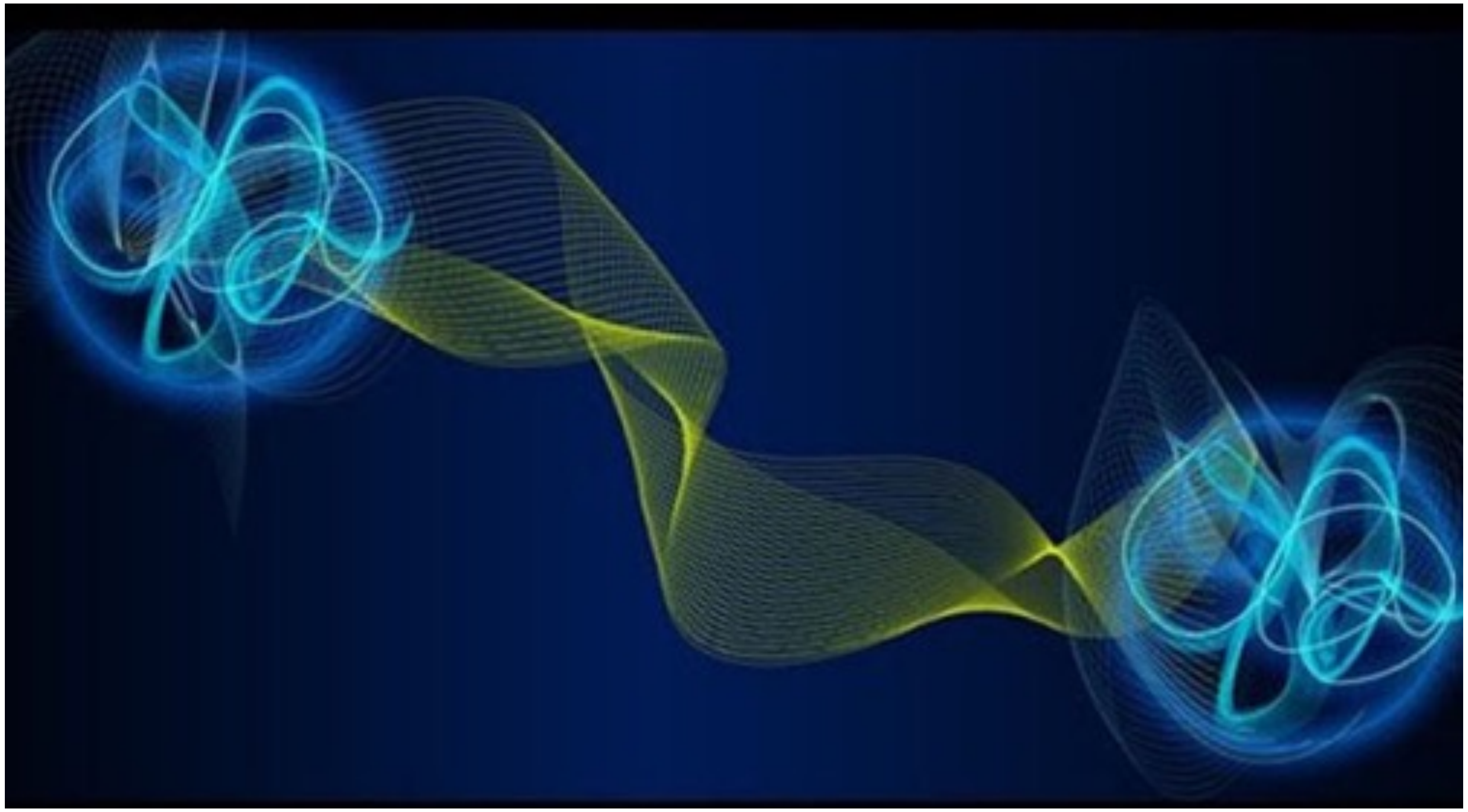
Entanglement & Symmetry



S. Beane, DBK, N. Klco, M. Savage, PRL 122, 102001 (2019), arXiv: 1812.03138

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  - ✦ UV fixed point
- IR more symmetric than UV
  - ✦ IR fixed point
  - ✦ Accidental symmetries
    - protection from radioactive decay
    - Lorentz symmetry from the lattice

There are times when a theorist wishes for more symmetry:

- hierarchy
- mass textures
- suppression of FCNC

Are there other ways that approximate symmetries can emerge?

Some evidence in low energy hadronic physics:

- "Schrödinger symmetry" (nonrelativistic conformal symmetry)
- spin-flavor symmetries

This talk: these approximate symmetries can be correlated with minimization of entanglement in scattering.

Emergent symmetries seen in the baryons:

i. SU(4), SU(6) spin-flavor symmetry

ii. SU(4) Wigner symmetry

iii. Schrödinger symmetry

iv. SU(16) (?!) in baryon octet

SU(4), SU(6) spin-flavor symmetry (1960s)

SU(4):

$$4 = \begin{pmatrix} u\uparrow \\ u\downarrow \\ d\uparrow \\ d\downarrow \end{pmatrix}$$

SU(6):

$$6 = \begin{pmatrix} u\uparrow \\ u\downarrow \\ d\uparrow \\ d\downarrow \\ s\uparrow \\ s\downarrow \end{pmatrix}$$

- Symmetry of non-relativistic quark model
- Approximate symmetry apparent in nature:
  - masses
  - magnetic moments & transitions
  - semi-leptonic currents
  - meson-baryon couplings
  - NN scattering

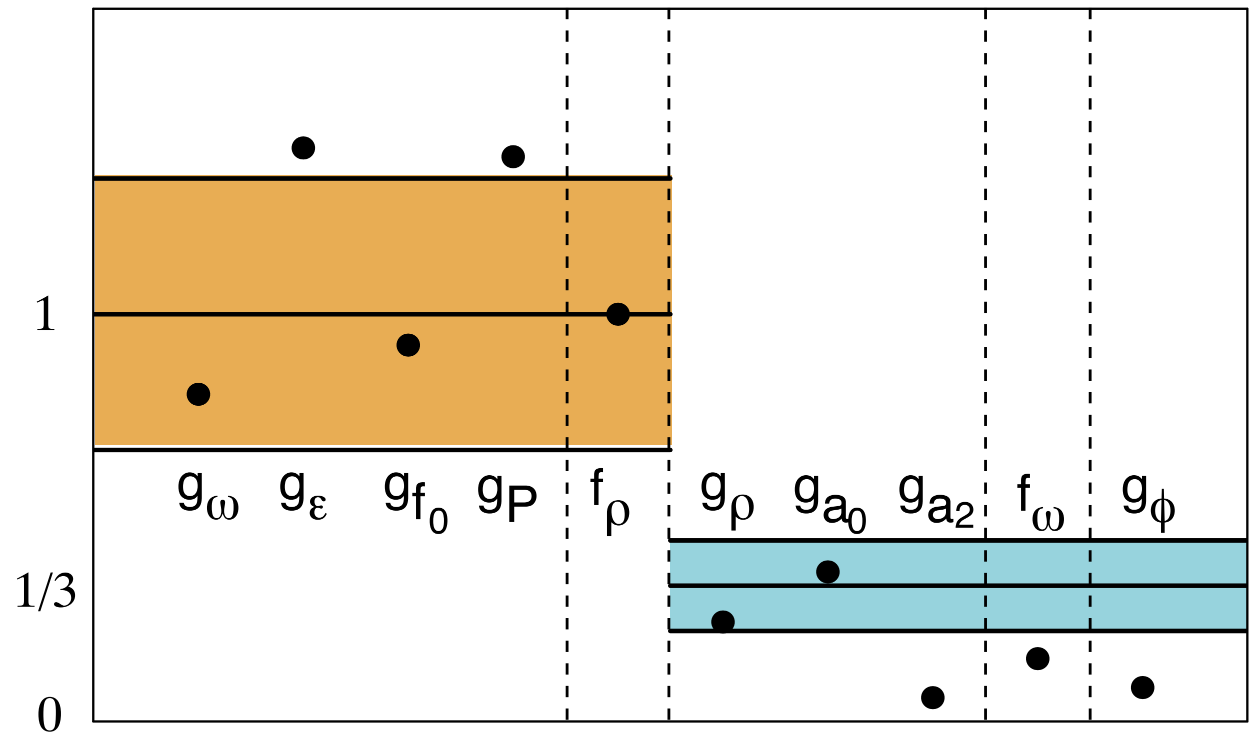
SU(4), SU(6) spin-flavor symmetry (1960s)

- Cannot be a symmetry of relativistic QFT (Coleman-Mandula)
- For baryon-meson couplings, does follow from QCD in large-N_c limit

Gervais, Sakita (1984); Dashen, Manohar (1993), Dashen, Jenkins, Manohar (1994)

(a) (b)

- Large-N_c seems to also work in nuclear physics:



DBK, A. Manohar (1996)

- ...and implies spin-flavor symmetries in low energy baryon-baryon interactions...

SU(2Nf) spin-flavor symmetries in low energy baryon-baryon interactions also follows from large-Nc DBK, M.J. Savage (1995)

- $N_f = 2$ (nucleons & Δ)

$$\mathcal{L}_6 = -\frac{1}{f_\pi^2}\left[a(\Psi^\dagger_{\mu\nu\rho}\Psi^{\mu\nu\rho})^2 + b\Psi^\dagger_{\mu\nu\sigma}\Psi^{\mu\nu\tau}\Psi^\dagger_{\rho\delta\tau}\Psi^{\rho\delta\sigma}\right]$$

$$\Psi^{(\alpha i)(\beta j)(\gamma k)} = \Delta^{ijk}_{\alpha\beta\gamma} + \frac{1}{\sqrt{18}}\left(N^i_\alpha \epsilon^{jk}\epsilon_{\beta\gamma} + N^j_\beta \epsilon^{ik}\epsilon_{\alpha\gamma} + N^k_\gamma \epsilon^{ij}\epsilon_{\alpha\beta}\right)$$

- $N_f = 2$ (restricted to nucleons)

$$\mathcal{L}_6 = -\tfrac{1}{2}C_S(N^\dagger N)^2 - \tfrac{1}{2}C_T(N^\dagger\vec{\sigma}N)^2$$ ➢ general Weinberg (1990)

$$C_S = \frac{2(a - b/27)}{f_\pi^2}\,, \qquad C_T = 0$$ ➢ SU(4) prediction

Is this an approximate symmetry of nature?

Predicts equal scattering lengths for $^1S_0$, $^3S_1$ NN scattering lengths

$^1S_0$ scattering length = -23.7 fm ~ 1/8 MeV

$^3S_1$ scattering length = + 5.4 fm ~ 1/35 MeV

$$\mathcal{A} \simeq \frac{4\pi}{M} \frac{1}{\left(-\frac{1}{a} + i\sqrt{ME}\right)}$$

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Better diagnostic: accidental SU(4)$_{\text{Wigner}}$ symmetry

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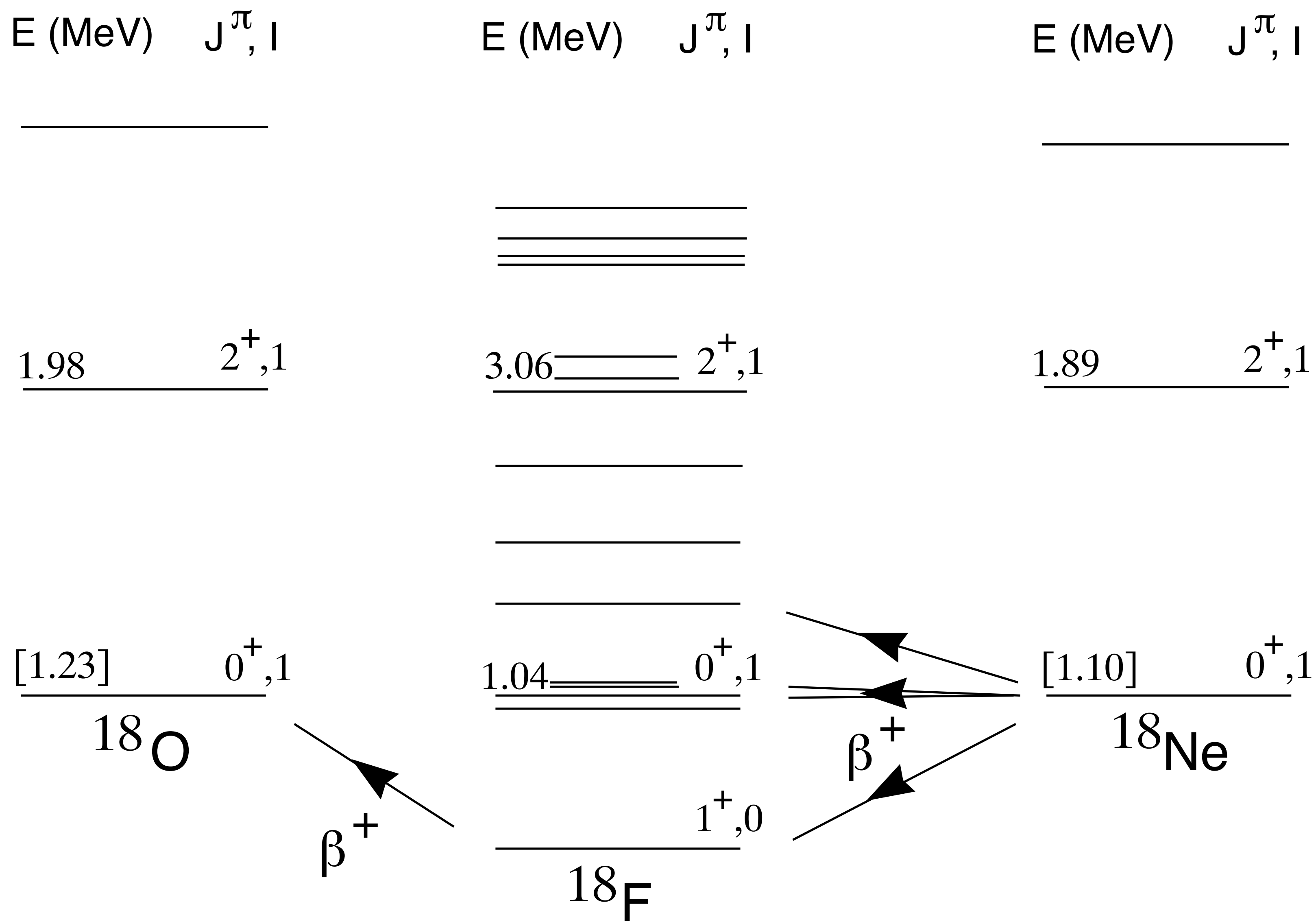
Example of evidence for $SU(4)_{Wigner}$: β-decay in A=18 isobars



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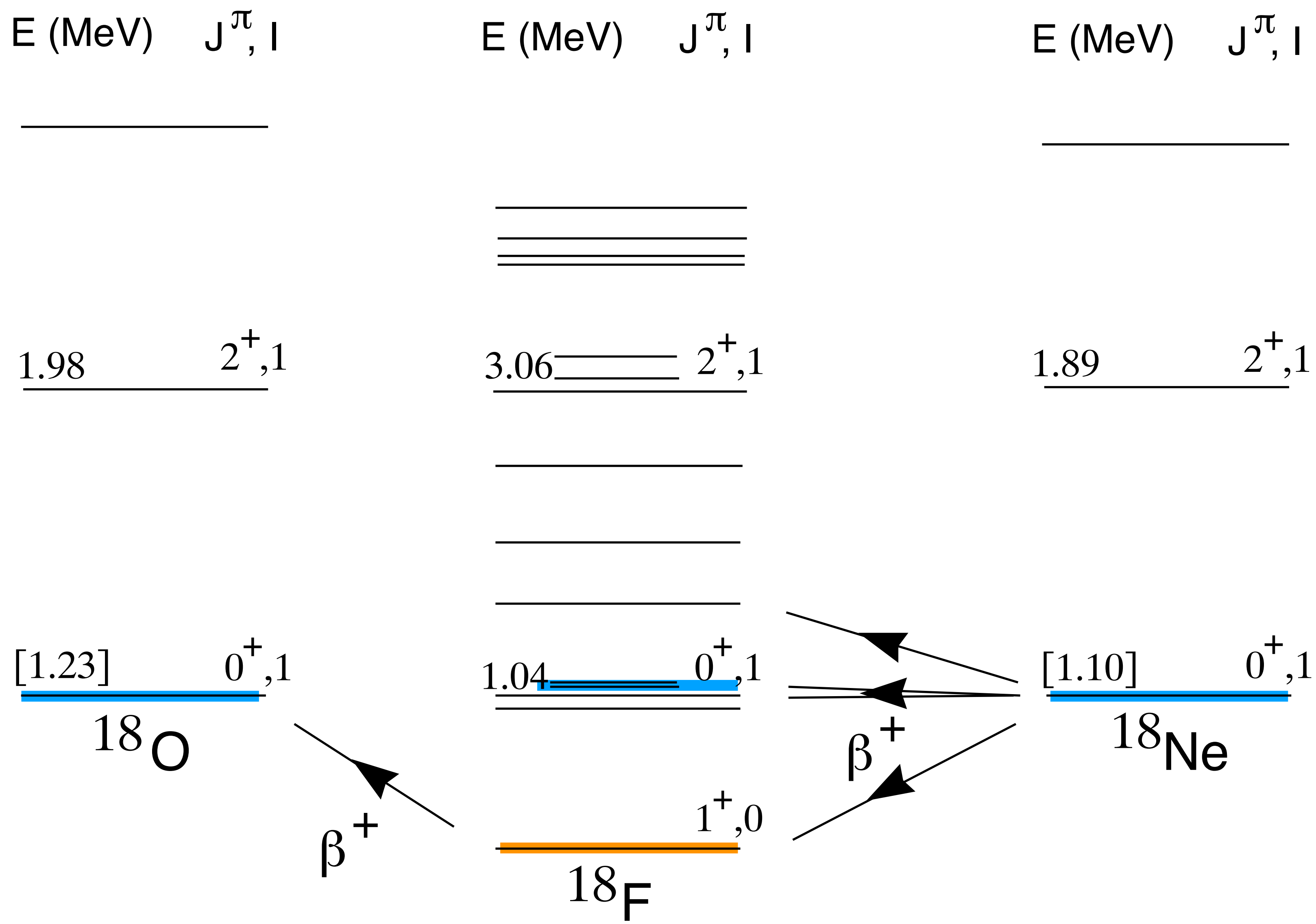

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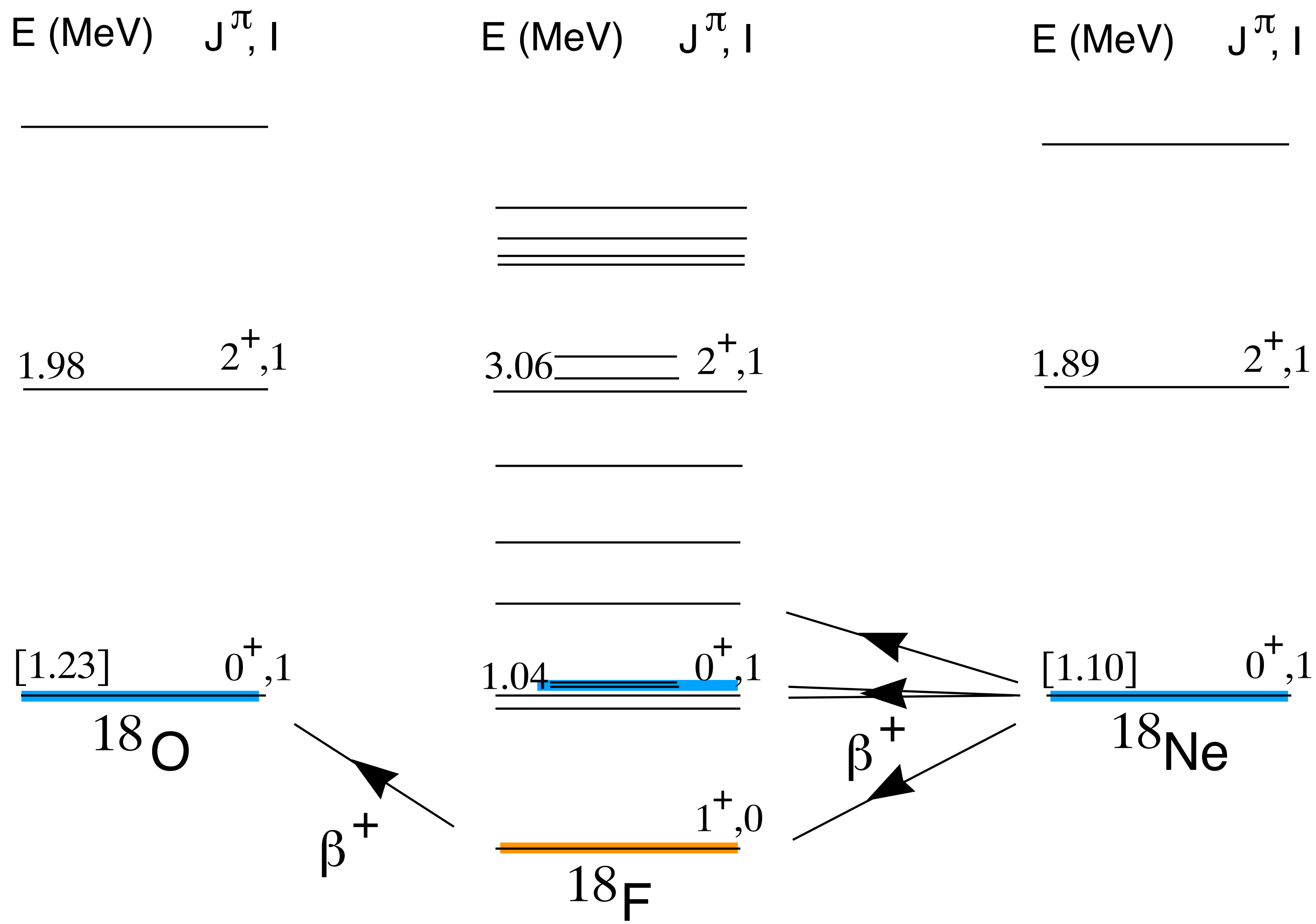

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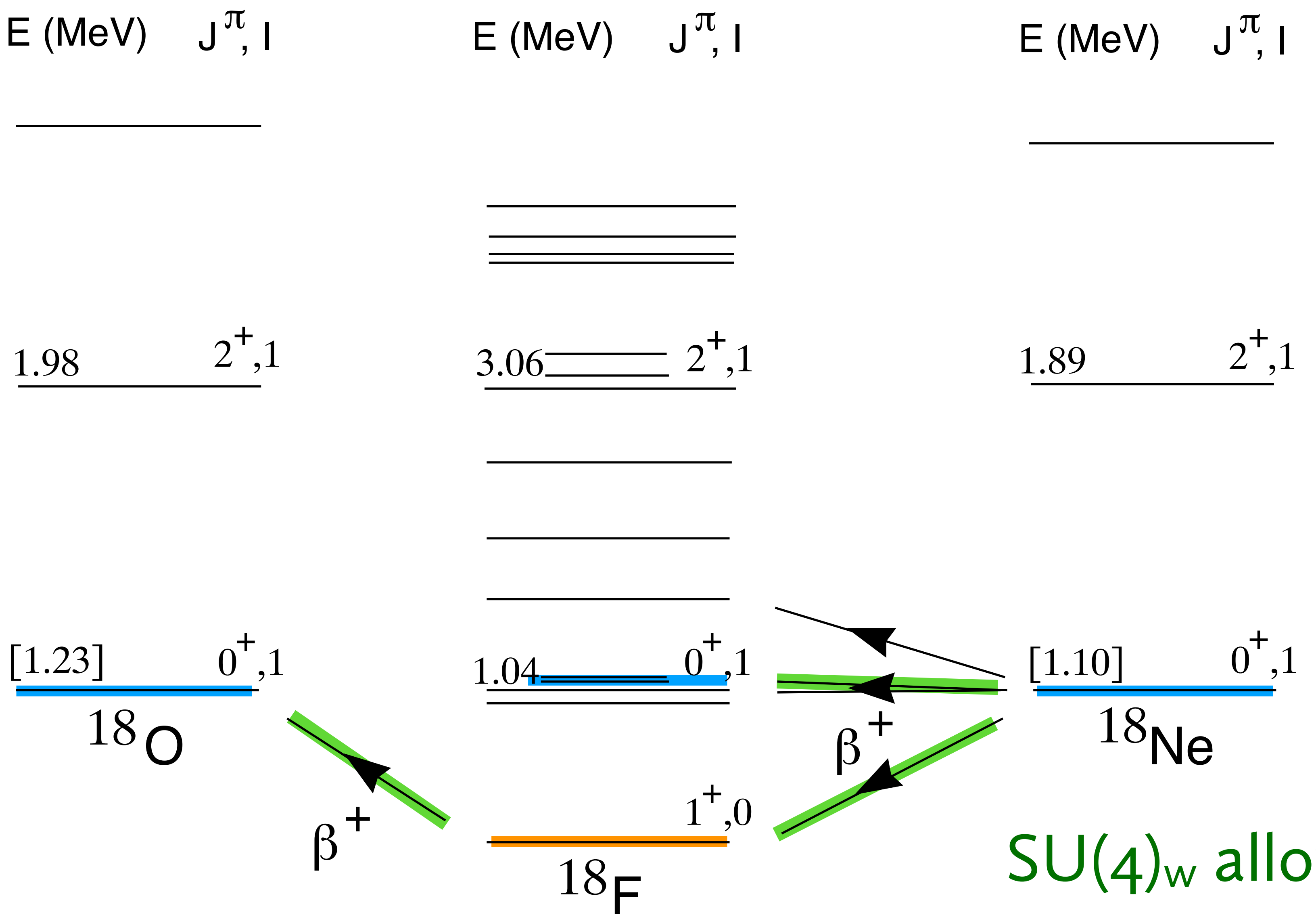

$SU(4)_W$ allowed transitions

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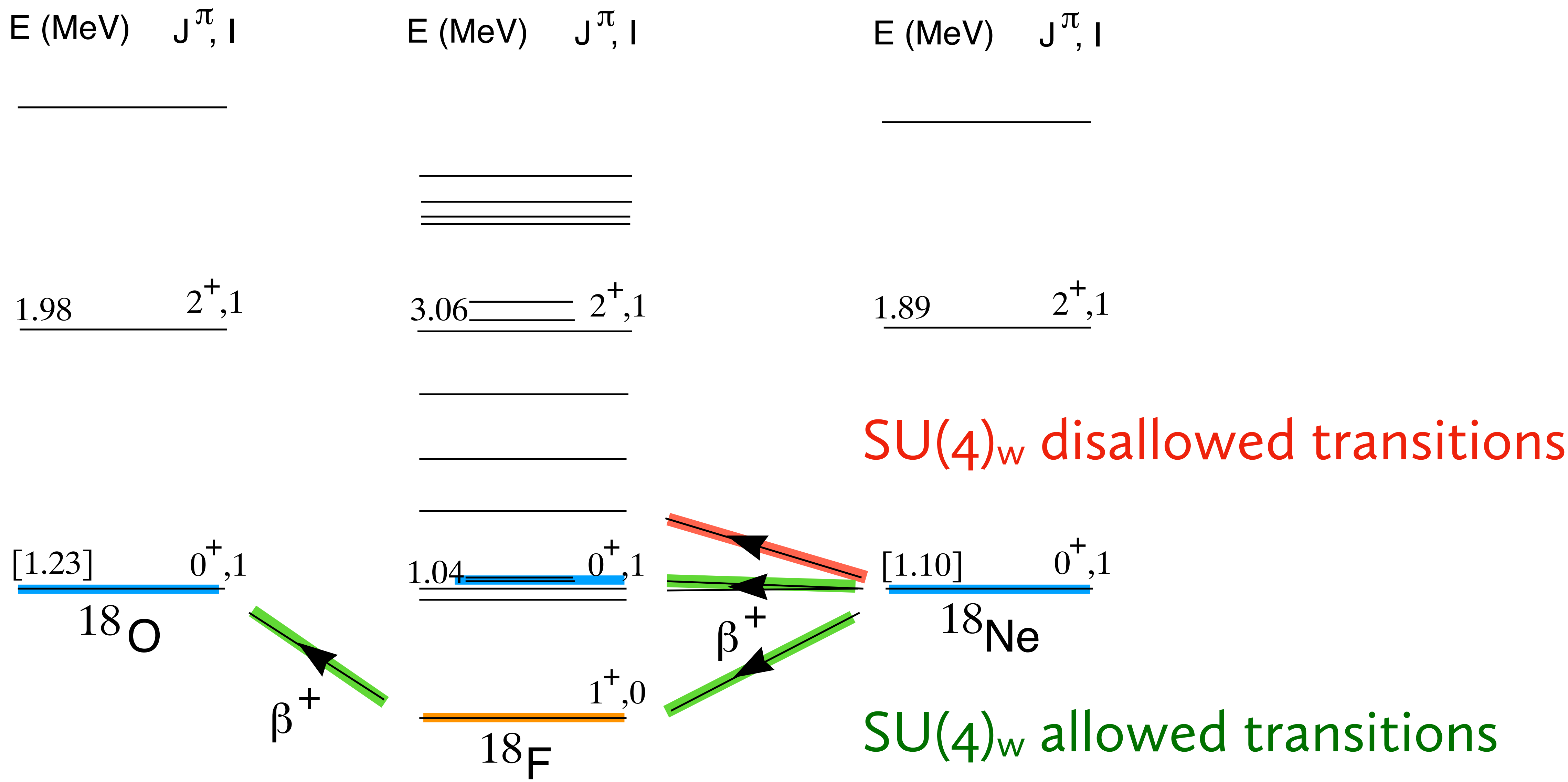

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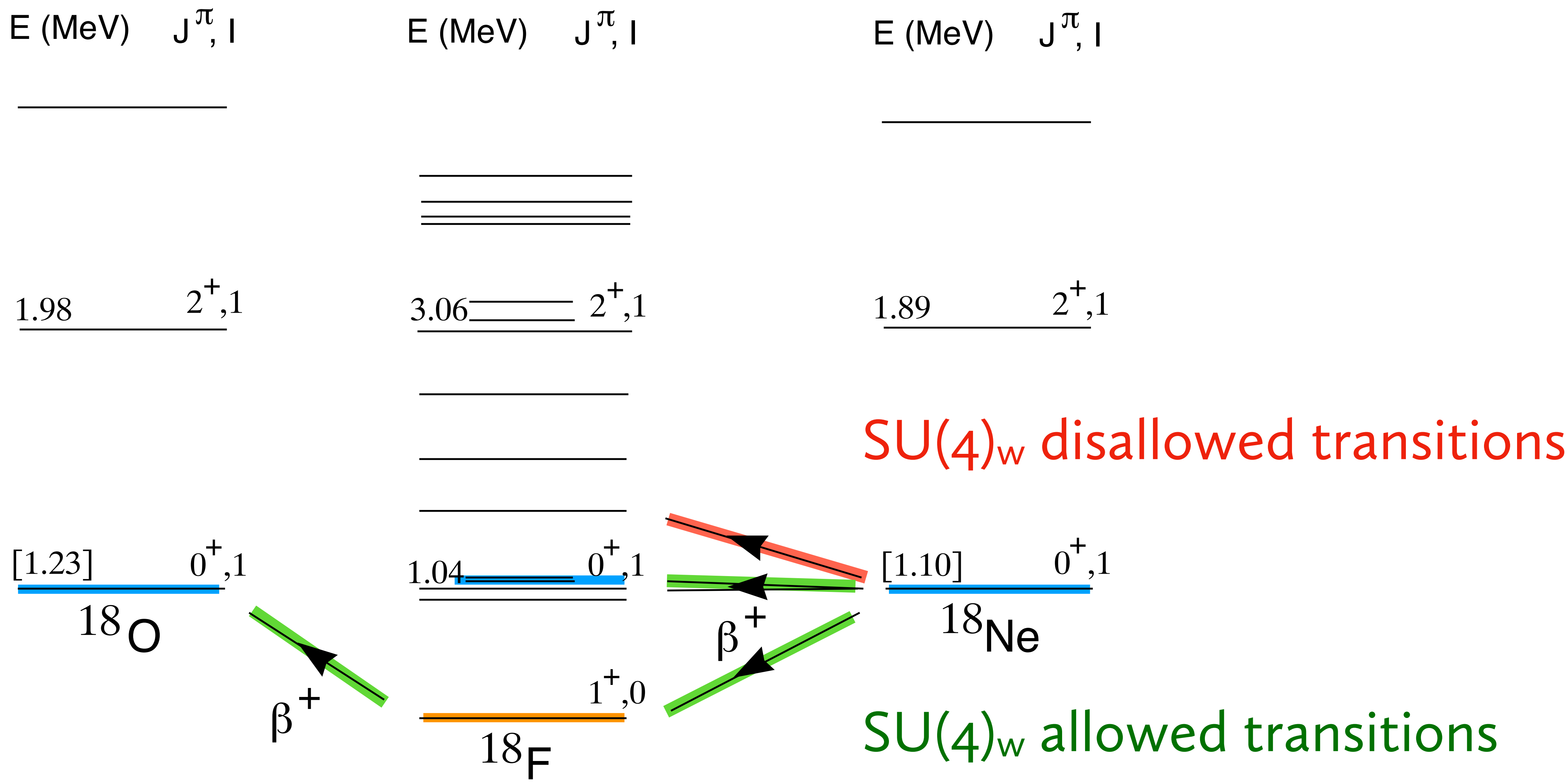

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SU(4)$_W$ allowed matrix elements ~**10 x greater** than SU(4)$_W$ disallowed

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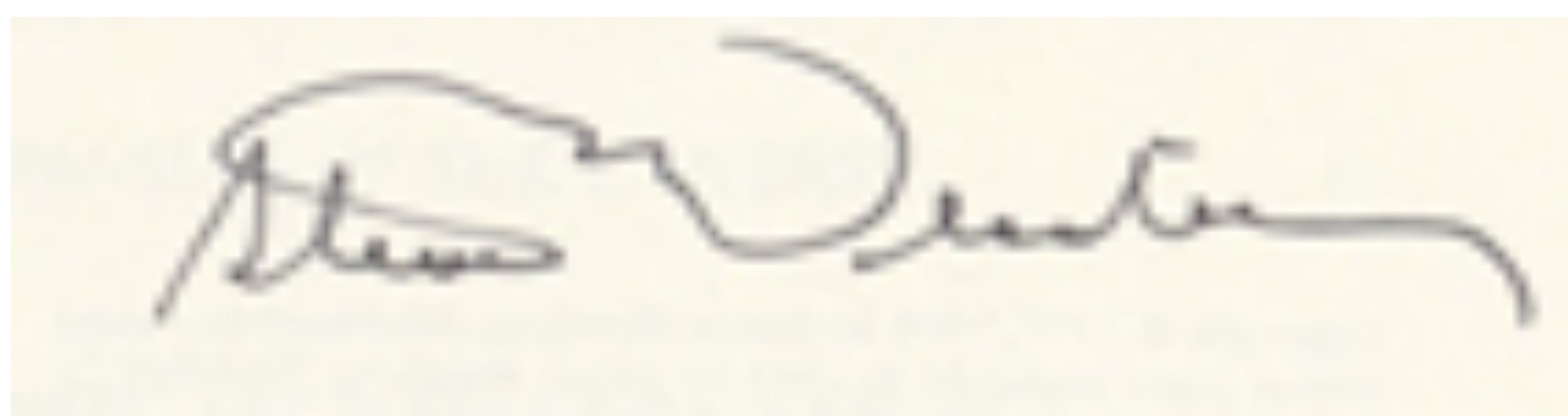

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Baryon-baryon interactions N_f=3 / SU(6) spin-flavor symmetry (also predicted by large-N_c)

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baryon decuplet ($T^{ijk}_{\alpha\beta\gamma}$)

baryon octet (B)

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baryon decuplet ($T^{ijk}_{\alpha\beta\gamma}$) baryon octet ($B^j_{m,\beta}$)

Low-energy EFT for just the octet:

M.J. Savage, M.B. Wise (1995)

$$\begin{aligned}\mathcal{L} = & -c_1 \mathrm{Tr}\, B_i^\dagger B_i B_j^\dagger B_j - c_2 \mathrm{Tr}\, B_i^\dagger B_j B_j^\dagger B_i - c_3 \mathrm{Tr}\, B_i^\dagger B_j^\dagger B_i B_j \\ & -c_4 \mathrm{Tr}\, B_i^\dagger B_j^\dagger B_j B_i - c_5 \mathrm{Tr}\, B_i^\dagger B_i \mathrm{Tr}\, B_j^\dagger B_j - c_6 \mathrm{Tr}\, B_i^\dagger B_j \mathrm{Tr}\, B_j^\dagger B_i\end{aligned}$$

$$B_i = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}_i$$

$i = \uparrow, \downarrow$

$$\mathcal{L} = -c_1 \mathrm{Tr}\, B_i^\dagger B_i B_j^\dagger B_j - c_2 \mathrm{Tr}\, B_i^\dagger B_j B_j^\dagger B_i - c_3 \mathrm{Tr}\, B_i^\dagger B_j^\dagger B_i B_j$$
$$-c_4 \mathrm{Tr}\, B_i^\dagger B_j^\dagger B_j B_i - c_5 \mathrm{Tr}\, B_i^\dagger B_i \mathrm{Tr}\, B_j^\dagger B_j - c_6 \mathrm{Tr}\, B_i^\dagger B_j \mathrm{Tr}\, B_j^\dagger B_i$$

SU(6) prediction:

$$c_1 = -\frac{7}{27}b\,, \qquad c_2 = \frac{1}{9}b\,, \qquad c_3 = \frac{10}{81}b\,,$$
$$c_4 = -\frac{14}{81}b\,, \qquad c_5 = a + \frac{2}{9}b\,, \qquad c_6 = -\frac{1}{9}b\,.$$

Does this work? Look at lattice data

- NPLQCD collaboration, 2015
- equal quark masses
- m_π = 806 MeV

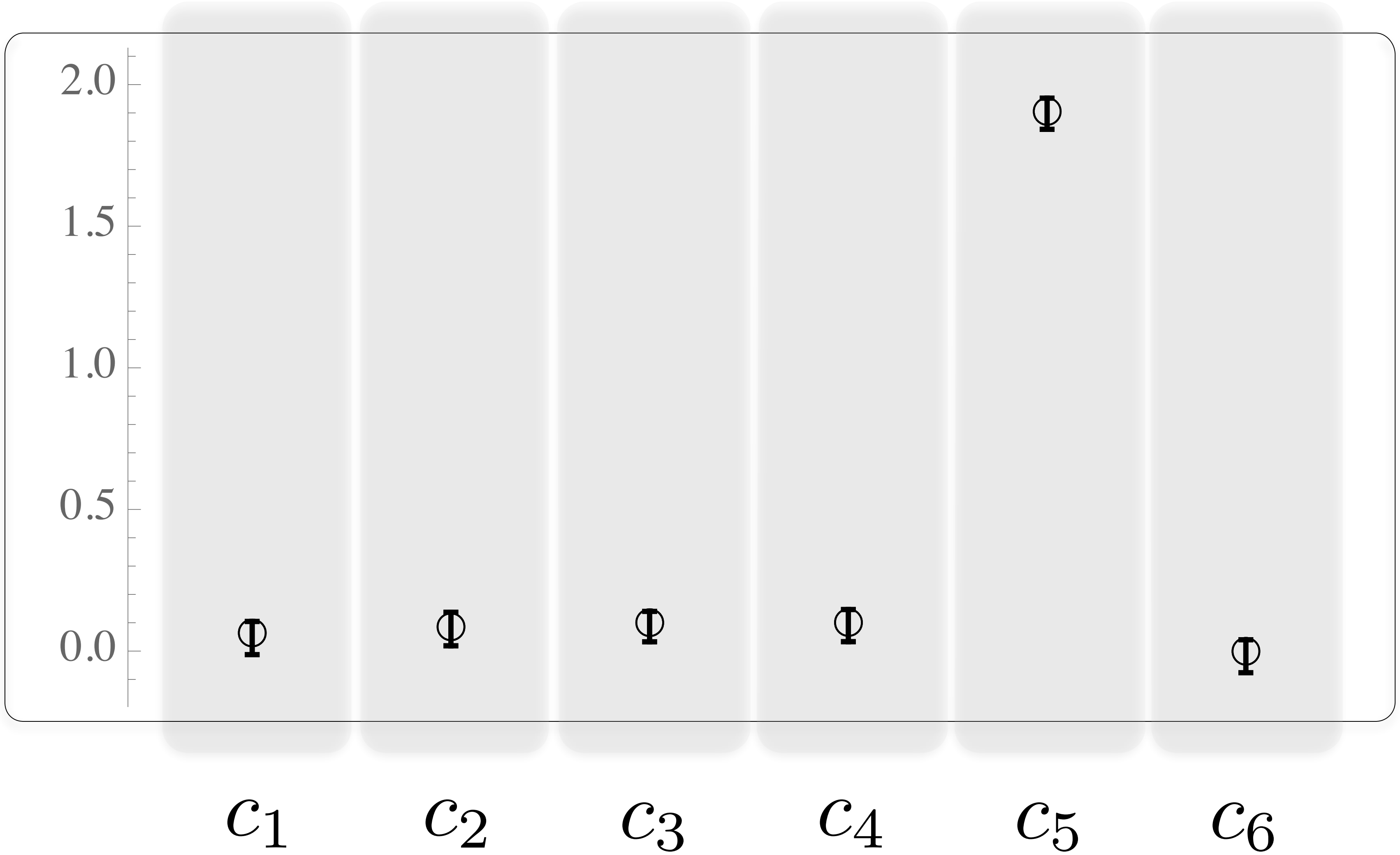
PHYSICAL REVIEW D **96,** 114510 (2017)

Baryon-baryon interactions and spin-flavor symmetry from lattice quantum chromodynamics

Michael L. Wagman,[1,2] Frank Winter,[3] Emmanuel Chang,[2] Zohreh Davoudi,[4] William Detmold,[4]
Kostas Orginos,[5,3] Martin J. Savage,[1,2] and Phiala E. Shanahan[4]

(NPLQCD Collaboration)

$$m_\pi \approx 806\ \text{MeV}$$
$$\mu = m_\pi$$



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NPLQCD results:

- Only $c_5 \neq 0$
- Near critical value for large scattering lengths

Similar to N_f=2 large-N_c result: $\mathcal{L}_6 = -\frac{1}{2} C_S (N^\dagger N)^2 - \frac{1}{2} C_T (N^\dagger \vec{\sigma} N)^2$

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But:

- More symmetric than SU(6) [b ≈ 0]
- EFT possesses **SU(16)** analog of $SU(4)_{\mathrm{Wigner}}$
- Near critical value for large scattering lengths — *conformal symmetry*

Not large-N_c predictions

N_f=2	N_f=3
$SU(4)_{Wigner}$	$SU(16)_{NPLQCD}$
~conformal	~conformal

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No known reason for these symmetries

...but correlated with low entanglement

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

DISCUSSION OF PROBABILITY RELATIONS BETWEEN SEPARATED SYSTEMS

By E. SCHRÖDINGER

[Communicated by Mr M. Born]

[*Received* 14 August, *read* 28 October 1935]

When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or wave-functions) have become entangled.

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EPR LOCAL REALITY ASSUMPTION

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E.g. pure state: $$|\,\psi\,\rangle = |\,\uparrow_x\,\downarrow_y\,\rangle$$

$$\hat{\rho} = |\,\psi\,\rangle\langle\,\psi\,|$$

$$\rho = \begin{pmatrix} 1 & 0 & \dots \\ 0 & 0 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

➤ rank 1

S = 0

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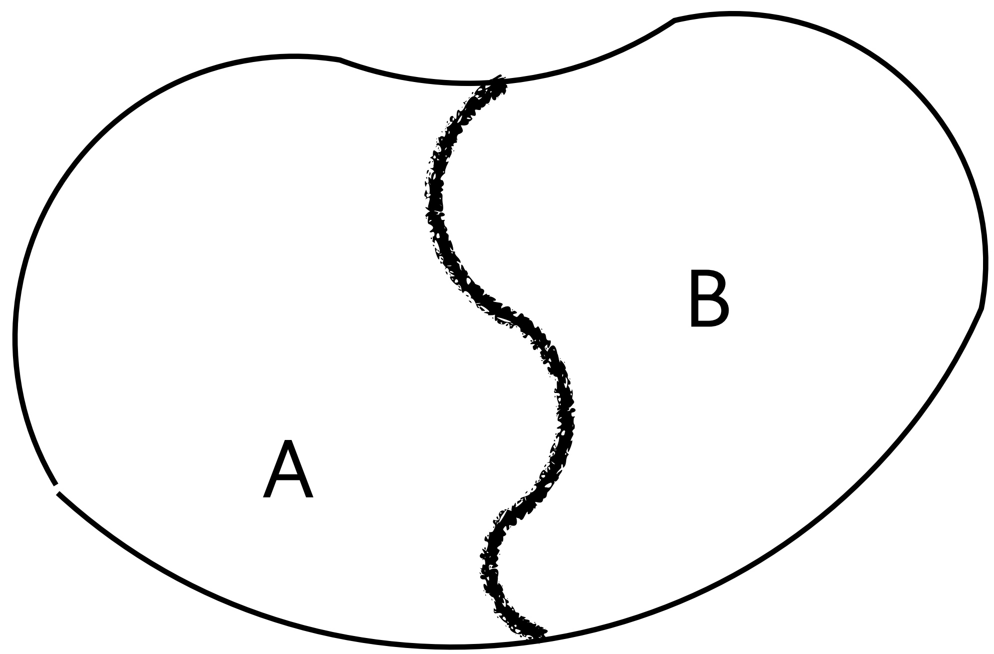

Factorizable Hilbert space: $\mathcal{H} = \mathcal{H}_A \times \mathcal{H}_B$

Reduced density matrix:

$$\rho_A = \mathrm{Tr}_B\, \rho$$
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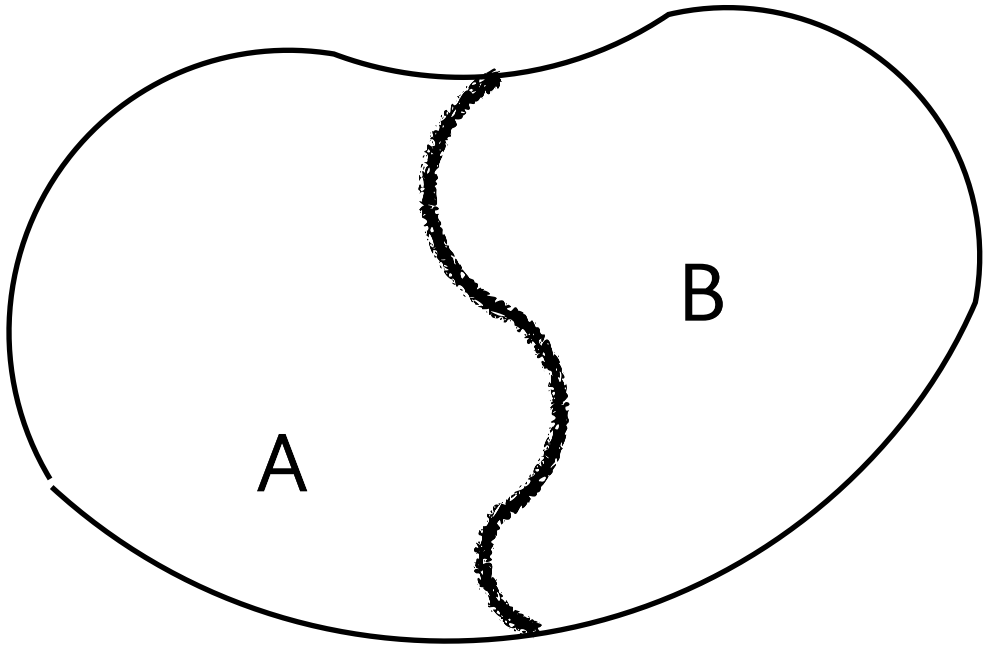

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Pure state on $\mathcal{H}$ — typically ρ_A, ρ_B will represent mixed states, reflected in entropy:

$$S = 0\ , \qquad S_A = S_B \neq 0$$

Shows that systems A and B are entangled

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A B

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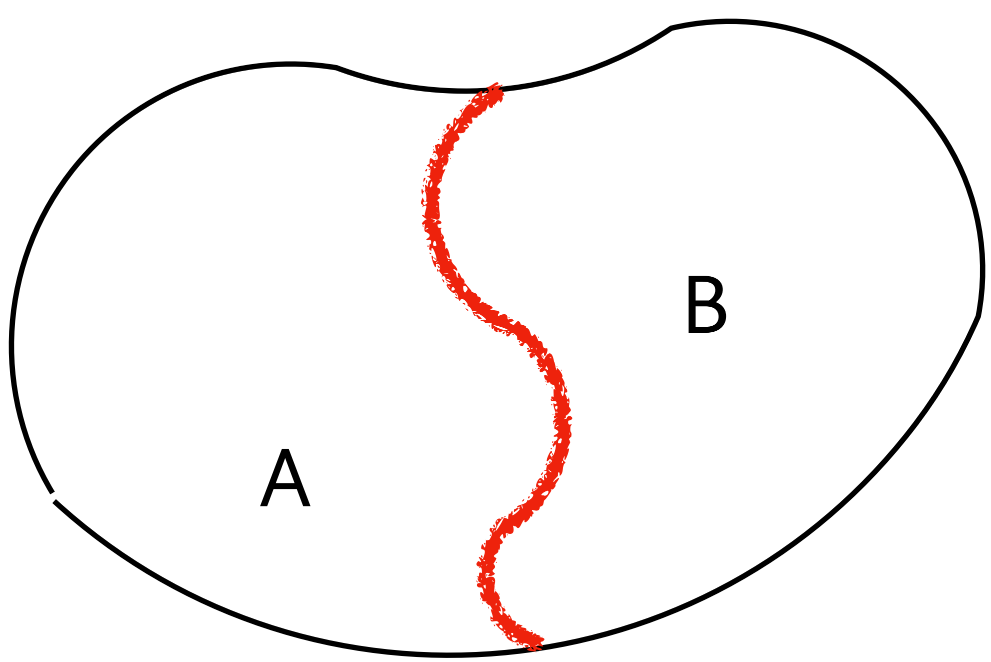
Observed that ground states seem to obey area-law entanglement

$$S_A = S_B \propto \text{area of shared boundary}$$



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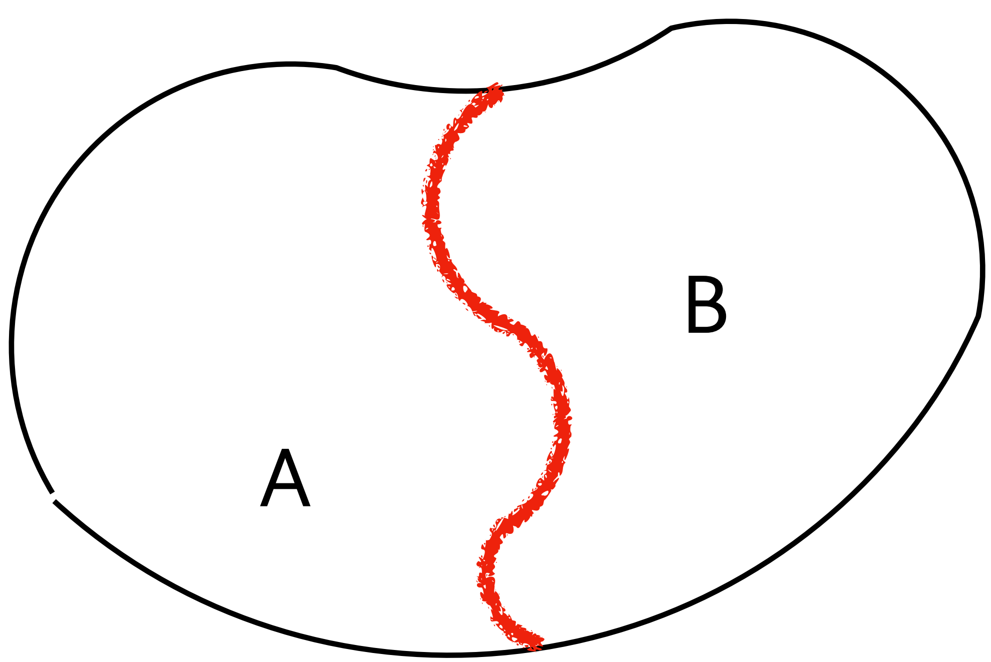

What is special about position coordinates?

Hamiltonian is local
Correlations fall off with |x-y|

Entanglement knows about dynamics

In a strongly coupled system with composite particles (eg, QCD) can entanglement help determine their wave functions and interactions (and hence their symmetries)?

Quantify the amount of entanglement in the S-matrix

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- Define the entanglement power for the S-matrix:

$$\mathcal{E}(\hat{\mathbf{S}}) = 1 - \int \frac{d\Omega_1}{4\pi} \frac{d\Omega_2}{4\pi} \, \mathrm{Tr}_1 \left[\, \hat{\rho}_1^2 \, \right]$$

Entanglement power in s-wave nucleon-nucleon scattering:

$$\hat{\mathbf{S}} = \frac{1}{4}\left(3e^{i2\delta_1} + e^{i2\delta_0}\right)\hat{\mathbf{1}} \; + \; \frac{1}{4}\left(e^{i2\delta_1} - e^{i2\delta_0}\right)\hat{\boldsymbol{\sigma}}\cdot\hat{\boldsymbol{\sigma}}$$

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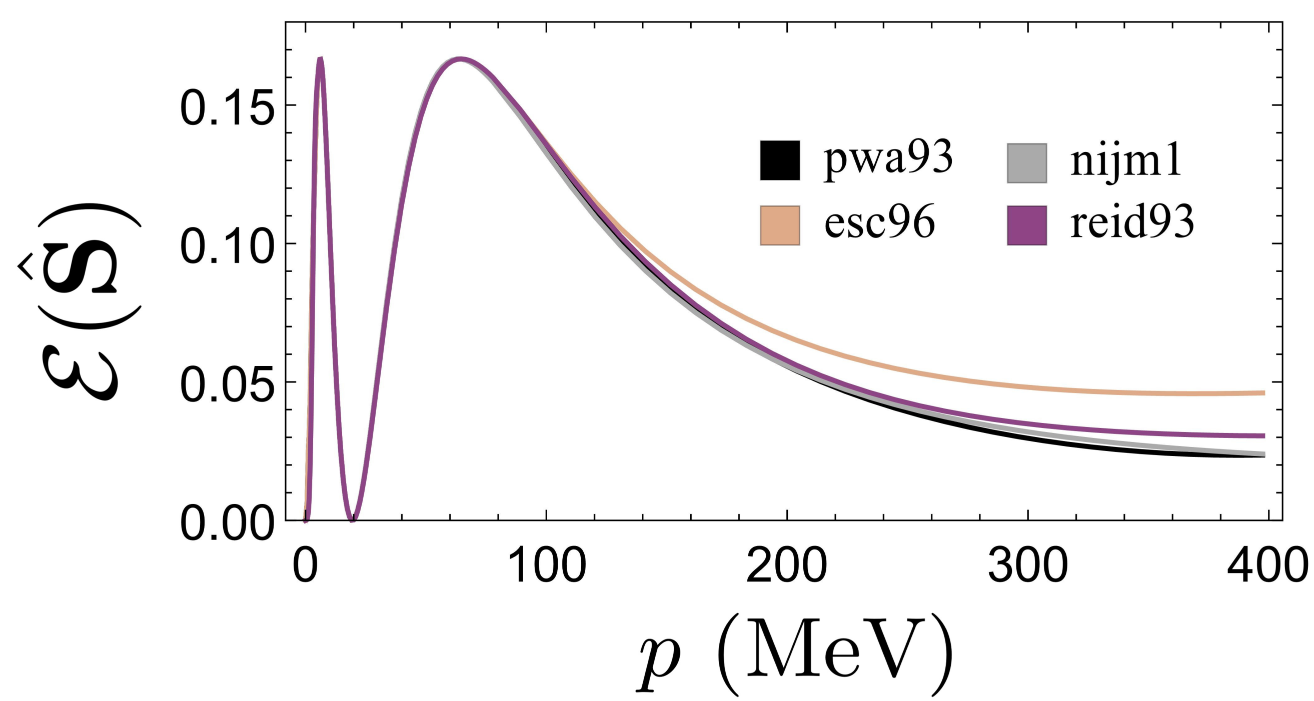
Use models which fit the phase shift data accurately:

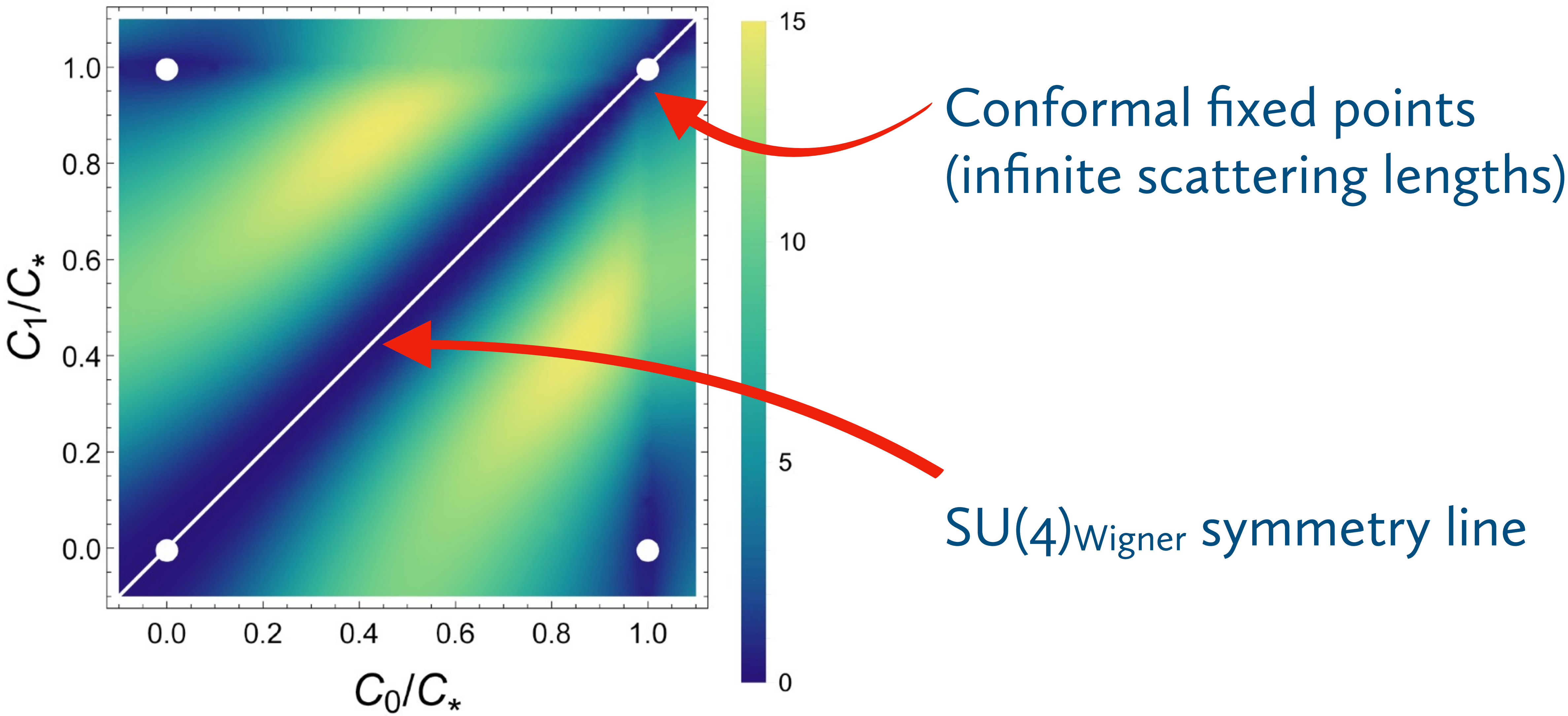


Alternatively: look at the space of low energy EFTs for $p_{cm} < m_\pi/2$

$$\mathcal{L}_6 = -\tfrac{1}{2}C_S(N^\dagger N)^2 - \tfrac{1}{2}C_T(N^\dagger\vec{\sigma}N)^2$$

$$^1S_0: \qquad \bar{C}_0 = (C_S - 3C_T)$$

$$^3S_1: \qquad \bar{C}_1 = (C_S + C_T)$$



Other definitions of entanglement power also explored

Can show in each case:

- $SU(4)_{Wigner}$ for N_f=2
- SU(16) for N_f=3
- Conformal symmetry, for N_f=2,3

Are sufficient to ensure zero entanglement power for the S-matrix

... and probably necessary.

Conclusions:

There are apparent approximate symmetries w/o explanation in the strong interactions:

- non-quark spin-flavor symmetries
- NR conformal (Schrödinger) symmetries

Can ascribe an “entanglement power” to the S-matrix which knows about flavor & spin changing interactions

Entanglement is minimized for flavors & spin diagonal interactions, as well as for conformal fixed points

Can symmetries be explained by dynamical systems “wanting” to minimize entanglement?

Need to examine more examples; model examples with feedback mechanism