Multicritical Nambu-Goldstone Modes and Nonrelativistic Naturalness

❀ Petr Hořava ❀

Berkeley Center for Theoretical Physics

Bay Area Particle Theory Seminar

SFSU, San Francisco, CA

October 10, 2014

work with Tom Griffin, Kevin Grosvenor, Ziqi Yan

Puzzles of Naturalness

Some of the most fascinating open problems in modern physics are all problems of naturalness:

- The cosmological constant problem
- The Higgs mass hierarchy problem
- The linear resistivity of strange metals, the regime above T_c in high-T_c superconductors [Bednorz&Müller '86; Polchinski '92]

In addition, the first two •s – together with the recent experimental facts – suggest that we may live in a strangely simple Universe . . .

Naturalness is again in the forefront
(as are its possible alternatives: landscape? . . .?)
If we are to save naturalness, we need new surprises!

What is Naturalness?

Technical Naturalness: 't Hooft (1979)

"The concept of causality requires that macroscopic phenomena follow from microscopic equations."

"The following dogma should be followed: At any energy scale μ, a physical parameter or a set of physical parameters $\alpha_i(\mu)$ is allowed to be very small only if the replacement $\alpha_i(\mu) = 0$ would increase the symmetry of the system."

Example: Massive $\lambda\phi^4$ in $3+1$ dimensions.

$$\lambda \sim \varepsilon, \quad m^2 \sim \mu^2\varepsilon, \quad \mu \sim m/\sqrt{\lambda}.$$

Symmetry: The constant shift $\phi \to \phi + a$.

"Pursuing naturalness beyond 1000 GeV will require theories that are immensely complex compared with some of the grand unified schemes."

Gravity without Relativity

(a.k.a. gravity with anisotropic scaling, or Hořava-Lifshitz gravity)

Field theories with anisotropic scaling:

$$x^i \to \lambda x^i, \quad t \to \lambda^z t.$$

z: dynamical critical exponent – characteristic of RG fixed point.

Many interesting examples: $z = 1,\ 2,\ \ldots,\ n,\ \ldots$
fractions: 3/2 (KPZ surface growth in $D = 1$), ..., $1/n$, ...
families with z varying continuously ...

Condensed matter, dynamical critical phenomena, quantum critical systems, ...

... and now gravity as well, with propagating gravitons, formulated as a quantum field theory of the metric.

Spontaneous Symmetry Breaking

Global internal symmetry breaking leads to Nambu-Goldstone modes. Phenomenon is remarkably universal, across many fields dealing with many-body systems.

But how many NG modes, and what is their low-energy dispersion relation?

- Relativistic case: All questions answered by Goldstone's theorem: One NG per broken generator, gapless=massless, $z = 1$ dispersion $\omega = k$.

- Nonrelativistic case: Classify NG modes by classifying their low-energy effective QFTs [Murayama&Watanabe, '12,'13].

Let's focus, for definiteness, on systems with Lifshitz symmetries. Write down possible EFT's for NG modes π^I.

Nonrelativistic Goldstone Theorem?

Assume Lifshitz symmetry. Then [Murayama&Watanabe]:
the EFTs are

$$S = \int dt\, d^D\mathbf{x} \left(\Omega_I(\pi)\dot{\pi}^I + g_{IJ}\dot{\pi}^I\dot{\pi}^I - h_{IJ}\partial_i\pi^I\partial_i\pi^J + \ldots \right).$$

Hence, this yields two types of NG modes:

- Type A, $z = 1$ dispersion $\omega = ck$ (those unpaired by Ω, with no T-reversal breaking). As in the relativistic case, one Type A NG mode per one broken generator.

- Type B, dispersion $\omega \sim k^2$. Each associated with a *pair* of broken symmetry generators, as paired by Ω. Minimal T-reversal symmetry is broken.

Anything else would be fine tuning . . . or would it?

Is there a gap in the argument? Consider $z = 2$ theory:

$$S_{\text{eff}} = \int dt\, d^D\mathbf{x}\left(g_{IJ}\dot{\pi}^I\dot{\pi}^J - \tilde{g}_{IJ}\Delta\pi^I\Delta\pi^J - c^2\hat{g}_{IJ}\partial_i\pi^I\partial_j\pi^J\right).$$

If the relevant deformation is not generated, we can have new NG modes, with $z = 2$, associated with just one broken symmetry and not a pair, and with no time reversal breaking.

Example: Start with $z = 2$ $O(N)$ LSM in $3+1$ dimensions,

$$S_{\text{eff}} = \int dt\, d^3\mathbf{x}\left(\dot{\phi}\cdot\dot{\phi} - \Delta\phi\cdot\Delta\phi - g(\phi^2)^2\partial_i\phi\partial_i\phi - \ldots - \lambda_5(\phi^2)^5\right.$$

$$\left. - \ldots - c^2\partial_i\phi\cdot\partial_i\phi - \ldots - \lambda(\phi^2)^2 + m^4\phi^2\right).$$

Consider this theory in the broken phase.
Keep it interacting, but make it superrenormalizable, like this:

Is there a gap in the argument? Consider $z = 2$ theory:

$$S_{\text{eff}} = \int dt\, d^D\mathbf{x}\, \left(g_{IJ}\dot{\pi}^I\dot{\pi}^J - \tilde{g}_{IJ}\Delta\pi^I\Delta\pi^J - c^2\hat{g}_{IJ}\partial_i\pi^I\partial_j\pi^J \right).$$

If the relevant deformation is not generated, we can have new NG modes, with $z = 2$, associated with just one broken symmetry and not a pair, and with no time reversal breaking.

Example: Start with $z = 2$ $O(N)$ LSM in $3 + 1$ dimensions,

$$S_{\text{eff}} = \int dt\, d^3\mathbf{x} \left(\dot{\phi}\cdot\dot{\phi} - \Delta\phi\cdot\Delta\phi - g(\phi^2)^2\partial_i\phi\partial_i\phi - \ldots - \lambda_5(\phi^2)^5 \right.$$

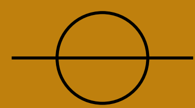
$$\left. - \ldots - c^2\partial_i\phi\cdot\partial_i\phi - \ldots - \lambda(\phi^2)^2 + m^4\phi^2 \right).$$

The effective speed counterterm for the NG modes is not generated, quantum correction is (finite and) small.
(To maintain control at low energies, can take large N.)

Naturalness of Slow NG Modes

How small is the quantum correction δc^2?

Consider the LSM, for simplicity in the unbroken phase. The first quantum correction to $\delta c^2 = 0$ comes at two loops, from



$$\sim \left(\frac{\lambda^2}{m^4}\right) |\mathbf{k}|^2 + \ldots,$$

and it is finite. What does this mean?

Assume a hidden symmetry, broken by ε at scale μ:

$$\lambda \sim \mu^3 \varepsilon, \quad m^4 \sim \mu^4 \varepsilon, \quad c^2 \sim \mu^2 \varepsilon.$$

This implies $\mu \sim m/\lambda$ and $c^2 \sim \lambda^2/m^4$, just as we found by the explicit calculation!

Polynomial Shift Symmetry

So, there must be a new symmetry at play:

the “quadratic shift” symmetry

$$\pi^I(t,x^i) \to \pi^I(t,x^i) + a^I_{ij} x^i x^j + a^I_j x^j + a_0.$$

Note it depends only on spatial coordinates, not on time.

This construction naturally iterates:

The higher “polynomial shift symmetry,”

$$\pi^I(t,x^i) \to \pi^I(t,x^i) + a^I_{j_1 j_2 \cdots j_{2z-2}} x^{j_1} x^{j_2} \ldots x^{j_{2z-2}} + \ldots$$

protects the $\omega \sim k^z$ low-energy dispersion for Type A modes (and the $\omega \sim k^{2z}$ low-energy dispersion for Type B modes).

Refining the Classification of Nonrelativistic NG Modes

Refined classification of technically natural NG modes with Lifshitz spacetime symmetries:

• Type A tower of multicritical NG modes with $z = 1, 2, \ldots$, until one hits against the multicritical analog of the Coleman Mermin-Wagner theorem at $z = D$;

• Type B tower of multicritical NG modes with $z = 2, 4, \ldots$ (and no analog of the MCW theorem).

These IR fixed points describe the free limit of multicritical NG modes, and imply low-energy theorems for scattering etc.

Generic interactions break the polynomial shift symmetry to the constant shift. But: Corrections are controllably small, if couplings are small.

Coleman-Mermin-Wagner Theorem & Cascading Multicriticality

Recall relativistic CMW theorem: In $1+1$ dimensions, SSB which would require a NG mode ϕ can never happen, since ϕ does not exist: $\langle\phi(0)\phi(x)\rangle \sim \log(\mu_{\rm IR}x)$.

Multicritical analog of CMW theorem: Type A $D=z$ NG modes do not exist as quantum objects at the fixed point.

Take say $z=3$ in $3+1$ dimensions, below some scale μ. Naive CMW theorem: no symmetry breaking, no condensate?

Novelty: Cascading hierarchical multicriticality of NG modes.

At some *physical* crossover scale $\mu_{\rm IR} \ll \mu$, turn on a $z<3$ deformation. The theory self-regulates in IR, with $\omega \sim |\mathbf{k}|^3 + \ldots \mu_{\rm IR}^2|\mathbf{k}|$. And SSB is possible, after all!

Lab implications in condensed matter? Cosmology?

Field Theories with Polynomial Shift Symmetries

So, we found examples where a new symmetry

$$\phi(t,x^i) \to \phi(t,x^i) + a_{j_1 j_2 \dots j_P} x^{j_1} x^{j_2} \dots x^{j_P} + \dots$$

protects the smallness of leading terms in the dispersion relation, and protects hierarchies.

In the examples shown, the symmetry is broken by interactions.

Now we can turn this around, and ask for the classification of scalar theories in which the polynomial shift symmetry is exact.

This is a very cute mathematical problem!

The simplest case of linear shift is related to the Galileon.

Polynomial Shift Invariants

It is natural to organize the invariants by their dimension at the free RG fixed point.

Task: Classify all terms in the Lagrangian containing n fields and $\Delta \equiv 2m$ derivatives, invariant under the degree-P shift symmetry up to a total derivative:

$$\delta_P L = \partial_i L_i.$$

This is essentially a cohomological problem. It defines a vector space of invariants $H_{P,n,\Delta,D}$.

How to solve it?

use Graph Theory!

Graph Theory

Represent each $\partial_{i_1} \ldots \phi \ldots \partial_{i_{2m}} \phi$ term by a graph:

(1) Each field ϕ is represented by a vertex ($\bullet$);

(2) Each pair $\partial_i [\ldots] \partial_i$ is represented by a link: $\overline{\qquad\qquad}$

Consider only "loopless graphs" – classification up to integration by parts.

To formulate $\delta_P L = \partial_i L_i$, two more vertices needed:

(3) each $a_{j_1 \ldots j_P} x^{j_1} \ldots x^{j_P}$ is represented by "$\otimes$";

(4) The ∂_i on the RHS is represented by a "free end": $\star$.

Examples: Galileons and Beyond

Start with $P = 1$, linear shift symmetry.

Problem of (minimal) n-point 1-invariants is equivalent to Galileons.

The minimal value for Δ is $\Delta = n - 1$, the invariant is unique:

$$L_n \propto T^{i_1 \dots i_{n-1} j_1 \dots j_{n-1}} \, \partial_{i_1}\phi \, \partial_{j_1}\phi \, \partial_{i_2 j_2}\phi \dots \partial_{i_{n-1} j_{n-1}}\phi,$$

where

$$T^{i_1 \dots i_{n-1} j_1 \dots j_{n-1}} = \varepsilon^{i_1 \dots i_{n-1} k_n \dots k_D} \, \varepsilon^{j_1 \dots j_{n-1}}{}_{k_n \dots k_D}.$$

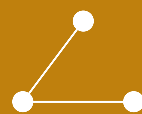
These are known. Yet, the graph-theory representation reveals new patterns in these 1-invariants.

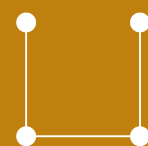
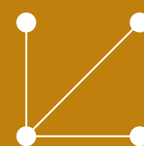
Examples: Galileons and Beyond

The n-point 1-invariants are:

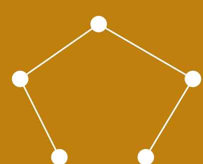
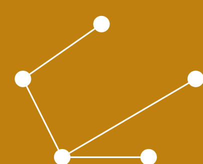
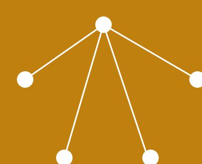
$L_1 = \phi = \bullet,$

$L_2 = \partial_i\phi\partial_i\phi =$  ,

$L_3 = 3\partial_i\phi\partial_j\phi\partial_i\partial_j\phi = 3$  ,

$L_4 = 12[i][ij]jk][k] + 4[i][j][k][ijk] = 12$  $+ 4$  ,

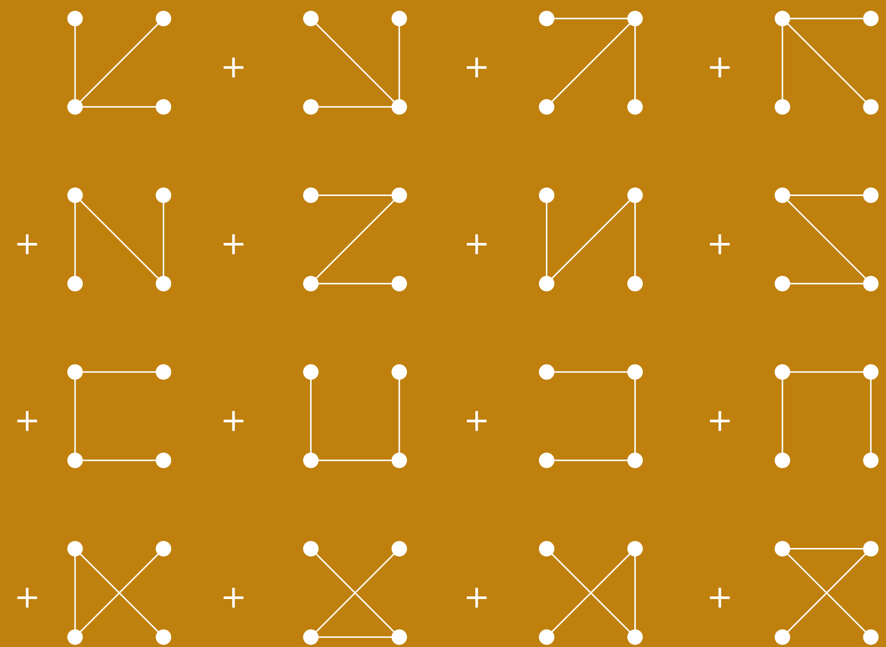
$L_5 = 60[i][ij][jk][kl][l] + 60[i][ij][jkl][k][l] + 5[i][j][k][l][ijkl]$

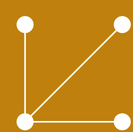
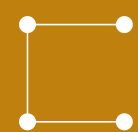
$= 60$  $+ 60$  $+ 5$  .

In graph theory, these unique 1-invariants correspond to
the sum over all spanning trees, with equal weight!

Examples: Galileons and Beyond

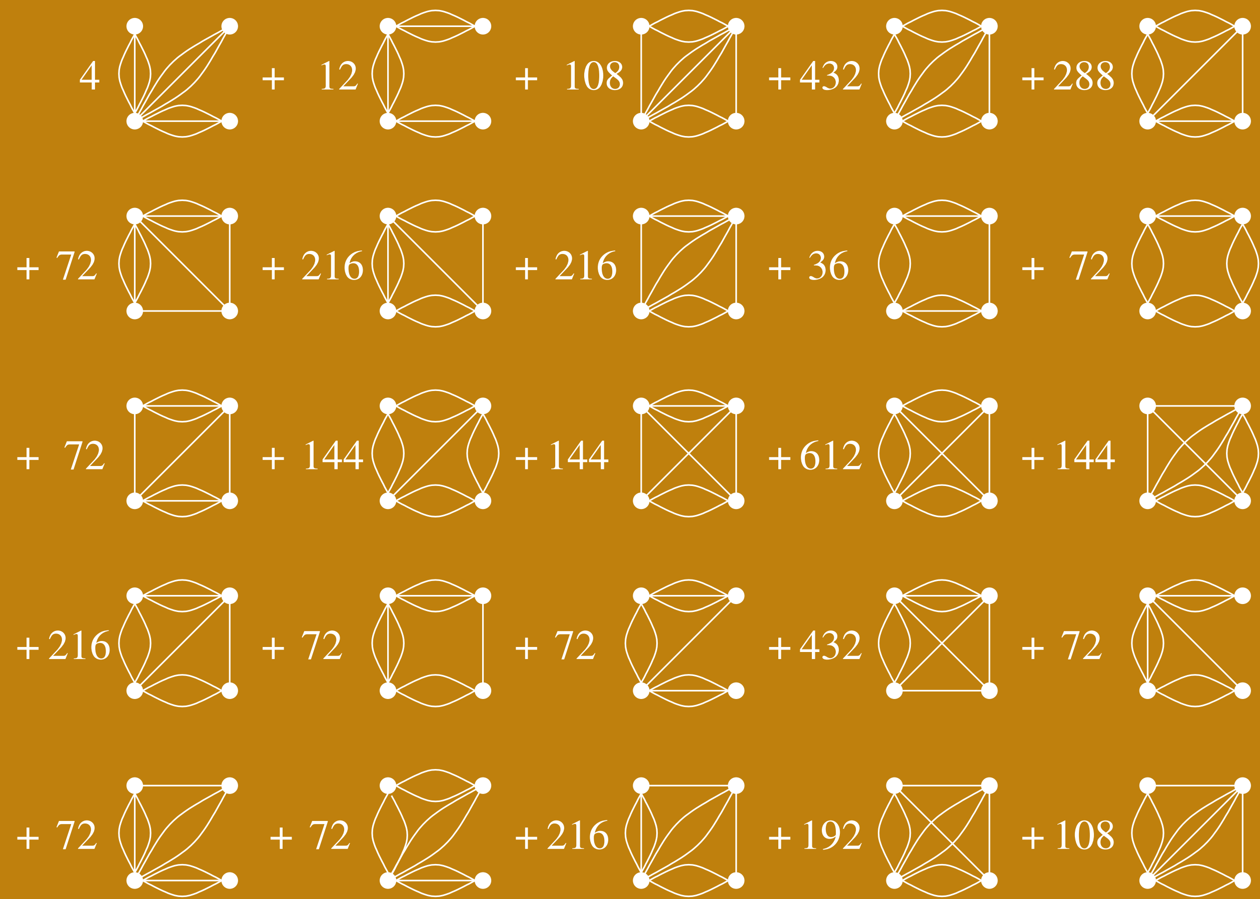
For example, the 4-point 1-invariant L_4 is:



$= 4$  $+ 12$ 

$P > 0$ Invariants: Superposition of Trees

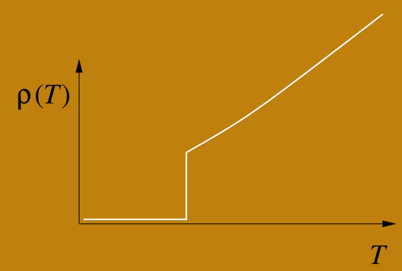
Example: The most relevant quintic-shift 4-pt invariant is



Back to Naturalness in Physics

In high-T_c superconductors, the phase above T_c exhibits "unnatural" properties:

Resistivity $\rho(T) \sim T$ over several orders of magnitude in temperature:



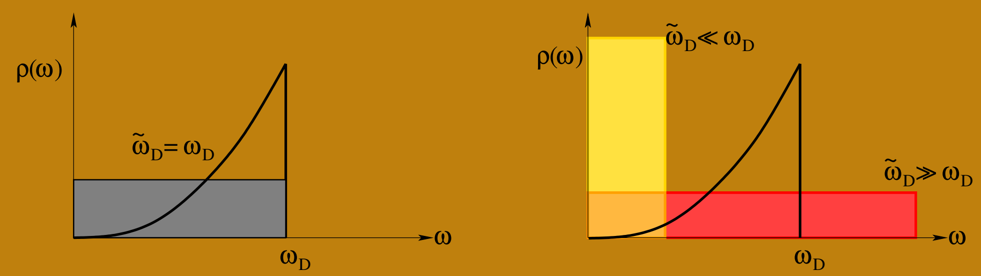
Use EFT: What gives $\rho \sim T$? Nothing!

- Electron-phonon interactions (the main mechanism for pairing in BCS): $\rho \sim T^5$
- Electron-electron interactions: $\rho \sim T^2$
- Electron-impurities: $\rho \sim \mathrm{const}$

A Simple Model of Strange Metals

Phonons are NG modes of SSB of space translations.

Consider Debye model with multicritical phonons: $\omega = \zeta|\mathbf{k}|^z$.
Phonon spectrum cut off at the Debye frequency $\tilde{\omega}_{\mathrm{D}}$.
Lower critical dimension: $D = z$. Density of states:



Couple to the Fermi surface, minimally:

$$g \int dt\, d^D\mathbf{x}\, Q\Psi^\dagger\Psi \equiv g \int dt\, d^D\mathbf{x}\, \partial_i Q_i \Psi^\dagger\Psi.$$

This coupling breaks polynomial shift, generates relevant deformations, and a natural pairing mechanism.

Resistivity in Strange Metals

Transport: Use Bloch-Boltzmann theory. In metals, this gives the Bloch-Grüneisen formula (with $\tau(\mathbf{k})$ the relaxation time):

$$\rho \sim \frac{1}{\tau(\varepsilon_{\mathrm{F}})} \sim \int_0^{\varepsilon_{\mathrm{F}}/T} |g_k|^2 n(k) k^2 \, k dk$$

with $n(k) = \dfrac{1}{\exp(\omega_k/T) - 1}$ the phonon distribution function,

and $g_k = g\dfrac{k}{\sqrt{\omega_k}}$ the electron-phonon vertex.

$3+1$ dimensions, $z=1$ phonons: $\rho \sim T^5$.

Resistivity in Strange Metals

Transport: Use Bloch-Boltzmann theory. In metals, this gives the Bloch-Grüneisen formula (with $\tau(\mathbf{k})$ the relaxation time):

$$\rho \sim \frac{1}{\tau(\varepsilon_{\mathrm{F}})} \sim \int_0^{\varepsilon_{\mathrm{F}}/T} |g_k|^2 n(k) k^2 \, k dk$$

with $n(k) = \dfrac{1}{\exp(\omega_k/T) - 1}$ the phonon distribution function,

and $g_k = g \dfrac{k}{\sqrt{\omega_k}}$ the electron-phonon vertex.

$3+1$ dimensions, general z phonons: $\rho \sim T^{(6-z)/z}$.

Resistivity in Strange Metals

Transport: Use Bloch-Boltzmann theory. In metals, this gives the Bloch-Grüneisen formula (with $\tau(\mathbf{k})$ the relaxation time):

$$\rho \sim \frac{1}{\tau(\varepsilon_{\rm F})} \sim \int_0^{\varepsilon_{\rm F}/T} |g_k|^2 n(k) k^2 \, k dk$$

with $n(k) = \dfrac{1}{\exp(\omega_k/T) - 1}$ the phonon distribution function,

and $g_k = g\dfrac{k}{\sqrt{\omega_k}}$ the electron-phonon vertex.

$D+1$ dimensions, general z phonons: $\rho \sim T^{(3+D-z)/z}$.

Outlook

Naturalness can still bring surprises, especially in non-relativistic theories where our hep-th intuition is often misleading

Applications to condensed matter (high-T_c superconductivity?)

Our results touch on hep-th, gr-qc, cond-mat, math-ph, hep-ph . . .

Keeping the original motivation in mind:

The two leading Naturalness puzzles are in astro-ph and hep-ph . . . can some new, perhaps yet undiscovered surprises about Naturalness help resolve them?