Chaos and symmetry breaking in low-dimensional AdS/CFT

Bay Area Particle Theory Seminar - March 2017

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based on:
- arXiv:1605.06098 (PRL 117 (2016), 11, 116601)

An old hope

Question: what is the minimal example of AdS/CFT?

Equivalently: what is the simplest consistent theory of quantum gravity on AdS?

Pure 3d gravity

Natural candidate: 3d pure gravity with negative cc

$$S_{3d} = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left(R + \frac{2}{L^2} \right)$$

Classical 3d gravity is simple:

Pure 3d gravity

Natural candidate: 3d pure gravity with negative cc

Classical 3d gravity is simple:

1. All solutions classified: boundary gravitons plus BTZ
2. May be recast as $SO(2,1) \times SO(2,1)$ Chern-Simons theory

$$c = \frac{3L}{2G} = 24k$$

$$S_{3d} = \frac{k}{4\pi} \int \mathrm{tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$

Pure 3d gravity

Natural candidate: 3d pure gravity with negative cc

$$S_{3d} = \frac{k}{4\pi} \int \mathrm{tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) \qquad c = \frac{3L}{2G} = 24k$$

Witten 2007: Conjecture that for k=1,2,.., dual to

"extremal CFT" with torus $Z = |\chi_k(q)|^2$

$$\chi_k(q) = q^{-k} \prod_{n=2}^{\infty} (1 - q^n)^{-1} + O(q)$$

Pure 3d gravity

Natural candidate: 3d pure gravity with negative cc

$$Z = |\chi_k(q)|^2 \qquad c = \frac{3L}{2G} = 24k$$

k=1: $\chi_1(q) = J(q)$ Modular J-function

this Z corresponds to known CFT! [Frenkel, Lepowsky, Meurman]

So-called Monster CFT

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Lightest states: 196883 operators of dimension 2

Identifying as BH creation ops, get

$S = \ln 196883 \approx 12.19$ vs. $S_{BH}(k=1) = 4\pi \approx 12.57$

Pure 3d gravity

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$$Z = |\chi_k(q)|^2 \qquad c = \frac{3L}{2G} = 24k$$

k=1: $\chi_1(q) = J(q)$ Modular J-function

k>1: not known if corresponding CFT exists,
some evidence against

What about AdS_2/CFT_1?

Simplest possible setting for the duality

And ubiquitous: AdS_2 generically appears as
near-horizon of SUSY black holes

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PROBLEM: Neither AdS_2 gravity nor CFT_1 exist

The problem

AdS_2: cannot support finite-energy excitations, $\langle E \rangle = 0$

CFT_1: One runs into a paradox of [Polchinski]

scale-invariant density of states in 1d is

$$\rho(E) = e^{S_1}\delta(E) + \frac{e^{S_2}}{E}$$

The problem

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Zero-energy states

No dynamics!

The problem

scale-invariant density of states in 1d is

$$\rho(E) = e^{S_1}\delta(E) + \frac{e^{S_2}}{E}$$

Divergent at E=0

Z does not exist!

Goal for this talk:

Obtain sensible notion of AdS_2/CFT_1

Outline

1. Motivation
2. $NAdS_2/NCFT_1$
3. Chaos
4. Parting words

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2. **$NAdS_2/NCFT_1$**
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Dilaton gravity

Pure 2d gravity is topological $\int d^2x\sqrt{-g}\,R = 4\pi\chi$

Reduction to 2d leads to dilaton gravity

$$S = \frac{1}{2\kappa^2}\int d^2x\sqrt{-g}\,(\varphi R + U[\varphi]) + S_{\mathrm{matter}}$$

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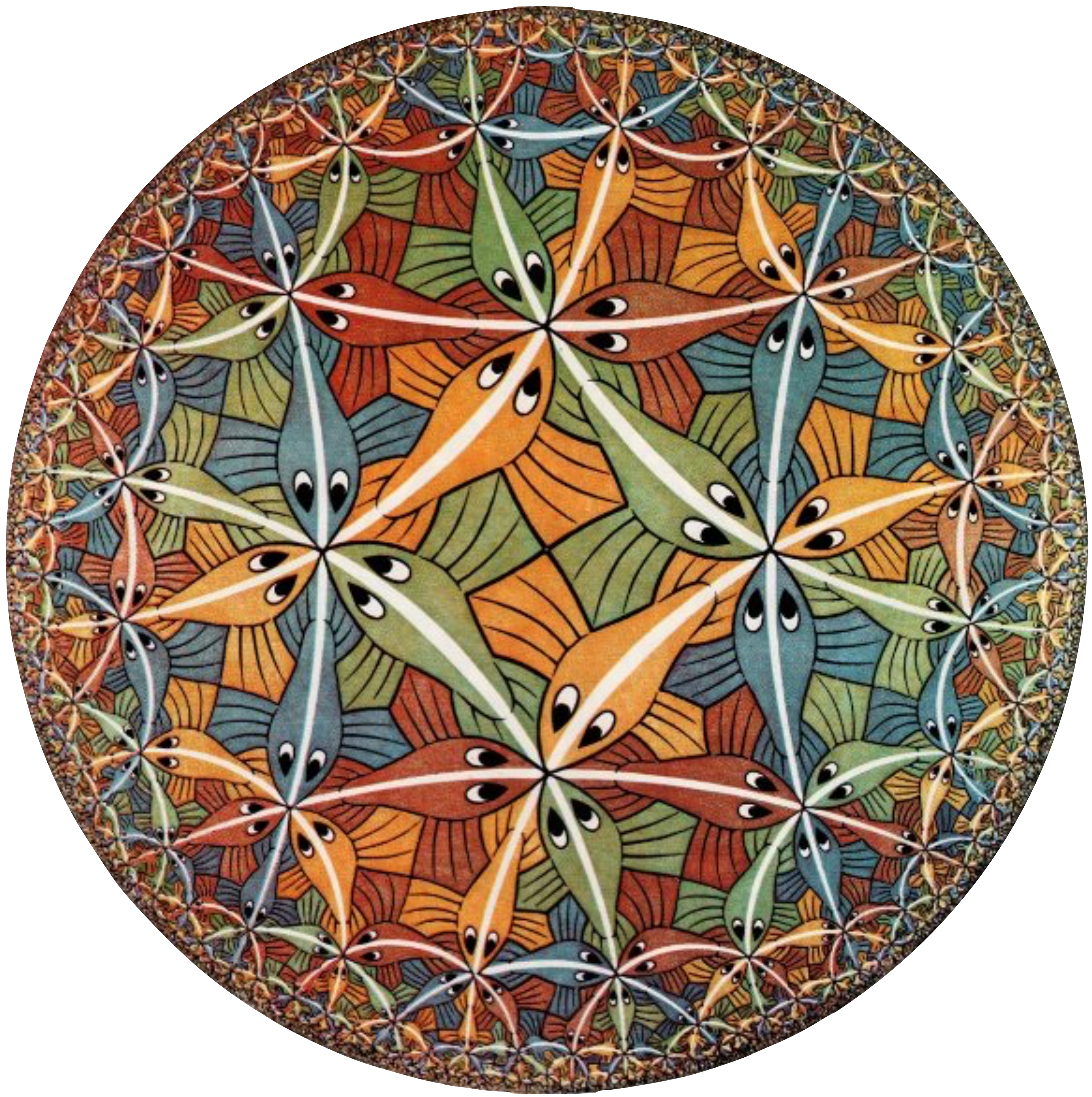
$$S = \frac{1}{2\kappa^2} \int d^2x \sqrt{-g} \left(\varphi R + U[\varphi] \right) + S_{\text{matter}}$$

General two-derivative theory
characterized by U

AdS_2 solutions with constant dilaton at roots of U

$$U[\varphi_0] = 0 \,, \qquad U'[\varphi_0] = \frac{2}{L^2}$$

AdS_2



Moduli of AdS_2

Most general AdS_2 spacetime characterized by a free function of boundary time

$$g = -r^2\left(1+\frac{h(t)}{r^2}\right)^2 dt^2 + \frac{dr^2}{r^2} \qquad \varphi = \varphi_0$$

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1. f(t) acts as conformal transformation on bdy
2. For any f(t), invariant under $PSL(2, \mathbb{R})$

$$f(t) \to \frac{af(t) + b}{cf(\tau) + d}$$

Moduli of AdS_2

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boundary conformal symmetry

= Virasoro symmetry with c=0

$NAdS_2$ spacetimes

Name due to [Maldacena, Stanford]

$U[\varphi] \approx 2(\varphi - \varphi_0)$ also admits linear dilaton solutions

$NAdS_2$ spacetimes

Name due to [Maldacena, Stanford]

$$U[\varphi] \approx 2(\varphi - \varphi_0) \longrightarrow \quad g = -r^2 dt^2 + \frac{dr^2}{r^2}$$

$$\varphi = \ell r + \varphi_0$$

Arises as IR endpoint of ANY holographic RG flow ending in AdS_2

$NAdS_2$ spacetimes

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$$U[\varphi] \approx 2(\varphi - \varphi_0) \longrightarrow \quad g = -r^2 dt^2 + \frac{dr^2}{r^2}$$

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Moduli f(t) become pseudo-moduli: Virasoro $\longrightarrow PSL(2, \mathbb{R})$

Dual to nearly conformal QM, dubbed $NCFT_1$

Fluid/gravity

Write NAdS_2 in infalling Eddington-Finkelstein coordinates

$$g = -\left(r^2 + 2\{f(t), t\}\right) dt^2 + 2dtdr + O(\varepsilon)$$

$$\varphi = \varphi_0 + \varepsilon \ell r + O(\varepsilon^2) \qquad\qquad T_{\mu\nu} = O(\varepsilon)$$

Solve bulk eoms exactly near AdS_2; can rewrite as equation on the boundary:

$$\boxed{\dot{E} = \dot{\lambda}\langle O\rangle} \quad \text{with} \quad \boxed{E = -\frac{\ell}{\kappa^2}\{f(t), t\} + (1-\Delta)\lambda\langle O\rangle}$$

Schwarzian action

Both follow from action where f(t) is fundamental field:

$$S = -\frac{\ell}{\kappa^2}\int dt\,\{f(t),t\} + W[\lambda(t);f(t)]$$

with $\{f(t),t\} = \frac{f'''(t)}{f'(t)} - \frac{3}{2}\left(\frac{f''(t)}{f'(t)}\right)^2$ = Schwarzian derivative

[KJ], [Maldacena, Stanford, Yang], [Engelsoy, Martens, Verlinde]

also obtained in the low-energy limit of Sachdev-Ye-Kitaev models [Maldacena, Stanford]

Schwarzian action

Both follow from action where f(t) is fundamental field:

$$S = -\frac{\ell}{\kappa^2}\int dt\{f(t),t\} + W[\lambda(t);f(t)] + O(\ell\lambda^2,\ell^2)$$

Leading interactions consistent with

Virasoro $\longrightarrow$ $PSL(2,\mathbb{R})$

The good and the ugly

(Very) good:

Rewrite two-derivative $NAdS_2$ gravity as 1d theory

The good and the ugly

(Very) good:

Rewrite two-derivative $NAdS_2$ gravity as 1d theory

Ugly:

Resulting Schwarzian theory is sick in isolation
[Stanford, Witten]

(situation recalls [Maloney, Witten] prescription for partition function of pure 3d gravity)

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2. $NAdS_2/NCFT_1$
3. **Chaos**
4. Parting words

Lyapunov exponent

Quantum mechanical analogue of Lyapunov exponent, characterizing early-time chaotic growth:

$$\langle [W(t), V(0)]^2 \rangle_\beta \sim e^{\lambda_L t}$$

Lyapunov exponent

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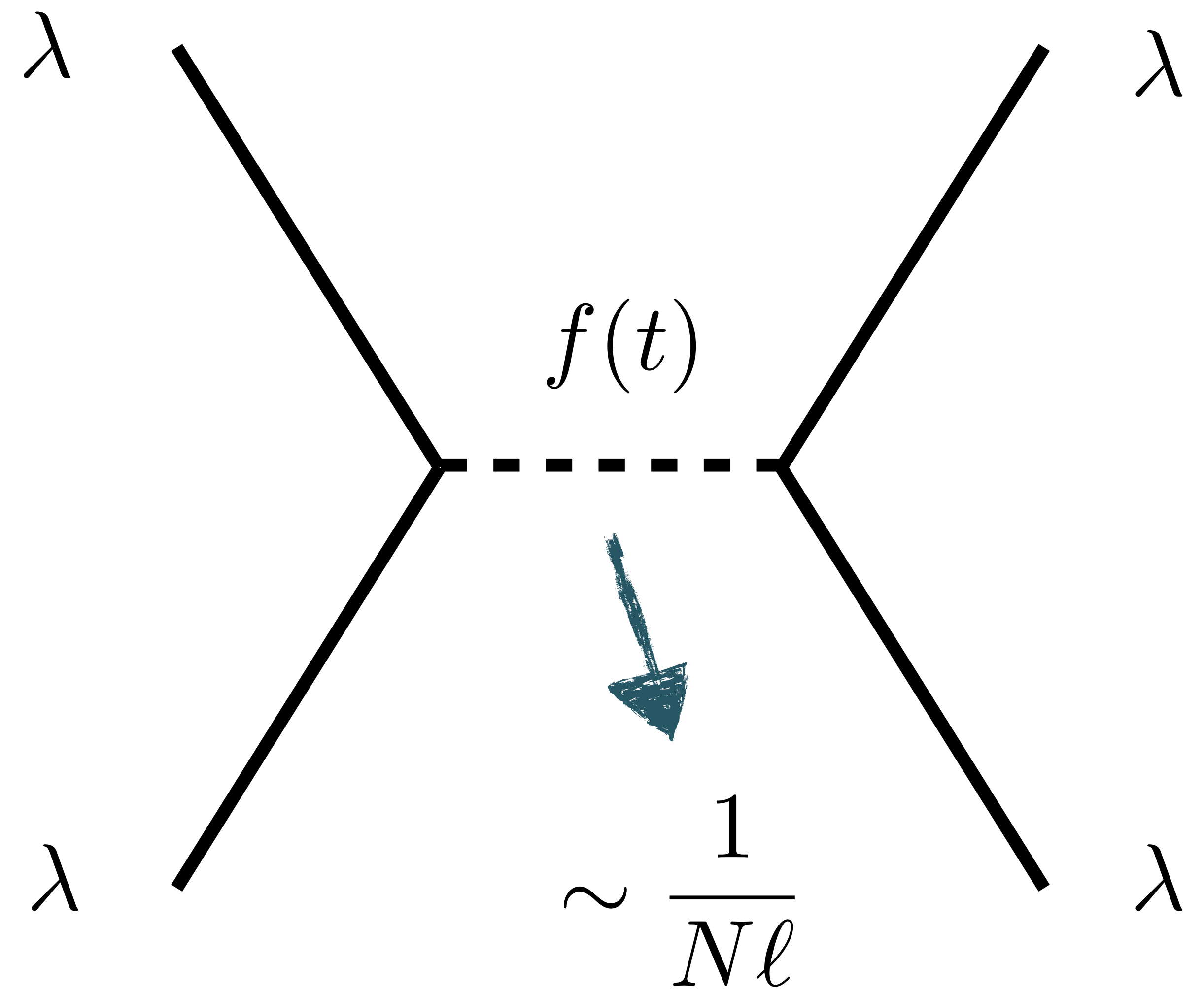
Two properties:

1. [Maldacena, Shenker, Stanford] $\lambda_L \leq \dfrac{2\pi}{\beta}$

2. Dual to Einstein gravity is maximally chaotic
[Shenker, Stanford]

Lyapunov exponent in $NAdS_2/NCFT_1$

Compute Euclidean four-point function at tree-level:



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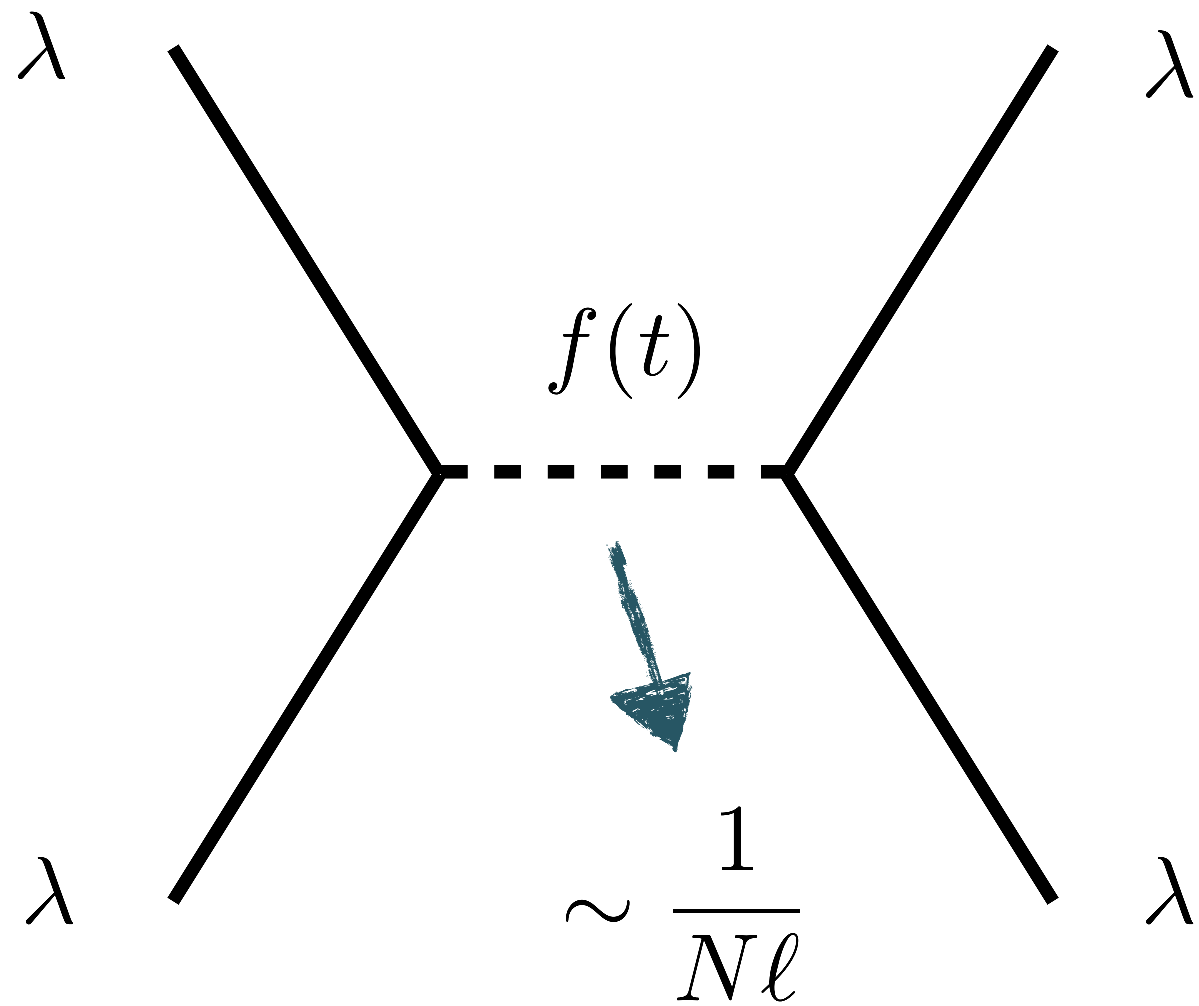

Analytically continue to get out-of-time-ordered four-point function

$$\lambda_L = \frac{2\pi}{\beta}$$

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A sigma model for AdS_3 gravity

WIP with Jordan Cotler (Stanford)

It is possible to rewrite pure classical AdS_3 gravity as an unconventional sigma model from $\mathcal{M}_2 \to \mathcal{M}_2$

A sigma model for AdS_3 gravity

WIP with Jordan Cotler (Stanford)

The action may be regarded as an action for an ``exact” hydrodynamics whose gradient expansion truncates at second order in derivatives

$$S = \frac{c}{24\pi} \int d^2\sigma \sqrt{-g} \left(4\pi^2 T^2 - \frac{(\partial T)^2}{T^2} - \ln T\, R \right)$$

A sigma model for AdS_3 gravity

WIP with Jordan Cotler (Stanford)

$$S = \frac{c}{24\pi}\int d^2\sigma\sqrt{-g}\left(4\pi^2 T^2 - \frac{(\partial T)^2}{T^2} - \ln T\, R\right)$$

Torus partition function?

Coupling to matter and Virasoro blocks?

Stay tuned!

Thank you!