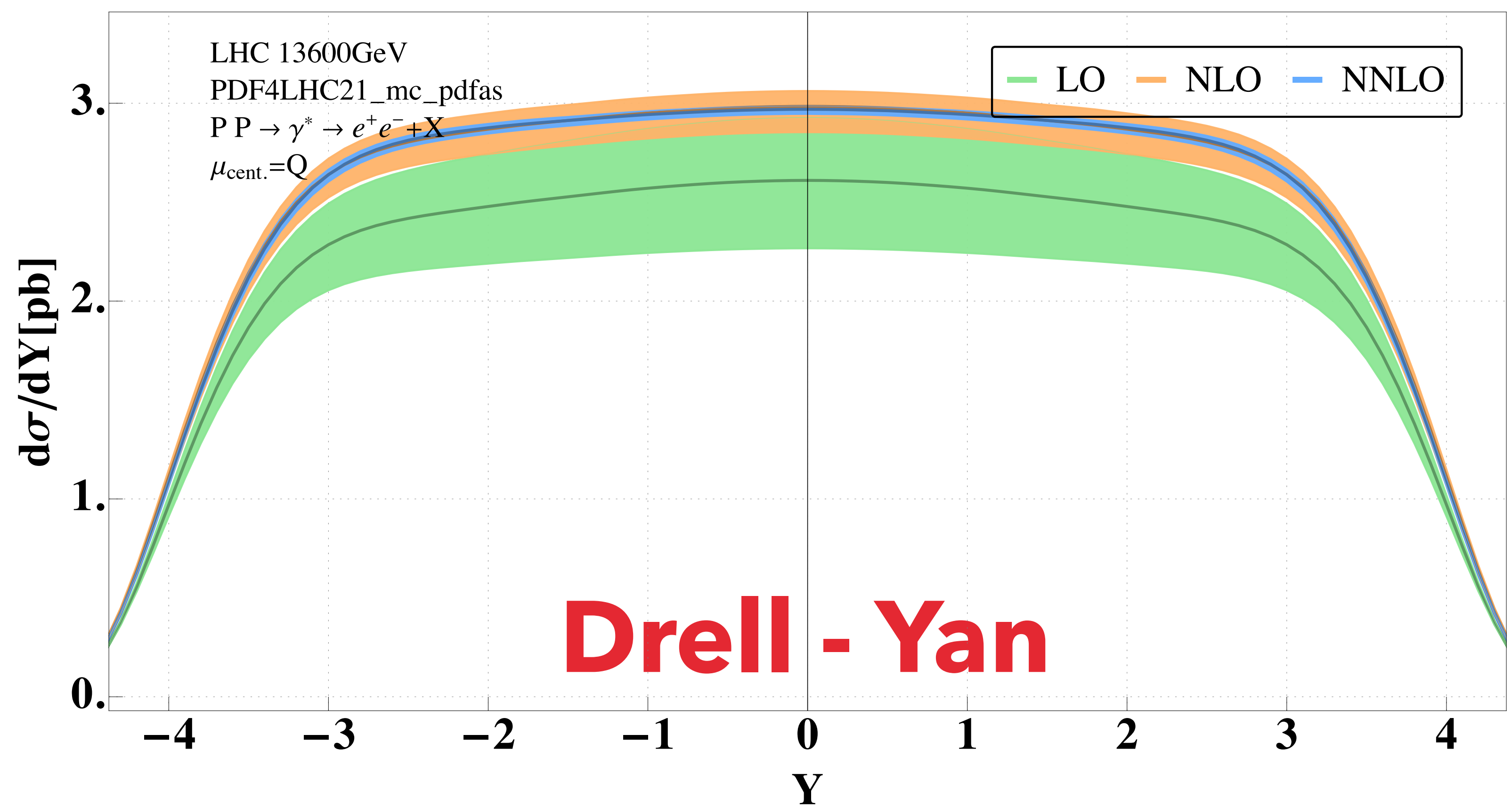
$$\frac{d\sigma}{dY} \sim \left.\frac{d\sigma}{dY}\right|_{C_1} + \left.\frac{d\sigma}{dY}\right|_{C_2} - \left.\frac{d\sigma}{dY}\right|_{C_1+C_2}$$

k || p1   k || p2   k || p1 & k || 2 = soft

RAPIDITY DISTRIBUTION AT NNLO

- Exact results available -> testing and validation
- See how well collinear expansion approximates exact results

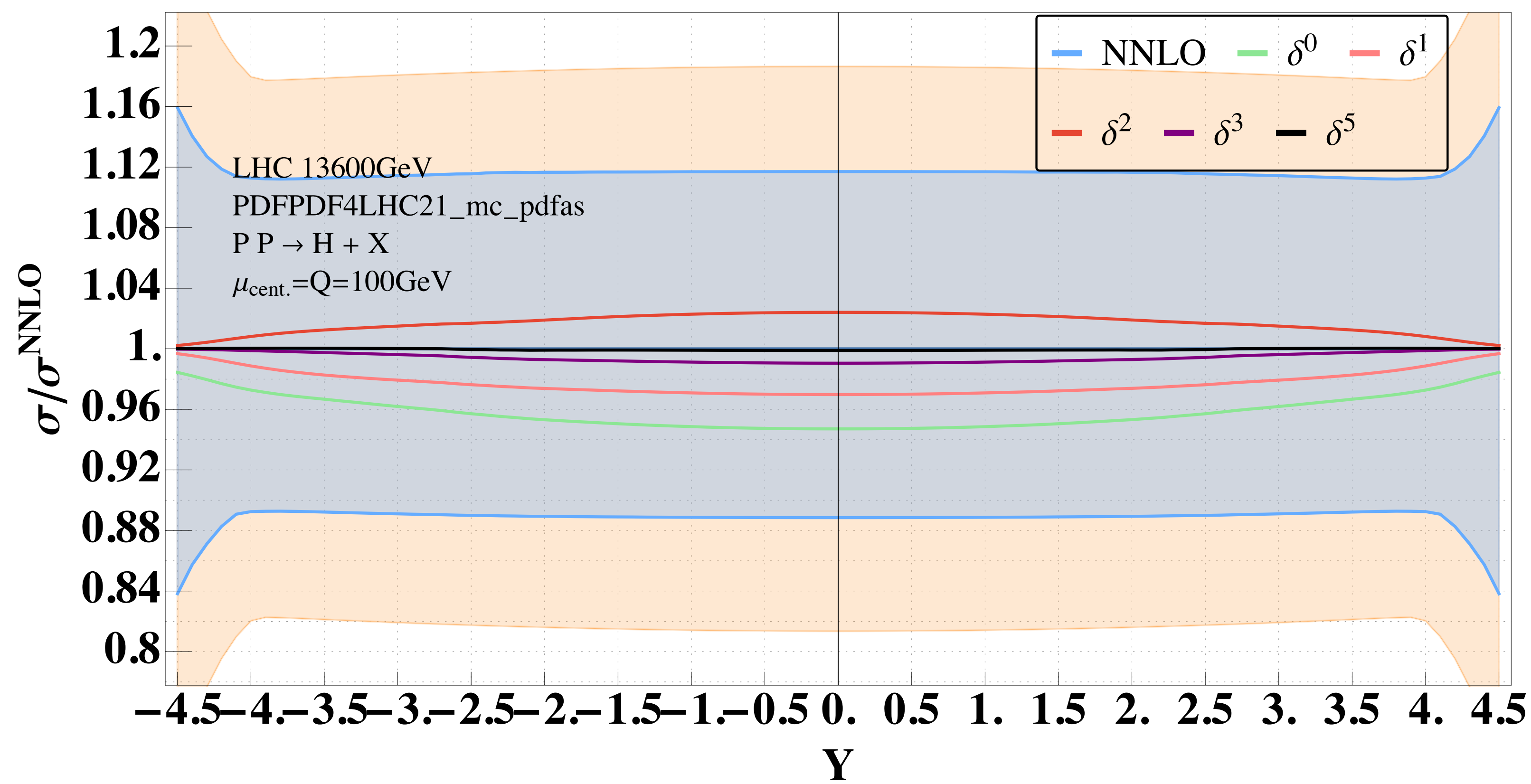


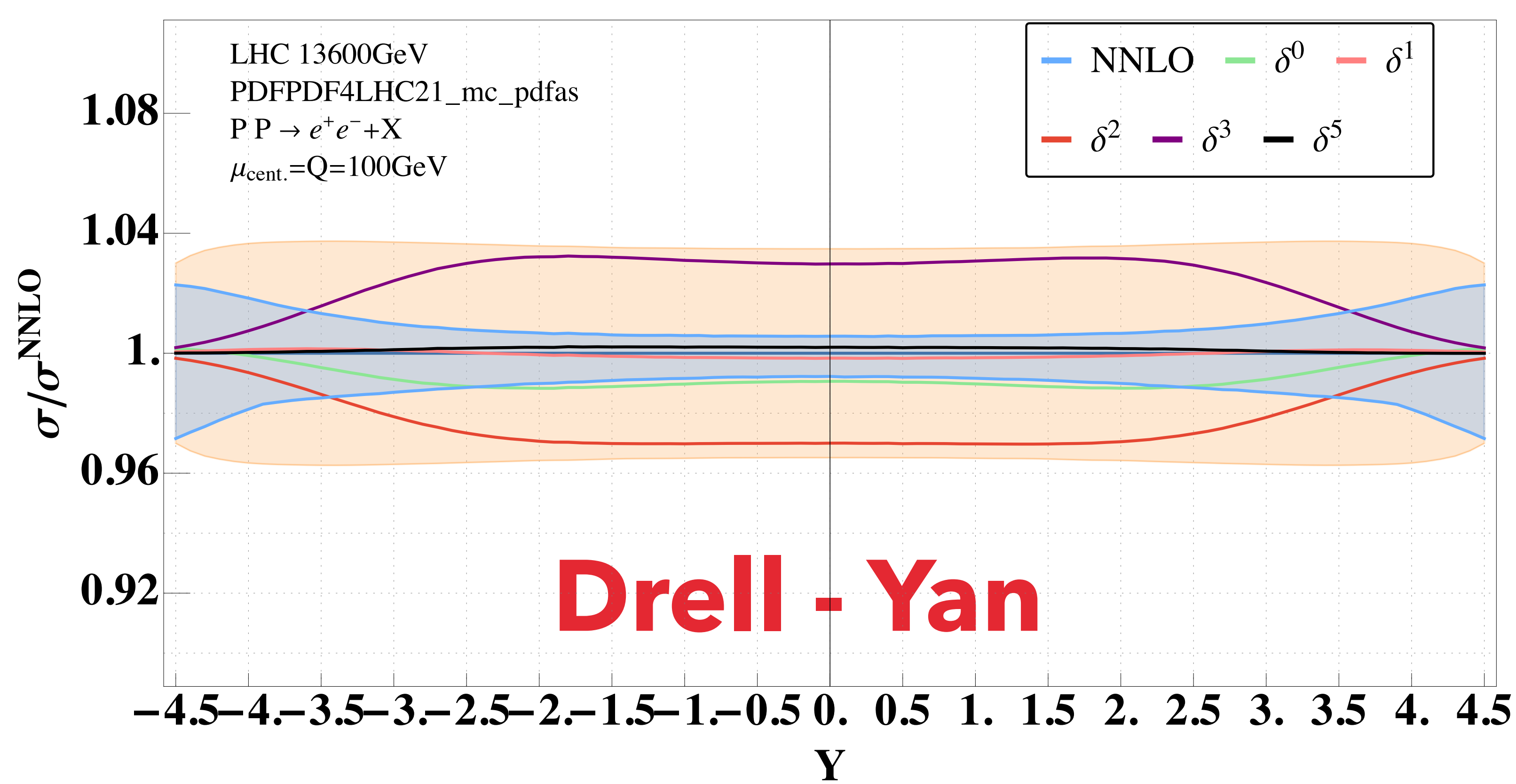


RAPIDITY DISTRIBUTION AT NNLO

- Q=100 GeV.
- Collinear expansion works well as an approximation for Higgs (gluon-induced) processes.
- Uncertainty on DY-type processes much smaller.
- Great in very forward region.

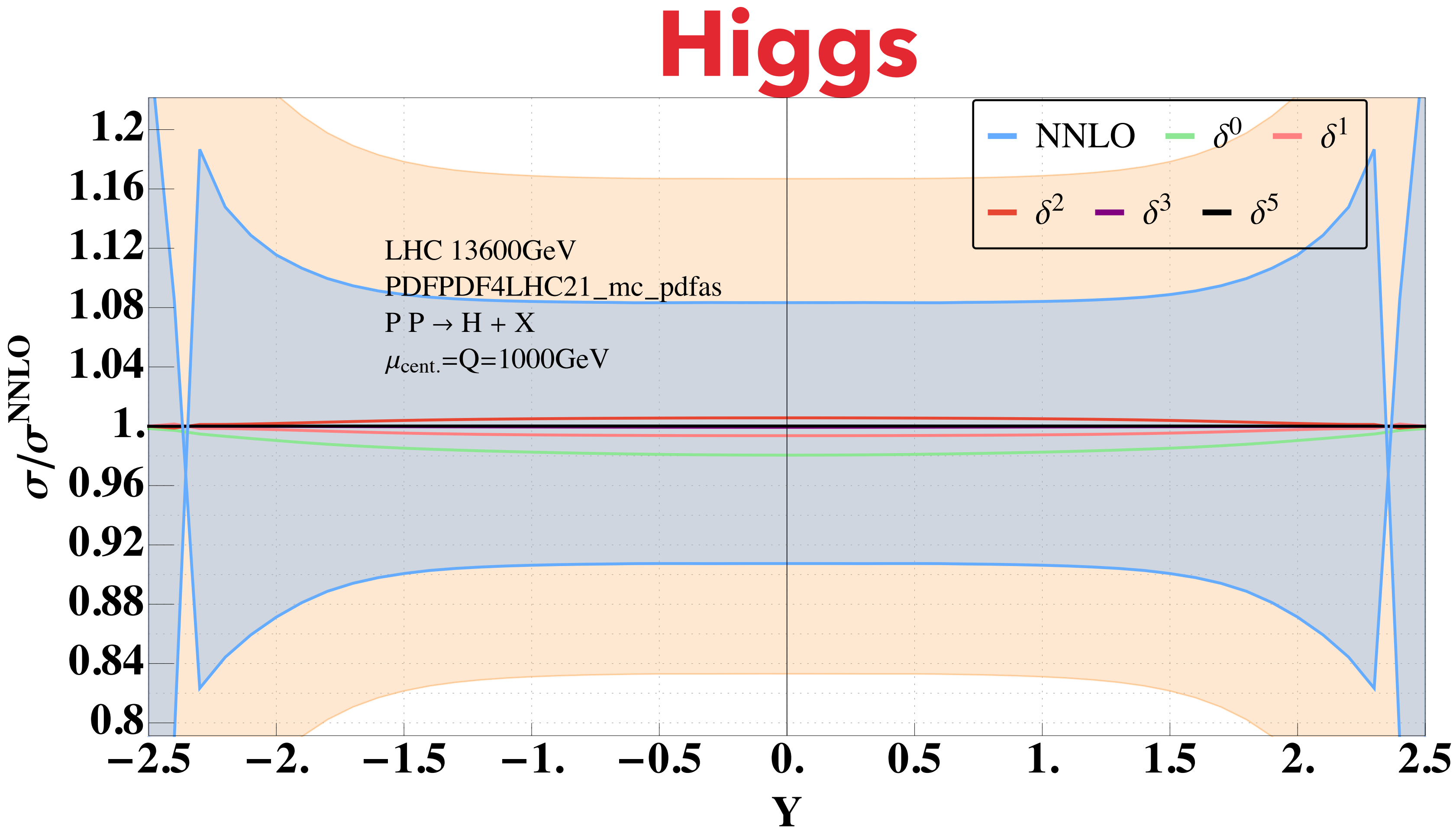
Higgs

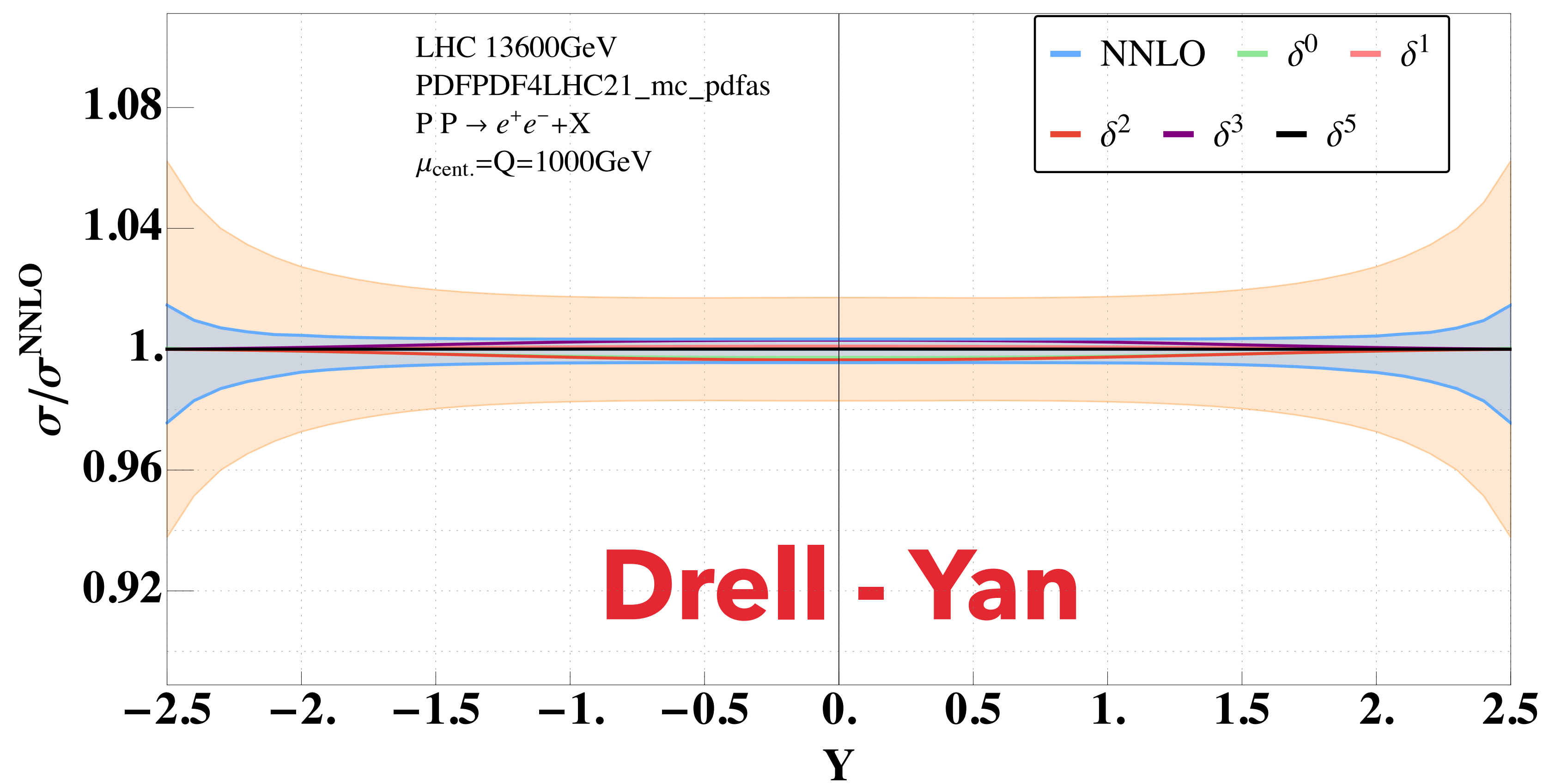




RAPIDITY DISTRIBUTION AT NNLO

- Q=1000 GeV.
- Collinear expansion works well for quark and gluon induced at high invariant mass.



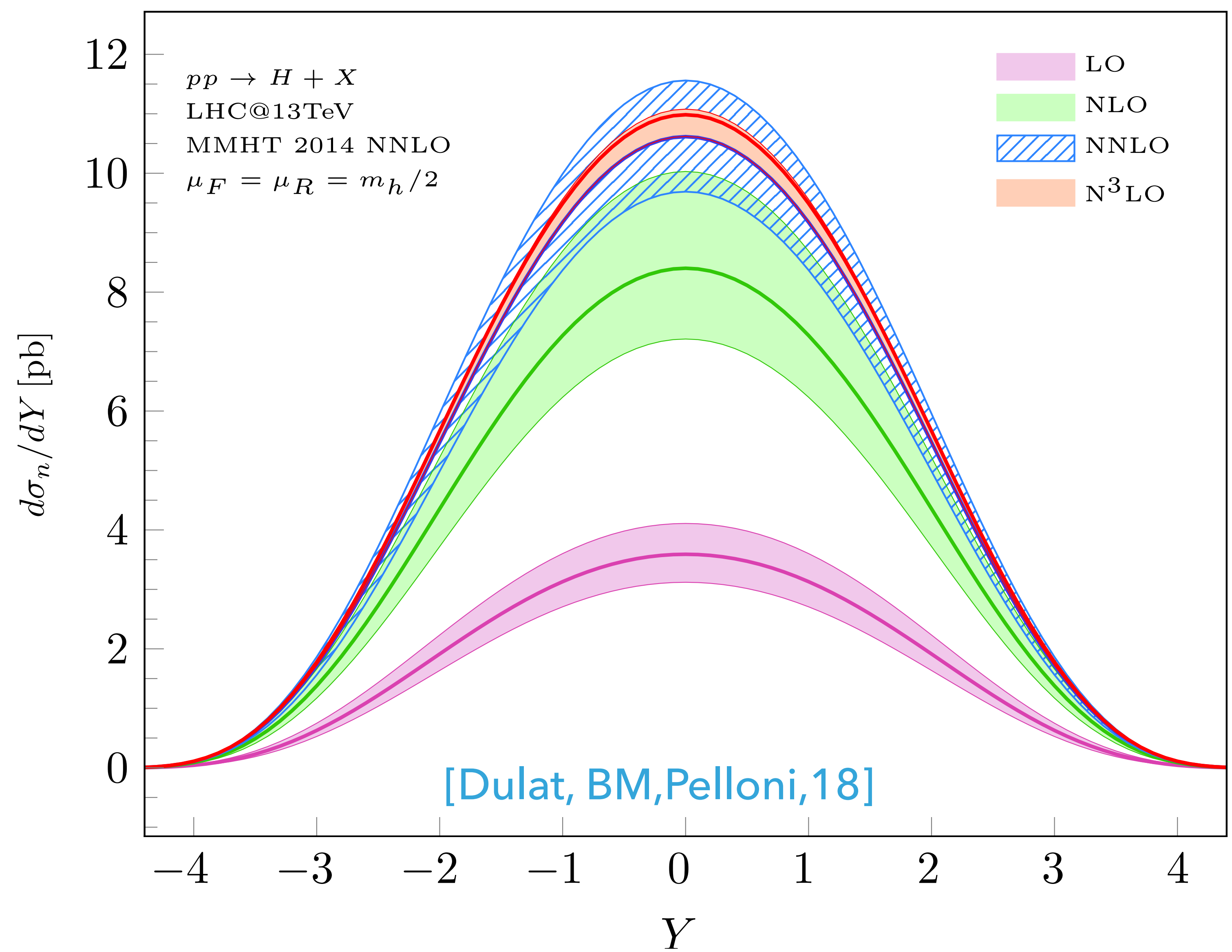


RAPIDITY DISTRIBUTION AT N3LO

- Created approximation for the ggF Higgs boson rapidity distribution at N3LO.

Ingredients:

- Collinear expansion*.
- Threshold Expansion (6 terms)
- Matching to inclusive cross section.
- Bonus Logs.



- Fully analytic:
  fast evaluation times!

FULLY DIFFERENTIAL HIGGS AT N3LO

- Predict fully differential distributions at the LHC:

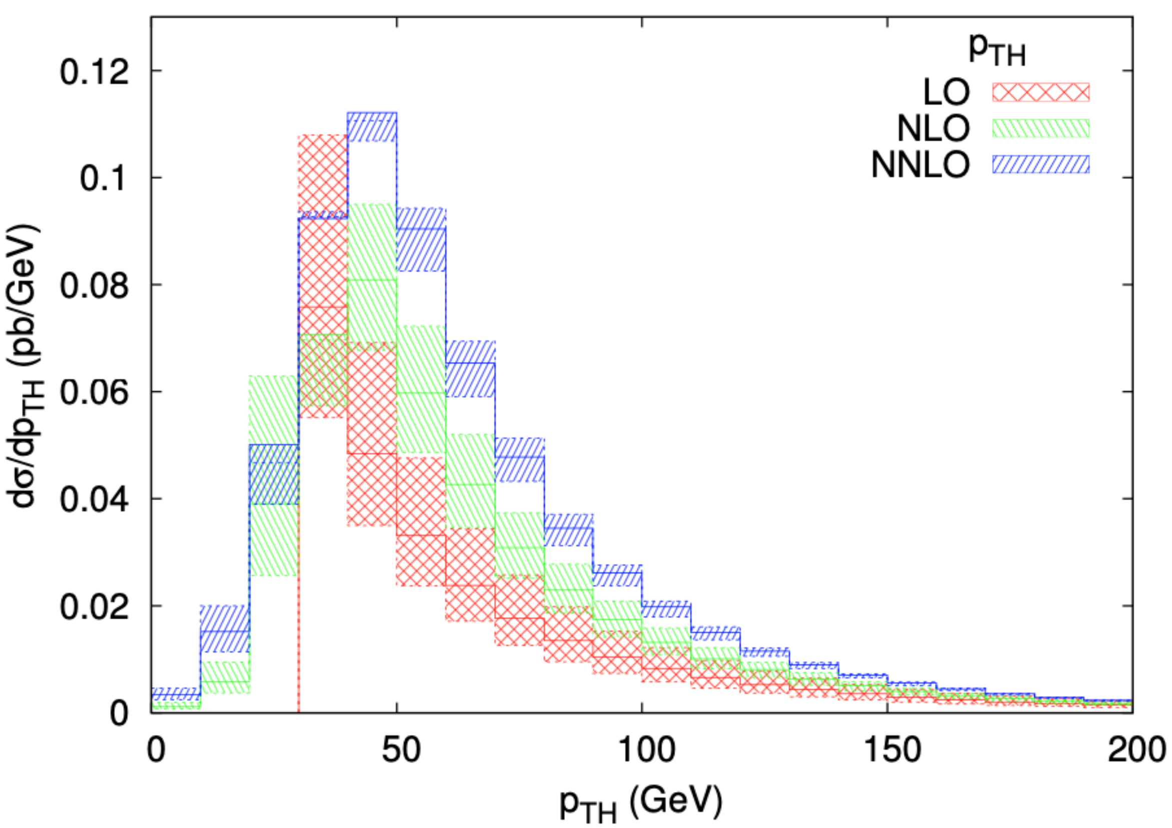
[Cacciari,Dreyer,Karlberg,Salam,Zanderighi; 15]

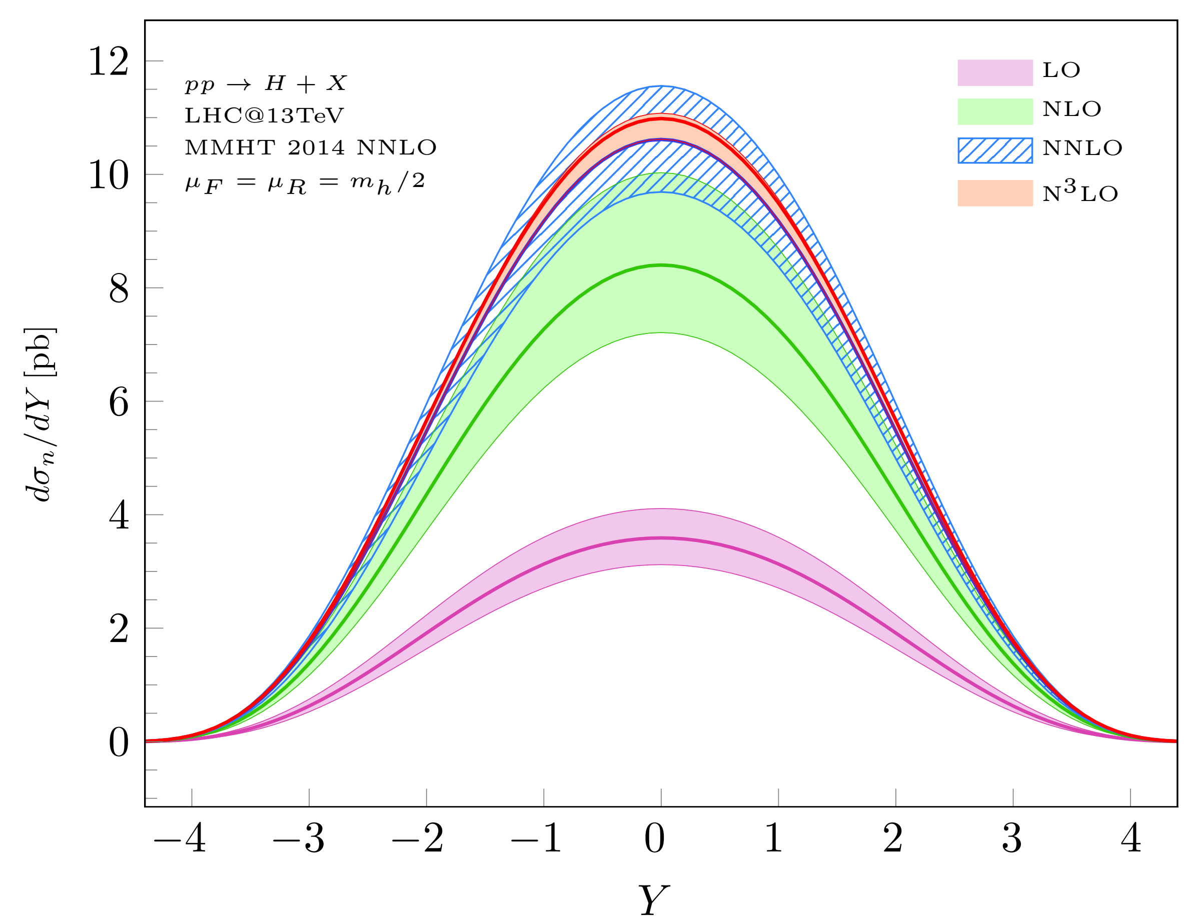
Projection-To-Born Subtraction

H+J at NNLO ∞ **Analytic Rapity Distribution of the Higgs**

[Chen, Gehrmann, Glover, Jaquier, 1408.5325]

[Dulat, BM,Pelloni,18]





FULLY DIFFERENTIAL HIGGS AT N3LO

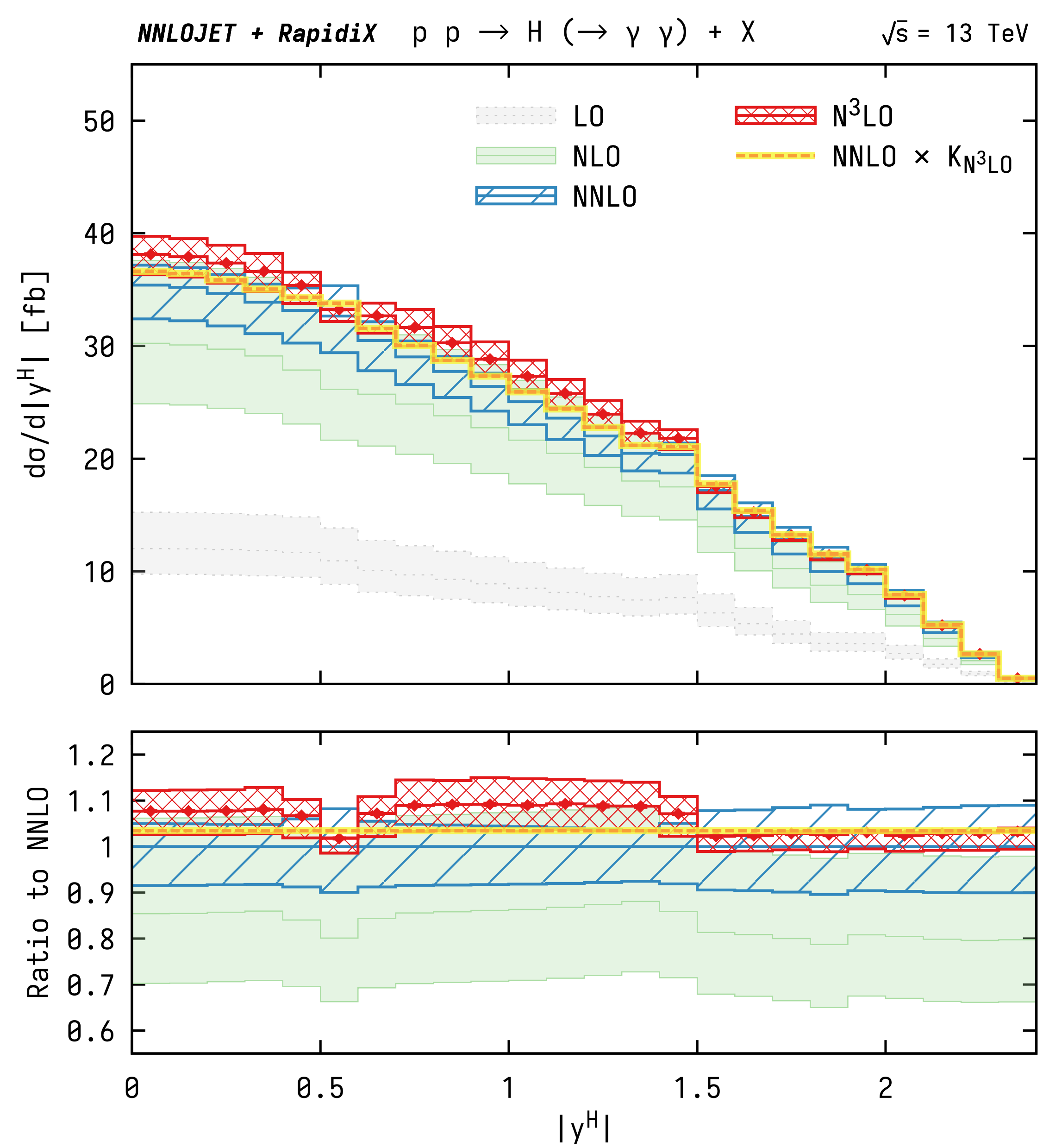
- Predict fully differential distributions at the LHC:

$$P\,P \to H + X \to \gamma\,\gamma + X$$

Chen, Gehrmann, Glover, Huss, BM, Pelloni [2102.07607]

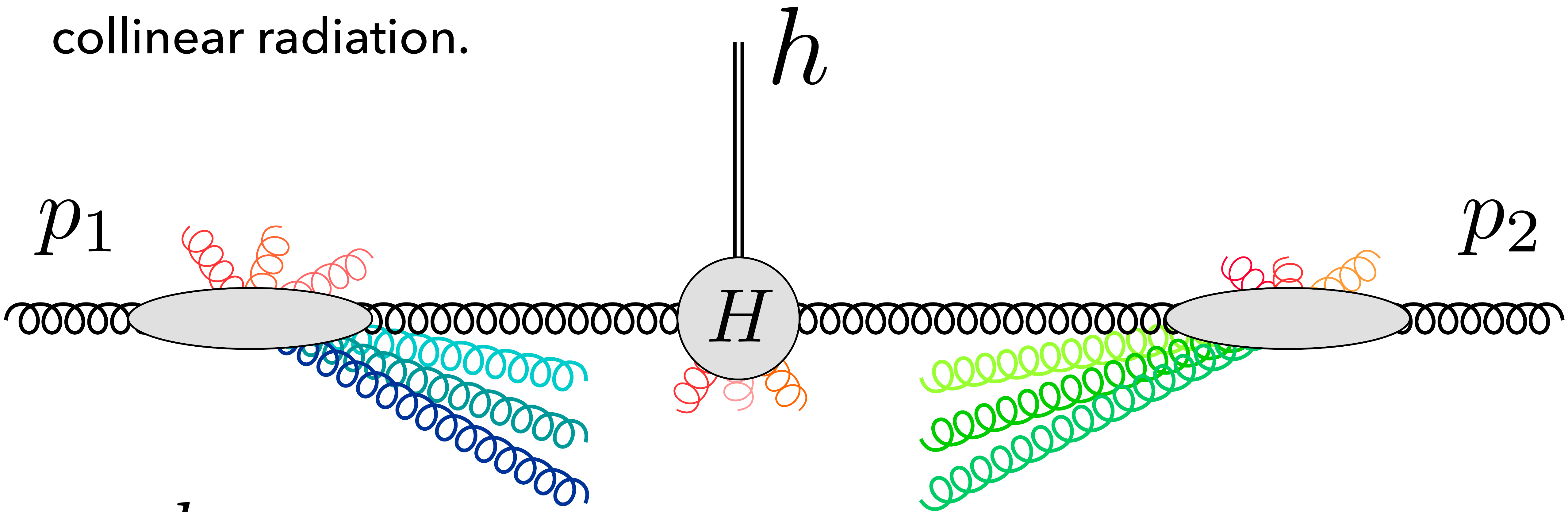
- First predictions for fully differential distributions at N3LO in perturbative QCD at the LHC.
- Corrections slightly larger than inclusive K - factor.
- Interesting features due to photon cuts.

Differential Rapidity



APPLICATIONS
FACTORIZATION AND RESUMMATION

- Let's pick an observable $\mathcal{T}$ that forces us to resolve infrared or collinear radiation.



$$\frac{d\sigma}{d\tau} = H \times [B_1(\tau) \otimes S(\tau) \otimes B_2(\tau)] + \mathcal{O}(\tau)$$

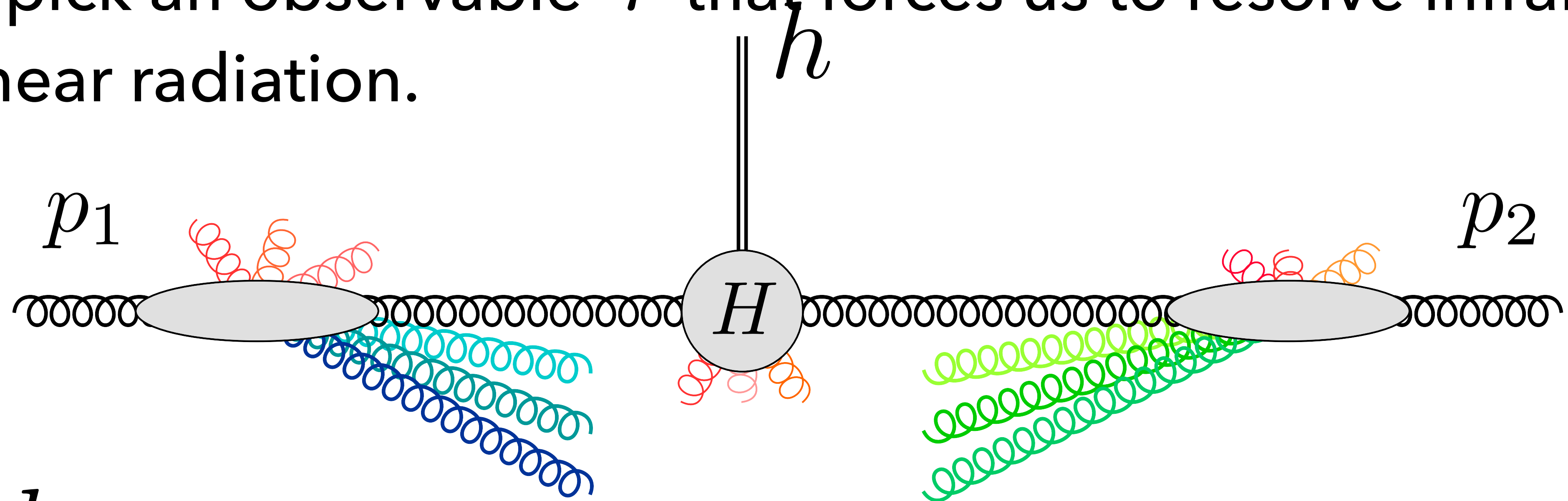
Hard **Collinear 1** **Soft** **Collinear 2**

- Example:

$$\tau = p_\perp^2$$

FACTORIZATION OF CROSS SECTIONS

- Let's pick an observable $\mathcal{T}$ that forces us to resolve infrared or collinear radiation.



$$\frac{d\sigma}{d\tau} = H \times \left[B_1(\tau) \otimes S(\tau) \otimes B_2(\tau)\right] + \mathcal{O}(\tau)$$

Only process dependence!

- Same ingredients for H, ZZ, WHZ, … 15W production!
- Very high degree of universality for color singlet production cross sections.
- Starting point for resummation of the perturbative expansion.

BEAM FUNCTIONS AT N3LO

- Collinear dynamics of factorization theorems captured by collinear expansions - relate to "beam functions".

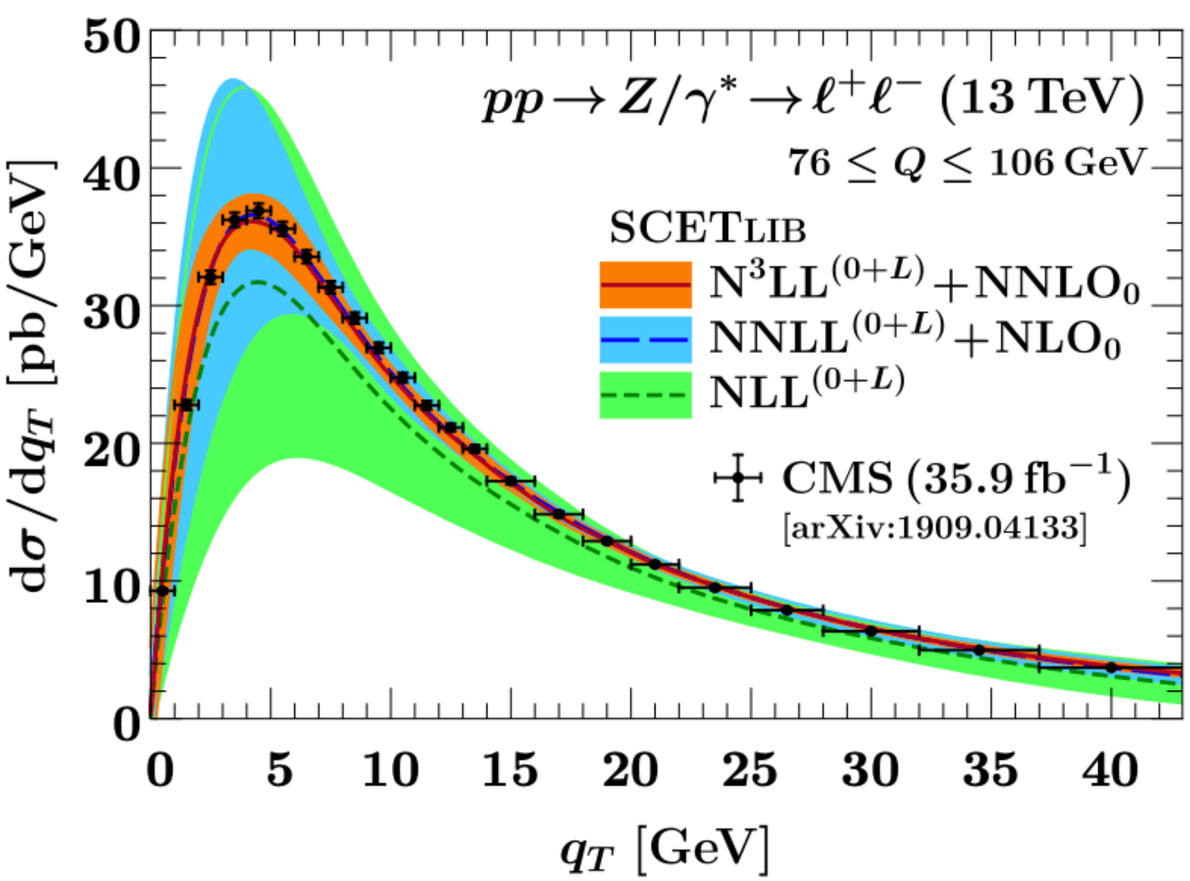
$$B_i(x_1^B,\mathcal{T}) = \sum_j \int_{x_1^B}^1 \frac{\mathrm{d}z_1}{z_1} f_j\left(\frac{x_1^B}{z_1}\right) \times \int_0^1 \mathrm{d}x \int_0^\infty \mathrm{d}w_1 \mathrm{d}w_2\, \delta\left[z_1 - (1-w_1)\right] \times \lim_{\text{strict } n-\text{coll.}} \left\{\delta\left[\mathcal{T} - \mathcal{T}(Q,Y,w_1,w_2,x)\right] \frac{\mathrm{d}\eta_{j\bar{i}}}{\mathrm{d}Q^2 \mathrm{d}w_1 \mathrm{d}w_2 \mathrm{d}x}\right\}.$$

- Beam functions: Probability to find a parton in a proton with momentum fraction x_1^B and observable value $\mathcal{T}$.
- We computed beam functions at N3LO:
  - N-Jettiness $\tau = \tau^N$ [Ebert,BM,Vita arXiv:2006.03056]
  - Transverse momentum $\tau = p_\perp^2$ [Ebert,BM,Vita arXiv:2006.05329]

Key ingredient for high precision resummation!

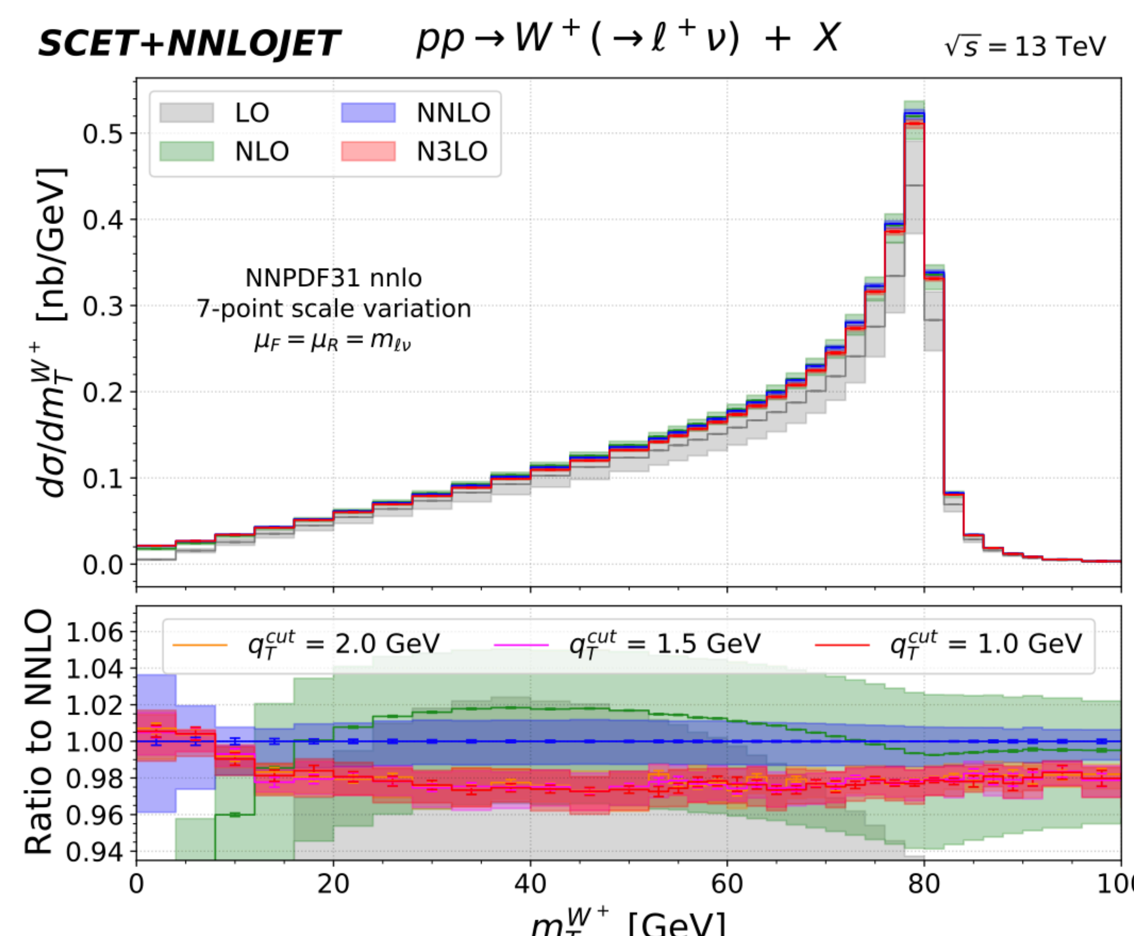
Last missing universal ingredient for qT-subtraction at N3LO.

Z boson transverse momentum:



[Ebert,Michel,Stewart,Tackmann 2006.11382]

W boson transverse mass:

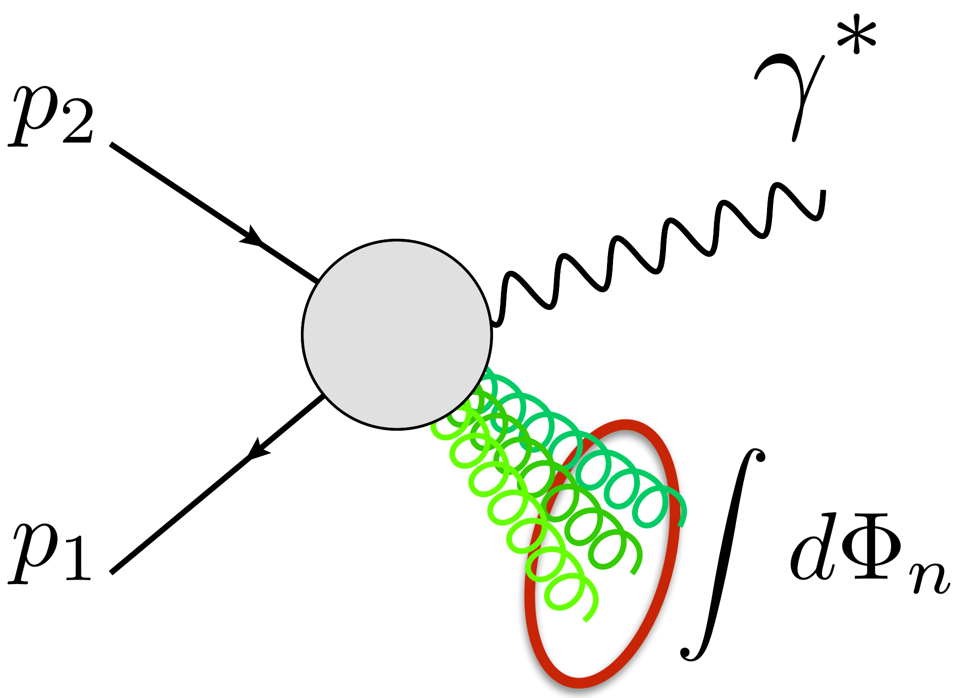


[Chen,Gehrmann,Glover,Huss, Yang,Zhu, 2205.11426]

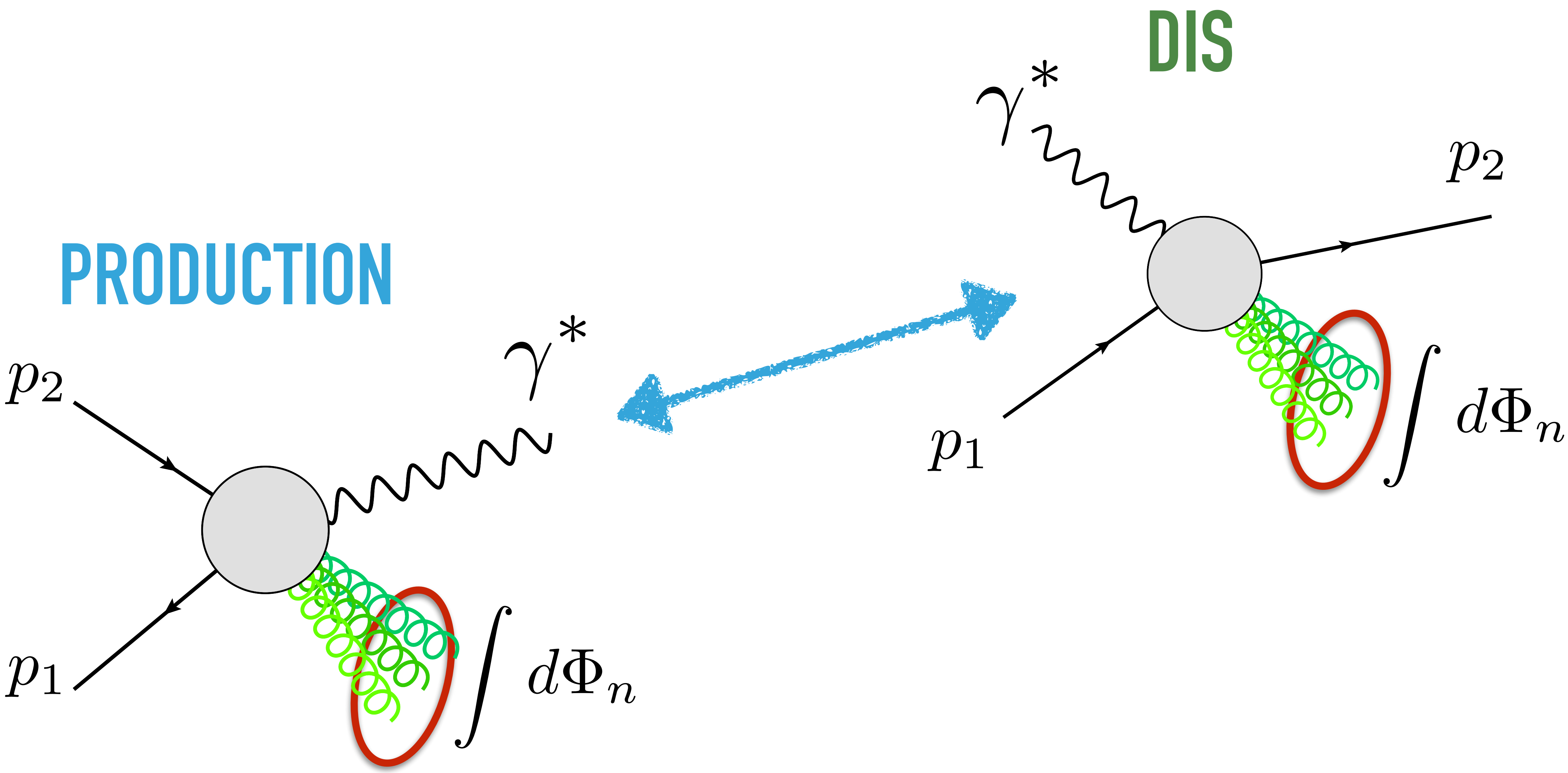
APPLICATIONS
CROSSING AND THE EEC

CROSSING

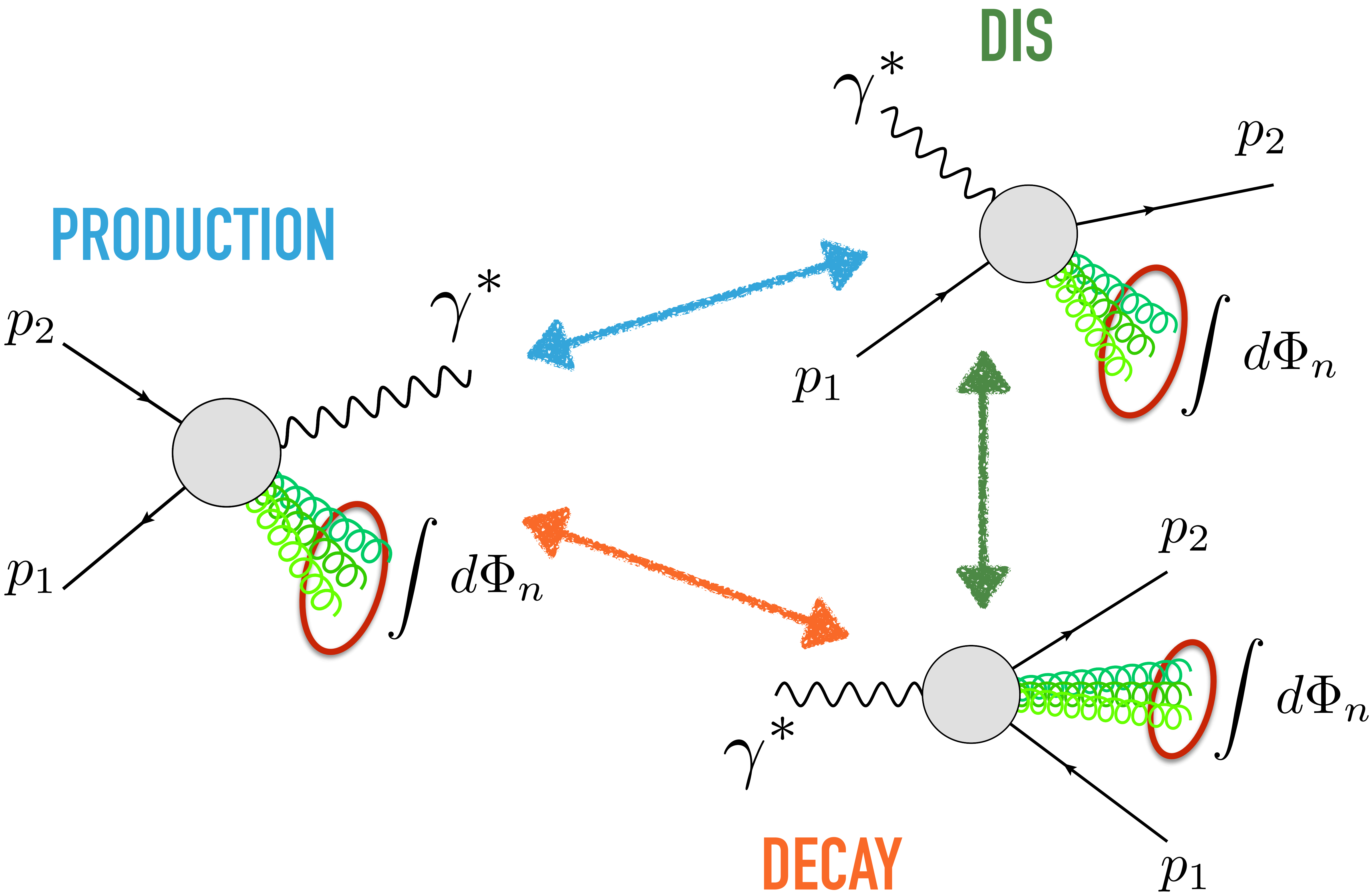
PRODUCTION



CROSSING



CROSSING



CROSSING

- We figured out how cross sections in IR sensitive limits are related: Production, DIS, Decay!

Production:

$$\frac{d\sigma}{d\tau} = H \times [B_1(\tau) \otimes S(\tau) \otimes B_2(\tau)] + \mathcal{O}(\tau)$$

Replace Beam Functions with Fragmentation Functions + Analytic Continuation!

Electron-Positron Annihilation

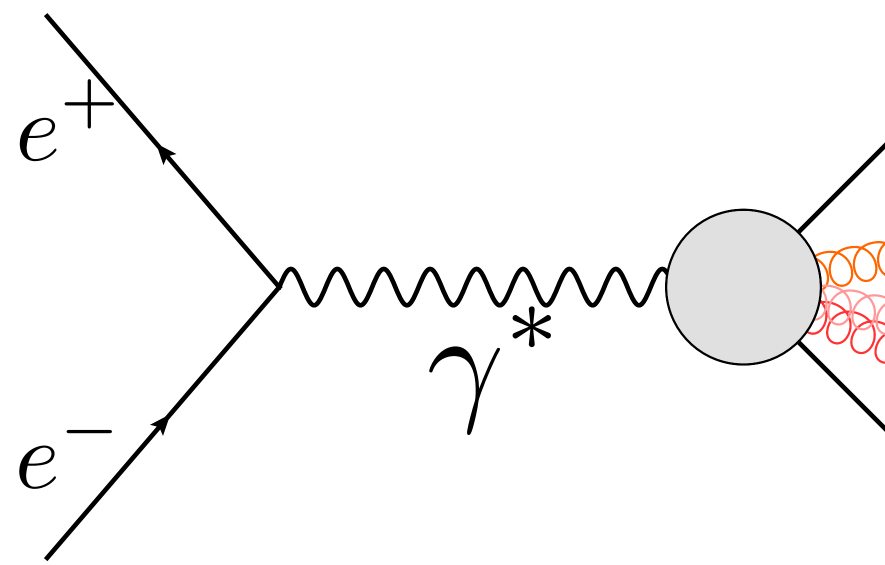
$$\frac{d\sigma}{d\tau} = H \times [F_1(\tau) \otimes S(\tau) \otimes F_2(\tau)] + \mathcal{O}(\tau)$$

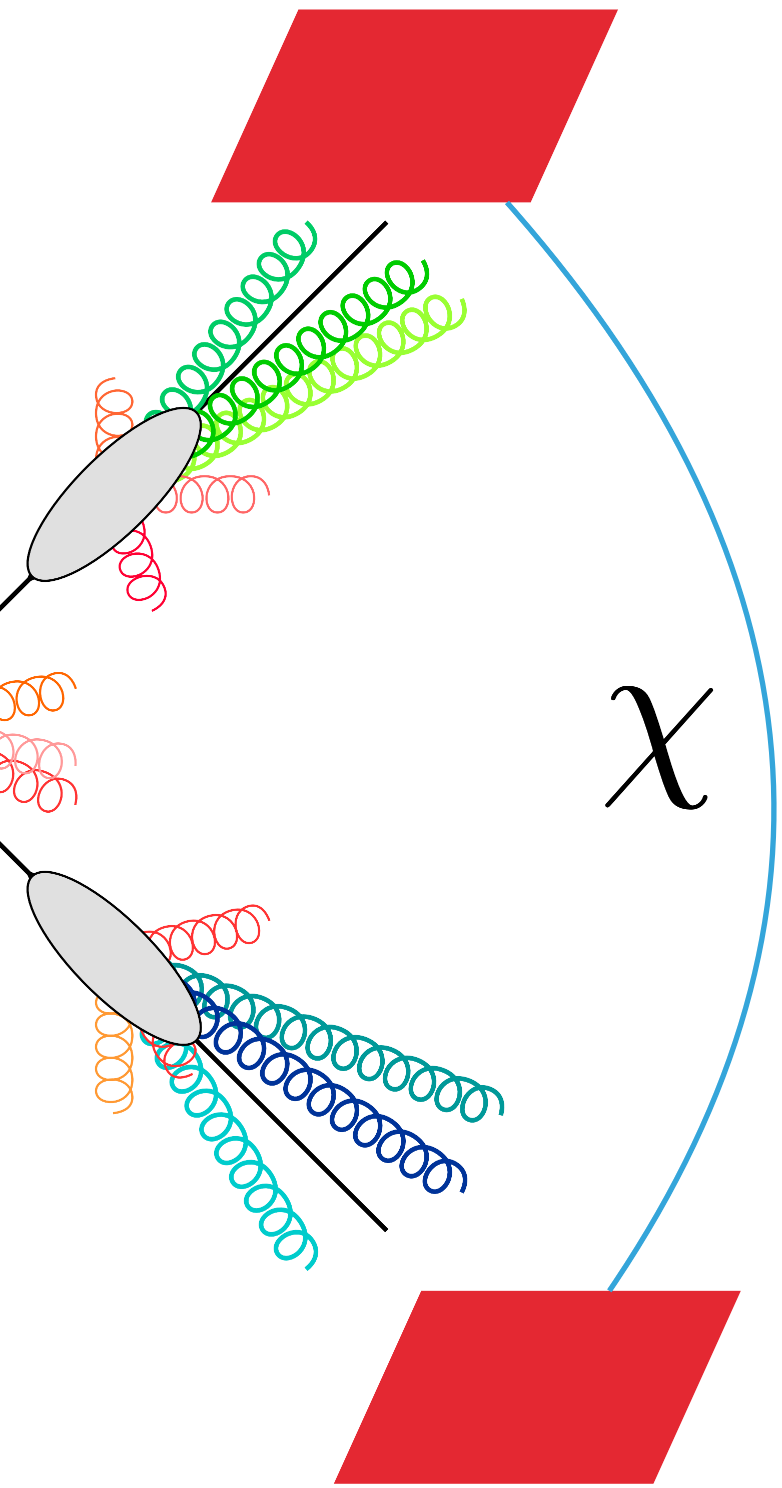
[Ebert,BM,Vita 2012.07853]

ENERGY-ENERGY-CORRELATION FUNCTIONS

$$\mathrm{EEC}(\chi) = \sum_{a,b} \int \mathrm{d}\sigma_{e^+e^- \to a+b+X} \, \frac{E_a E_b}{Q^2} \, \delta(\cos\chi_{ab} - \cos\chi)$$

- Look at large angular separation of detectors.
- Almost like DY in reverse!



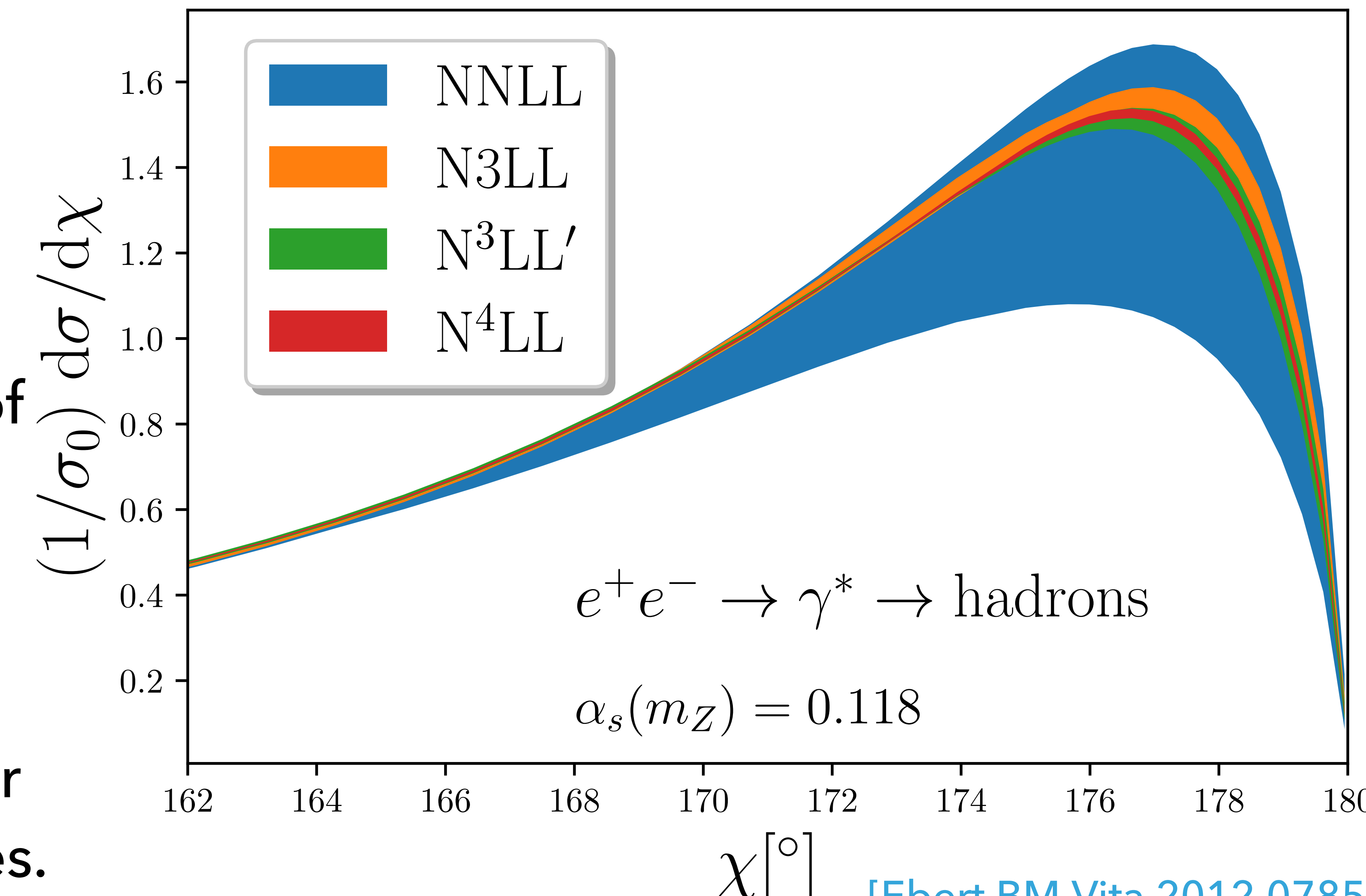


- Can be used to understand resummation in the limit $\chi \to 180^\circ$
- Our collinear expansions provides the required analytic input!

ENERGY-ENERGY-CORRELATION FUNCTIONS

$$\mathrm{EEC}(\chi) = \sum_{a,b} \int \mathrm{d}\sigma_{e^+e^- \to a+b+X} \frac{E_a E_b}{Q^2} \delta(\cos\chi_{ab} - \cos\chi)$$

- First resummation of an event shape at N4LL!
- Very high precision predictions for one of the most classical event shape observables.
- Versatility of collinear expansion techniques.

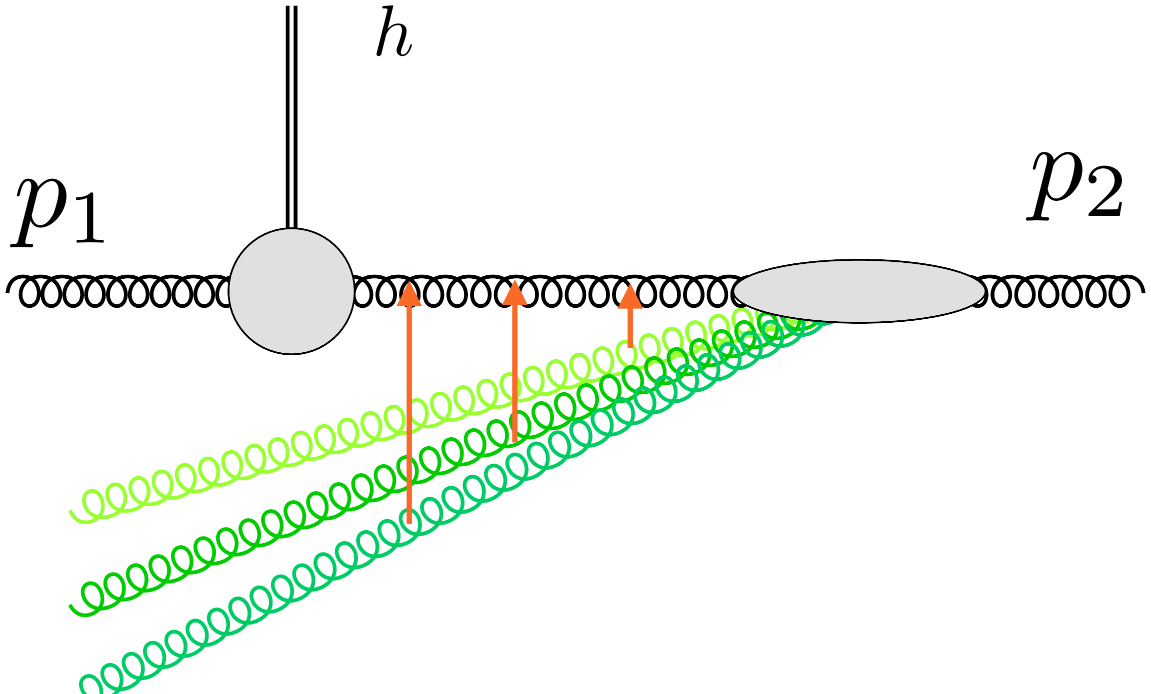


[Ebert,BM,Vita 2012.07859]

[Duhr,BM,Vita 2205.02242]

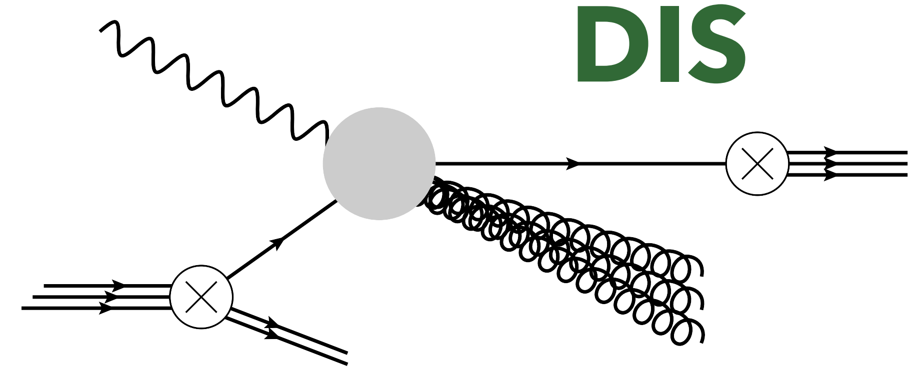
CONCLUSIONS

- Developed systematic expansion around collinear limit of color singlet LHC cross sections.
  - New ways to compute:
    Fully differential predictions for Higgs boson production at N3LO.
  - New universal ingredients:
    Beam functions at N3LO.
  - Crossing and resummation:
    The EEC at N4LL.



THANK YOU!

CROSSING: FRAGMENTATION FUNCTIONS



- Semi-Inclusive DIS in the small $P_{2,\perp}$ limit:

$$\frac{d\sigma_{P+h\to H+X}}{d\vec{P}^2_{2,\perp}\,d\xi} = \hat{\sigma}_0 \sum_{i,j} H_{ij} \times (Q^2,\mu) \tilde{f}_i(P_{2,\perp}) \otimes d_j(P_{2,\perp},\xi) \left[1 + \mathcal{O}\left(\vec{P}^2_{2,\perp}/Q^2\right)\right]$$

- Transverse Momentum Dependent Fragmentation Function (TMDFF)

The probability to find a Hadron inside a parton with a given longitudinal momentum fraction ξ and transverse momentum $P_{2,\perp}$

- Related to the longitudinal fragmentation function via

TMDFF

FF

$$\tilde{d}_j\left(\xi, b_T, \mu, \tau, \omega_b\right) = \sqrt{\tilde{S}(b_T,\mu,\tau)} \sum_j \int_\xi^1 \frac{d\zeta}{\zeta} \tilde{\mathcal{I}}^{\mathrm{FF}}_{ij}\left(\zeta, b_T, \mu, \tau, \omega_b\right) D_j\left(\frac{\xi}{\zeta}, \mu\right).$$

Soft Function

Matching Kernel

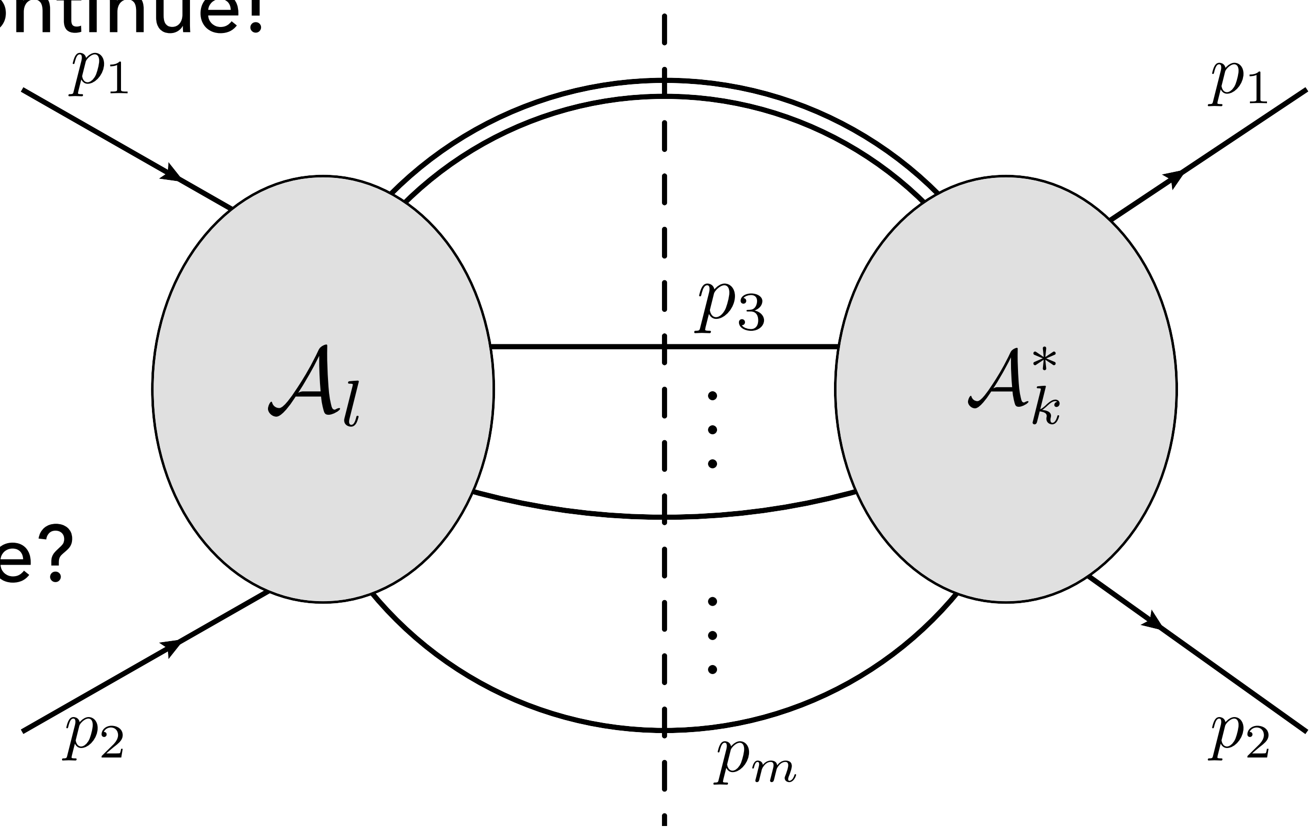
arXiv:2012.07853

CROSSING:

- To cross means to analytically continue!
- Our variables change sign:

$$\{s, w_1, w_2\}$$

- What is their branch cut structure?



[cross section]

$$\frac{\mathrm{d}\eta_{ij}^{(m+l+k)}}{\mathrm{d}Q^2\mathrm{d}w_1\mathrm{d}w_2\mathrm{d}x} = (sw_1w_2)^{-m\epsilon} \times \Bigg[\sum_{i_1,i_2=0}^{l} \sum_{j_1,j_2=0}^{k} \frac{\mathrm{d}\eta_{ij}^{(m+l+k,i_1,i_2,j_1,j_2)}}{\mathrm{d}Q^2\mathrm{d}w_1\mathrm{d}w_2\mathrm{d}x}$$

[real coefficients - complicated stuff]

$$\times \Re\left(\left[(-s)^{(i_1+i_2-l)\epsilon}(sw_1)^{-i_1\epsilon}(sw_2)^{-i_2\epsilon}\right]\left[(-s)^{(j_1+j_2-k)\epsilon}(sw_1)^{-j_1\epsilon}(sw_2)^{-j_2\epsilon}\right]^*\right)\Bigg].$$

[Mandelstam Invariants! - Continue!]