Virtual proton smashers for the LHC

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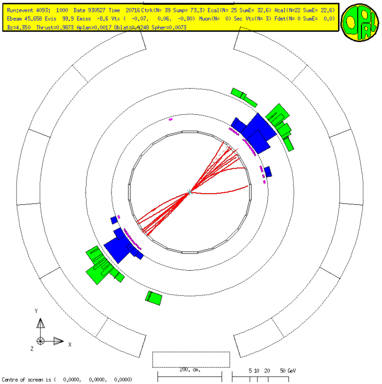
Bay Area Particle Theory Seminar

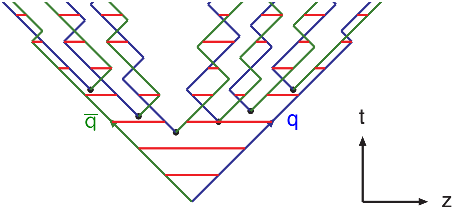
San Francisco, 10/14/2016

U.S. DEPARTMENT OF ENERGY
Office of Science

SLAC NATIONAL ACCELERATOR LABORATORY

[Andersson,Gustafson,Ingelman,Sjöstrand] Phys.Rept.97(1983)31





- Lund string model: $\sim$ like rubber band that is pulled apart and breaks into pieces, or like a magnet broken into smaller pieces.
- Complete description of 2-jet events in $e^+e^- \to$hadrons

Event generators in 1978

[Andersson,Gustafson,Ingelman,Sjöstrand] Phys.Rept.97(1983)31

```
      SUBROUTINE JETGEN(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      IFLSGN=(10-IFLBEG)/5
      W=2.*EBEG
      I=0
      IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
      IFL1=IABS(IFLBEG)
      PT1=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI1=6.2832*RANF(0)
      PX1=PT1*COS(PHI1)
      PY1=PT1*SIN(PHI1)
  100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
      IFL2=1+INT(RANF(0)/PUD)
      PT2=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI2=6.2832*RANF(0)
      PX2=PT2*COS(PHI2)
      PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
      K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
      ISPIN=INT(PS1+RANF(0))
      K(I,2)=1+9*ISPIN+K(I,1)
      IF(K(I,1).LE.6) GOTO 110
      TMIX=RANF(0)
      KM=K(I,1)-6+3*ISPIN
      K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
  110 P(I,5)=PMAS(K(I,2))
      P(I,1)=PX1+PX2
      P(I,2)=PY1+PY2
      PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
      X=RANF(0)
      IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
      P(I,3)=(X*W-PMTS/(X*W))/2.
      P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
  120 IPD=IPD+1
      IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
      IF(IPD.LT.I.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
      IFL1=IFL2
      PX1=-PX2
      PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
      W=(1.-X)*W
      IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
      N=I
      RETURN
      END
```

```
      SUBROUTINE LIST(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /DATA3/ CHA1(9), CHA2(19), CHA3(2)
      WRITE(6,110)
      DO 100 I=1,N
      IF(K(I,1).GT.0) C1=CHA1(K(I,1))
      IF(K(I,1).LE.0) IC1=-K(I,1)
      C2=CHA2(K(I,2))
      C3=CHA3((47-K(I,2))/20)
      IF(K(I,1).GT.0) WRITE(6,120) I, C1, C2, C3, (P(I,J), J=1,5)
  100 IF(K(I,1).LE.0) WRITE(6,130) I, IC1, C2, C3, (P(I,J), J=1,5)
      RETURN
  110 FORMAT(////T11,'I',T17,'ORI',T24,'PART',T32,'STAB',
     &T44,'PX',T56,'PY',T68,'PZ',T80,'E',T92,'M'/)
  120 FORMAT(10X,I2,4X,A2,1X,2(4X,A4),5(4X,F8.1))
  130 FORMAT(10X,I2,4X,1X,I2,2(4X,A4),5(4X,F8.1))
      END
```

```
      SUBROUTINE DECAY(IPD,I)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      COMMON /DATA2/ IDCO(12), CBR(29), KDP(29,3)
      DIMENSION U(3), BE(3)
C 1 DECAY CHANNEL CHOICE, GIVES DECAY PRODUCTS
      TBR=RANF(0)
      IDC=IDCO(K(IPD,2)-7)
  100 IDC=IDC+1
      IF(TBR.GT.CBR(IDC)) GOTO 100
      ND=(59+KDP(IDC,3))/20
      DO 110 I1=I+1,I+ND
      K(I1,1)=-IPD
      K(I1,2)=KDP(IDC,I1-I)
  110 P(I1,5)=PMAS(K(I1,2))
C 2 IN THREE-PARTICLE DECAY CHOICE OF INVARIANT MASS OF PRODUCTS 2+3
      IF(ND.EQ.2) GOTO 130
      SA=(P(IPD,5)+P(I+1,5))**2
      SB=(P(IPD,5)-P(I+1,5))**2
      SC=(P(I+2,5)+P(I+3,5))**2
      SD=(P(I+2,5)-P(I+3,5))**2
      TDU=(SA-SD)*(SB-SC)/(4.*SQRT(SB*SC))
      IF(K(IPD,2).GE.11) TDU=SQRT(SB*SC)*TDU**3
  120 SX=SC+(SB-SC)*RANF(0)
      TDF=SQRT((SX-SA)*(SX-SB)*(SX-SC)*(SX-SD))/SX
      IF(K(IPD,2).GE.11) TDF=SX*TDF**3
      IF(RANF(0)*TDU.GT.TDF) GOTO 120
      P(100,5)=SQRT(SX)
C 3 TWO-PARTICLE DECAY IN CM, TWICE TO SIMULATE THREE-PARTICLE DECAY
  130 DO 160 IL=1,ND-1
      IO=(IL-1)*100-(IL-2)*IPD
      I1=I+IL
      I2=(ND-IL-1)*100-(ND-IL-2)*(I+IL+1)
      PA=SQRT((P(IO,5)**2-(P(I1,5)+P(I2,5))**2)*
     &(P(IO,5)**2-(P(I1,5)-P(I2,5))**2))/(2.*P(IO,5))
  140 U(3)=2.*RANF(0)-1.
      PHI=6.2832*RANF(0)
      U(1)=SQRT(1.-U(3)**2)*COS(PHI)
      U(2)=SQRT(1.-U(3)**2)*SIN(PHI)
      TDA=1.-(U(1)*P(IO,1)+U(2)*P(IO,2)+U(3)*P(IO,3))**2/
     &(P(IO,1)**2+P(IO,2)**2+P(IO,3)**2)
      IF(K(IPD,2).GE.11.AND.IL.EQ.2.AND.RANF(0).GT.TDA) GOTO 140
      DO 150 J=1,3
      P(I1,J)=PA*U(J)
  150 P(I2,J)=-PA*U(J)
      P(I1,4)=SQRT(PA**2+P(I1,5)**2)
  160 P(I2,4)=SQRT(PA**2+P(I2,5)**2)
C 4 DECAY PRODUCTS LORENTZ TRANSFORMED TO LAB SYSTEM
      DO 190 IL=ND-1,1,-1
      IO=(IL-1)*100-(IL-2)*IPD
      DO 170 J=1,3
  170 BE(J)=P(IO,J)/P(IO,4)
      GA=P(IO,4)/P(IO,5)
      DO 190 I1=I+IL,I+ND
      BEP=BE(1)*P(I1,1)+BE(2)*P(I1,2)+BE(3)*P(I1,3)
      DO 180 J=1,3
  180 P(I1,J)=P(I1,J)+GA*(GA/(1.+GA)*BEP+P(I1,4))*BE(J)
  190 P(I1,4)=GA*(P(I1,4)+BEP)
      I=I+ND
      RETURN
      END
```

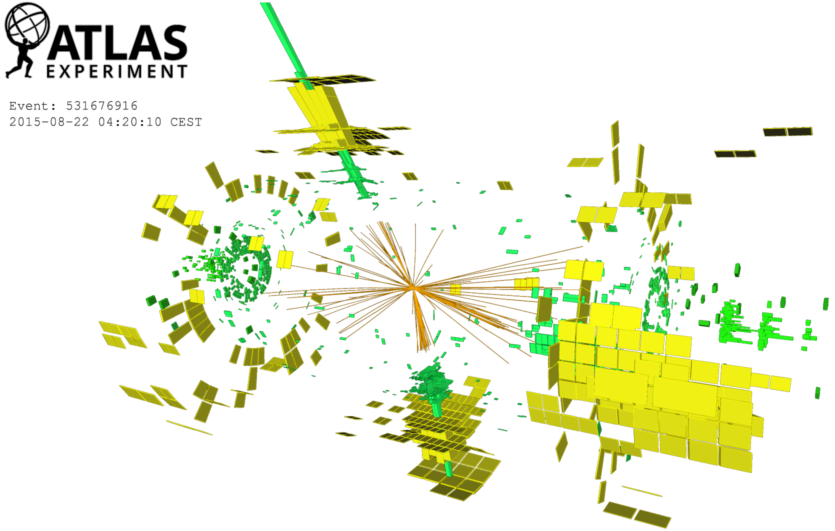
≈ 200 punched cards

Fortran code

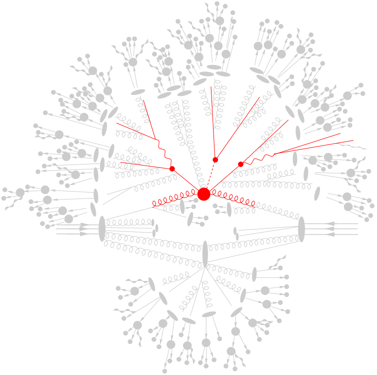
```
      SUBROUTINE EDIT(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /EDPAR/ ITHROW, PZMIN, PMIN, THETA, PHI, BETA(3)
      REAL ROT(3,3), PR(3)
C 1 THROW AWAY NEUTRALS OR UNSTABLE OR WITH TOO LOW PZ OR P
      I1=0
      DO 110 I=1,N
      IF(ITHROW.GE.1.AND.K(I,2).GE.8) GOTO 110
      IF(ITHROW.GE.2.AND.K(I,2).GE.6) GOTO 110
      IF(ITHROW.GE.3.AND.K(I,2).EQ.1) GOTO 110
      IF(P(I,3).LT.PZMIN.OR.P(I,4)**2-P(I,5)**2.LT.PMIN**2) GOTO 110
      I1=I1+1
      K(I1,1)=IDIM(K(I,1),0)
      K(I1,2)=K(I,2)
      DO 100 J=1,5
  100 P(I1,J)=P(I,J)
  110 CONTINUE
      N=I1
C 2 ROTATE TO GIVE JET PRODUCED IN DIRECTION THETA, PHI
      IF(THETA.LT.1E-4) GOTO 140
      ROT(1,1)=COS(THETA)*COS(PHI)
      ROT(1,2)=-SIN(PHI)
      ROT(1,3)=SIN(THETA)*COS(PHI)
      ROT(2,1)=COS(THETA)*SIN(PHI)
      ROT(2,2)=COS(PHI)
      ROT(2,3)=SIN(THETA)*SIN(PHI)
      ROT(3,1)=-SIN(THETA)
      ROT(3,2)=0.
      ROT(3,3)=COS(THETA)
      DO 130 I=1,N
      DO 120 J=1,3
  120 PR(J)=P(I,J)
      DO 130 J=1,3
  130 P(I,J)=ROT(J,1)*PR(1)+ROT(J,2)*PR(2)+ROT(J,3)*PR(3)
C 3 OVERALL LORENTZ BOOST GIVEN BY BETA VECTOR
  140 IF(BETA(1)**2+BETA(2)**2+BETA(3)**2.LT.1E-8) RETURN
      GA=1./SQRT(1.-BETA(1)**2-BETA(2)**2-BETA(3)**2)
      DO 160 I=1,N
      BEP=BETA(1)*P(I,1)+BETA(2)*P(I,2)+BETA(3)*P(I,3)
      DO 150 J=1,3
  150 P(I,J)=P(I,J)+GA*(GA/(1.+GA)*BEP+P(I,4))*BETA(J)
  160 P(I,4)=GA*(P(I,4)+BEP)
      RETURN
      END
```

```
      BLOCK DATA
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /EDPAR/ ITHROW, PZMIN, PMIN, THETA, PHI, BETA(3)
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      COMMON /DATA2/ IDCO(12), CBR(29), KDP(29,3)
      COMMON /DATA3/ CHA1(9), CHA2(19), CHA3(2)
      DATA PUD/0.4/, PS1/0.5/, SIGMA/350./, CX2/0.77/,
     &EBEG/10000./, WFIN/100./, IFLBEG/1/
      DATA ITHROW/1/, PZMIN/0./, PMIN/0./, THETA,PHI,BETA/5*0./
      DATA MESO/7,1,3,2,8,5,4,6,9,7,2,4,1,8,6,3,5,9/
      DATA CMIX/2*0.5,1.,2*0.5,1.,2*0.25,0.5,2*0.,1./
      DATA PMAS/0.,2*139.6,2*493.7,2*497.7,135.,548.8,957.6,
     &2*765.9,2*892.2,2*896.3,770.2,782.6,1019.6/
      DATA IDCO/0,1,6,11,12,13,15,17,19,21,22,25/
      DATA CBR/1.,0.381,0.681,0.918,0.969,1.,0.426,0.662,0.959,
     &0.980,1.,1.,1.,0.667,1.,0.667,1.,0.667,1.,0.667,1.,1.,
     &0.899,0.987,1.,0.486,0.837,0.984,1./
      DATA KDP/1,1,8,2,1,1,2,8,1,1,1,2,3,6,4,7,5,4,6,5,7,2,2,
     &1,2,4,6,2,1,1,1,8,3,2,1,3,8,17,18,1,8,8,2,8,3,8,3,8,2,8,
     &3,3,8,3,5,7,3,9,0,0,8,8,3,8,9,9,14*0,8,4*0,8,0/
      DATA CHA1/'UD','DU','US','SU','DS','SD','UU','DD','SS'/
      DATA CHA2/'GAMM','PI+','PI-','K+','K-','K0','KB0','PI0','ETA',
     &'ETAP','RHO+','RHO-','K*+','K*-','K*0','KB*0','RHO0','OMEG','PHI'/
      DATA CHA3/'    ','STAB'/
      END
```

Experimental situation in 2016



[Buckley et al.] arXiv:1101.2599



Need to cover large dynamic range

- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays

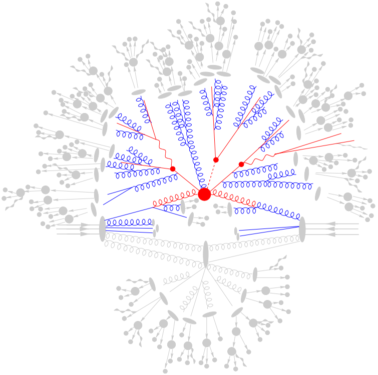
Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1 p_2 \to X} = \sum_{i,j\in\{q,g\}} \int \mathrm{d}x_1 \mathrm{d}x_2 \underbrace{f_{p_1,i}(x_1,\mu_F^2) f_{p_2,j}(x_2,\mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij\to X}(x_1 x_2,\mu_F^2)}_{\text{short distance}}$$

Event generators in 2016

[Buckley et al.] arXiv:1101.2599



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Event generators in 2016

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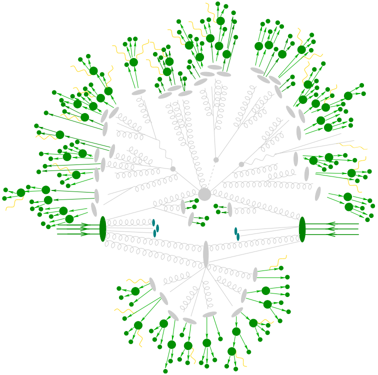

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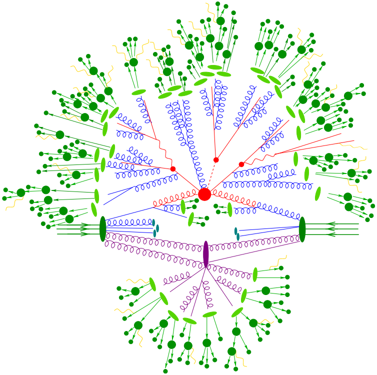

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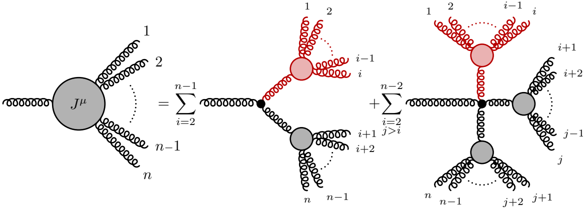
Computing short-distance cross sections – LO

[Berends,Giele] NPB306(1988)759, [Duhr,Maltoni,SH] hep-ph/0607057
[Caravaglios,Mangano,Moretti,Pittau] hep-ph/9807570

- Using Feynman diagrams gets complicated very quickly
  $gg \to gg$ – 4 diagrams, $gg \to 12g$ – 5,348,843,500 diagrams

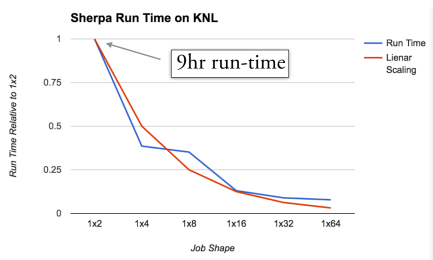


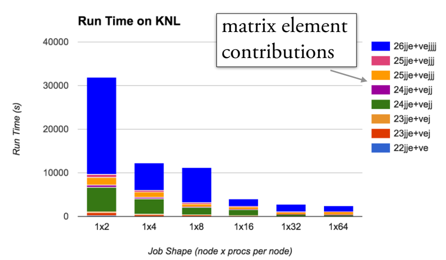
- Eliminate common subexpressions using DAG and parallelize calculation
  $\to$ High-multiplicity tree-level matrix element generators

σ [pb]	Number of jets						
$pp \to t\bar{t}$ + jets	0	1	2	3	4	5	6
Comix	754.8(8)	745(1)	518(1)	309.8(8)	170.4(7)	89.2(4)	44.4(4)
ALPGEN	755.4(8)	748(2)	518(2)	310.9(8)	170.9(5)	87.6(3)	45.1(8)
AMEGIC	754.4(3)	747(1)	520(1)				

[Childers,Uram,LeCompte,Benjamin,SH] CHEP 2016

- Challenge to make this work on tomorrow's (& today's) computers
- 2000's paradigm: Memory is free, Flops are expensive
  Example: 16-core Xeon, 20MB L2 Cache, 64GB RAM
- 2020's paradigm: Flops are free, Memory is expensive & must be managed
  Example: 68-core Xeon KNL, 34MB L2 Cache, 16GB HBM, 96GB RAM





[figures stolen from Taylor Childers' talk at CHEP]

Computing short-distance cross sections – NLO

- Tree amplitudes recycled into NLO real corrections and subtraction terms

$$\sigma_{\rm NLO} = \int d\Phi_{\rm B} \sum \left({\rm B} + \tilde{\rm V} + {\rm I} \right) + \int d\Phi_R \sum \left({\rm R} - {\rm S} \right)$$

- Virtual corrections computed separately ($\nearrow$ next slide)
- Various automated NLO frameworks based on this idea
  - HELAC [Bevilacqua,Czakon,Garzelli,vanHameren,Kardos,Papadopoulos,Pittau,Worek]
  - Herwig7 [Bellm,Fischer,Gieseke,Grellscheid,Harrendorf,Nail,Papaefstathiou, Plätzer,Rauch^2,Reuschle,Richardson,Schichtel,Seymour,Siódmok,Wilcock]
  - MadGraph5 [Alwall,Frederix,Frixione,Hirschi,Maltoni,Mattelaer,Shao,Stelzer,Torrielli,Zaro]
  - Sherpa [Bothmann,Krauss,Kuttimalai,Li,Schönherr,Schulz,Schumann,Siegert,SH]
  - Whizard [Chokoufe,Hoang,Kilian,Ohl,Reuter,Stahlhofen,Teubner,Weiss]

Computing short-distance cross sections – NLO

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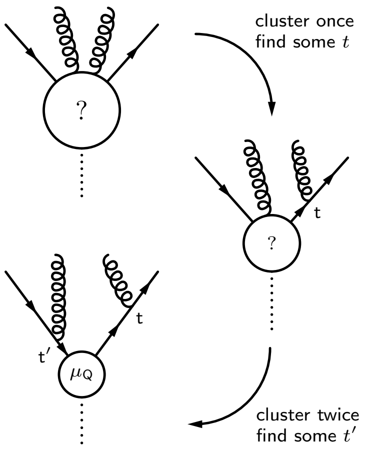
Amazing progress on 1-loop computations during last decade (NLO revolution)

- **Based on generalized unitarity** [Bern,Dixon,Dunbar,Kosower] hep-ph/9409265 [Ossola,Papadopoulos,Pittau] hep-ph/0609007, arXiv:0802.1876 [Forde] arXiv:0704.1835
  - BlackHat [Bern,Dixon,Febres-Cordero,Ita,Kosower,Maître,Ozeren,SH]
  - GoSam [Cullen,Greiner,Heinrich,Luisoni,Mastrolia,Ossola,Reiter,Tramontano]
  - HelacNLO [Bevilacqua,Czakon,Garzelli,vanHameren,Kardos,Papadopoulos,Pittau,Worek]
  - MadLoop [Hirschi,Frederix,Frixione,Garzelli,Maltoni,Pittau]
  - NJet [Badger,Biedermann,Uwer,Yundin]
  - OpenLoops [Cascioli,Maierhöfer,Pozzorini]
  - Rocket [Ellis,Giele,Kunszt,Melnikov,Zanderighi]
- **Based on tensor reduction** [Binoth,Guillet,Pilon,Heinrich,Schubert] hep-ph/0504267 [Denner,Dittmaier] hep-ph/0509141, [Denner,Dittmaier,Hofer] arXiv:1604.06792
  - Golem95 [Binoth,Cullen,Greiner,Guffanti,Guillet,Heinrich,Karg,Kauer,Reiter,Reuter]
  - MadGolem [Binoth,Goncalves Netto,Lopez-Val,Mawatari,Plehn,Wigmore]
  - MadLoop [Hirschi,Frederix,Frixione,Garzelli,Maltoni,Pittau]
  - OpenLoops [Cascioli,Maierhöfer,Pozzorini]

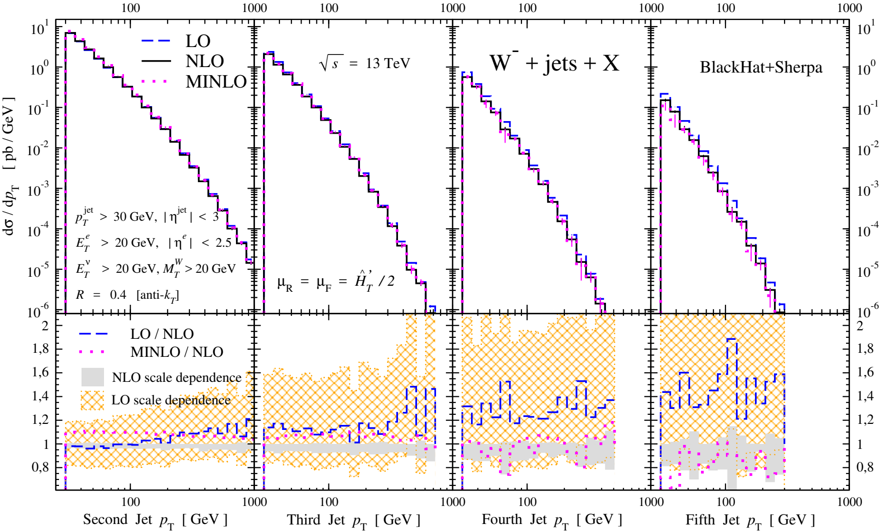
Scale uncertainties at NLO – A controversy?

- Renormalization/factorization scale typically used at very high multiplicity: sum of transverse mass $H_{T,m} = \sum m_\perp$
- Has been criticized for being 'too large' and insensitive to dynamics of process
- Very different scale defined by MINLO
  [Hamilton,Nason,Zanderighi] arXiv:1206.3572
  - Interpret event in terms of QCD branchings, like in a parton-shower
  - Assign transverse momentum scales q to splittings, evaluate one α_s at each of these scales
  - Multiply with NLL Sudakov factors, subtract first-order expansion
- MINLO scale probes detailed dynamics, typically very small $\rightarrow$ good candidate for comparison to $H_{T,m}$



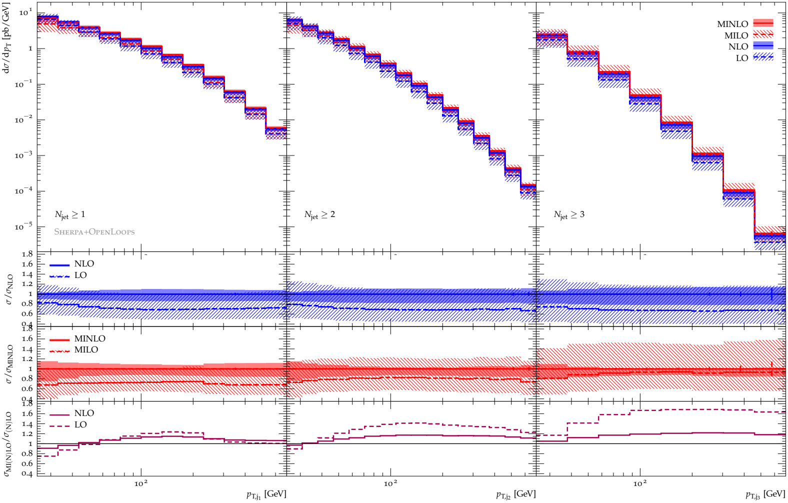
W+5 jets at NLO

[Bern,Dixon,Febres Cordero,Ita,Kosower,Maître,Ozeren,SH] arXiv:1304.1253
[13 TeV predictions courtesy of Fernando Febres-Cordero]



$t\bar{t}$+3jets at NLO QCD

[Maierhöfer,Moretti,Pozzorini,Siegert,SH] arXiv:1607.06934

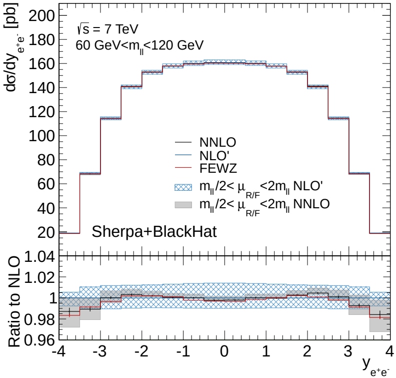


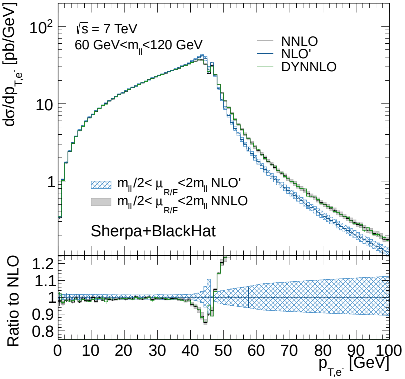
- NLO calculations conveniently recycle tree-level results into Born, real-emission and infrared subtraction terms
  $\rightarrow$ ideally recycle existing NLO results into parts of NNLO
- q_T cutoff method [Catani,Grazzini] hep-ph/0703012, [Gao,Li,Zhu] arXiv:1210.2808
  partitions phase-space into $q_T \approx 0$ bin and finite q_T region
- Prediction for zero-q_T bin from resummation
  [Becher,Neubert] arXiv:1007.4005, [Gehrmann,Lübbert,Yang] arXiv:1209.0682, arXiv:1403.6451
- Finite q_T region can be taken from from automated NLO frameworks

Drell-Yan at NNLO QCD

SLAC

[Li,Prestel,SH] arXiv:1405.3607



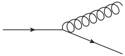


$E_{\rm cms}$	7 TeV	14 TeV	33 TeV	100 TeV
VRAP	$973.99(9)^{+4.70}_{-1.84}$ pb	$2079.0(3)\ ^{+14.7}_{-6.9}$ pb	$4909.7(8)\ ^{+45.1}_{-27.2}$ pb	$13346(3)\ ^{+129}_{-111}$ pb
SHERPA	$973.7(3)\ ^{+4.78}_{-2.21}$ pb	$2078.2(10)^{+15.0}_{-8.0}$ pb	$4905.9(28)^{+45.1}_{-27.9}$ pb	$13340(14)^{+152}_{-110}$ pb

Making life simpler – Parton showers

[Marchesini,Webber] NPB238(1984)1, [Sjöstrand] PLB157(1985)321

- Parton "decay" can occur in two ways:



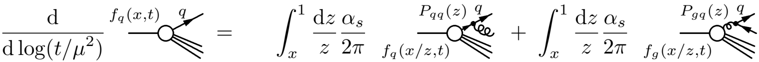
\- observed

\- unobserved

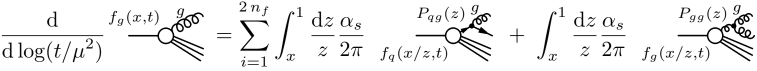
- Impose probability conservation $\Rightarrow$ observed $+$ unobserved $= 1$
  Splitting governed by Poisson statistics $\rightarrow$ survival probability

$$\Delta(t,t') := \mathcal{P}_{\text{nosplit}}(t,t') = \exp\left\{-\int_t^{t'} \frac{\mathrm{d}\bar{t}}{\bar{t}}\, \mathcal{P}_{\text{split}}(\bar{t}\,)\right\}$$

- Key to Monte-Carlo simulation of arbitrarily many emissions



$$\frac{\mathrm{d}}{\mathrm{d}\log(t/\mu^2)}\, f_q(x,t) = \int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}\, P_{qq}(z)\, f_q(x/z,t) + \int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}\, P_{gq}(z)\, f_g(x/z,t)$$

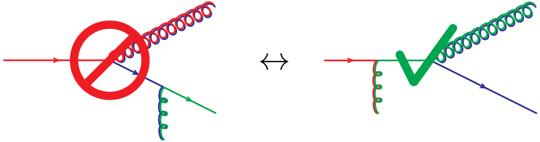


$$\frac{\mathrm{d}}{\mathrm{d}\log(t/\mu^2)}\, f_g(x,t) = \sum_{i=1}^{2n_f}\int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}\, P_{qg}(z)\, f_q(x/z,t) + \int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s}{2\pi}\, P_{gg}(z)\, f_g(x/z,t)$$

Color coherence and the dipole picture

[Marchesini,Webber] NPB310(1988)461

- Individual color charges inside a color dipole cannot be resolved by gluons of wavelength larger than the dipole size
  $\rightarrow$ emission off combined mother parton instead

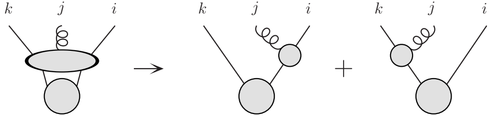


- Net effect is destructive interference outside cone with opening angle defined by emitting color dipole
  $\rightarrow$ Soft anomalous dimension halved due to reduced phase space
- Formerly implemented by angular ordering / angular veto
- Alternative description in terms of color dipoles
  [Gustafsson,Pettersson] NPB306(1988)746, [Kharraziha,Lönnblad] hep-ph/9709424
  [Winter,Krauss] arXiv:0712.3913

The midpoint between dipole and parton showers

- Angular ordered / vetoed parton shower does not fill full phase space
  Dipole shower lacks parton interpretation $\rightarrow$ prefer alternative to both
- Can preserve parton picture by partial fractioning soft eikonal
  $\leftrightarrow$ soft enhanced part of splitting function [Catani,Seymour] hep-ph/9605323

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} \rightarrow \frac{1}{p_i p_j}\frac{p_i p_k}{(p_i + p_k)p_j} + \frac{1}{p_k p_j}\frac{p_i p_k}{(p_i + p_k)p_j}$$



- "Spectator"-dependent kernels, singular in soft-collinear region only
  $\rightarrow$ capture dominant coherence effects (3-parton correlations)

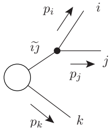
$$\frac{1}{1-z} \rightarrow \frac{1-z}{(1-z)^2+\kappa^2} \qquad \kappa^2 = \frac{k_\perp^2}{Q^2}$$

- For correct soft evolution, ordering variable must be identical at both "dipole ends" ($\rightarrow$ recover soft eikonal at integrand level)

The midpoint between dipole and parton showers

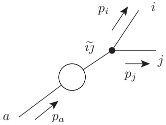
Choose parametrization such that soft term is $\frac{1-z}{(1-z)^2+\kappa^2}$ in all dipole types

(1) FF



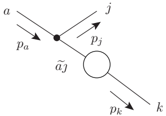
$$\kappa^2 = \frac{p_i p_j \, p_j p_k}{(p_{\widetilde{ij}} p_{\tilde{k}})^2} \qquad z_j = \frac{p_j p_k}{p_{\widetilde{ij}} p_{\tilde{k}}}$$

(2) FI



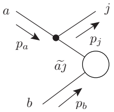
$$\kappa^2 = \frac{p_i p_j \, p_j p_a}{(p_{ij} p_a)^2} \qquad z_j = \frac{p_j p_a}{p_{ij} p_a}$$

(3) IF



$$\kappa^2 = \frac{p_a p_j \, p_j p_k}{(p_{jk} p_a)^2} \qquad z_j = \frac{p_j p_k}{p_{jk} p_a}$$

(4) II



$$\kappa^2 = \frac{p_a p_j \, p_j p_b}{(p_a p_b)^2} \qquad z_j = \frac{p_j p_b}{p_a p_b}$$

Preserve collinear anomalous dimensions & sum rules → splitting functions fixed

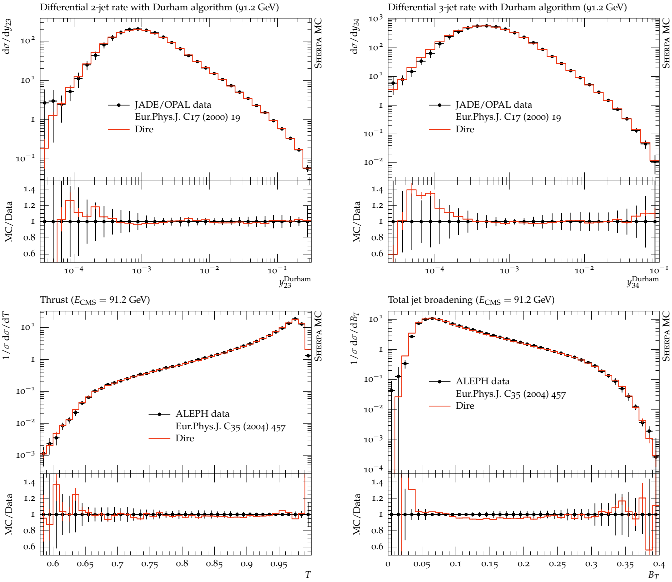
$$P_{qq}(z,\kappa^2) = 2\,C_F\left[\left(\frac{1-z}{(1-z)^2+\kappa^2}\right)_+ - \frac{1+z}{2}\right] + \gamma_q\,\delta(1-z)$$

$$P_{gg}(z,\kappa^2) = 2\,C_A\left[\left(\frac{1-z}{(1-z)^2+\kappa^2}\right)_+ + \frac{z}{z^2+\kappa^2} - 2 + z(1-z)\right] + \gamma_g\,\delta(1-z)$$

$$P_{qg}(z,\kappa^2) = 2\,C_F\left[\frac{z}{z^2+\kappa^2} - \frac{2-z}{2}\right] \qquad P_{gq}(z,\kappa^2) = T_R\left[z^2+(1-z)^2\right]$$

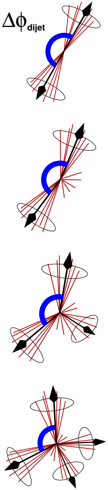
The midpoint between dipole and parton showers

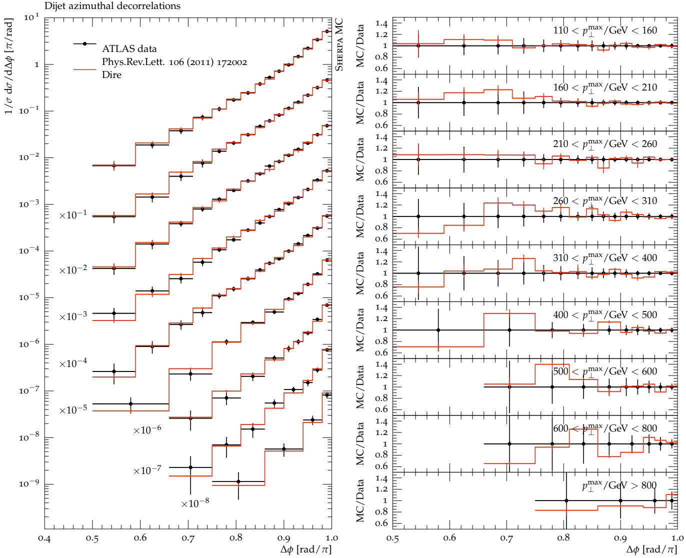
[Prestel,SH] arXiv:1506.05057



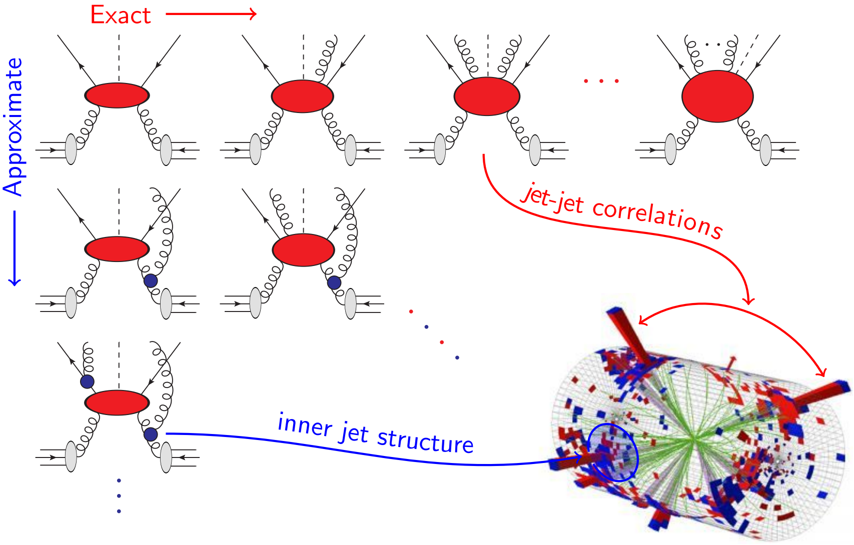
Predictions for the LHC

[Prestel,SH] arXiv:1506.05057

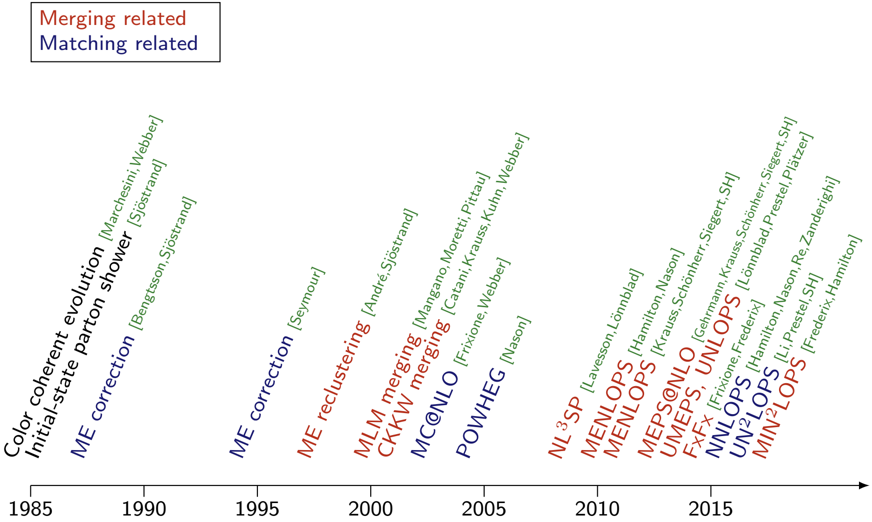




Parton-shower matching & merging



The long road to precision simulations



NLO matching

Two possible ways to match NLO calculations and parton showers

Additive (MC@NLO-like)

[Frixione,Webber] hep-ph/0204244

- Use parton-shower splitting kernel as NLO subtraction term
- Multiply LO event weight by Born-local K-factor including integrated subtraction term and virtual corrections
- Add hard remainder function consisting of subtracted real-emission correction

Multiplicative (POWHEG-like)

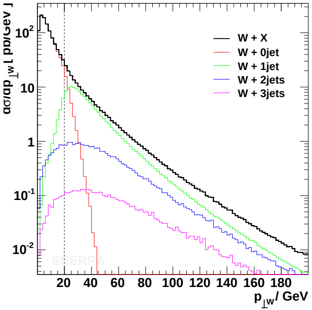
[Nason] hep-ph/0409146

- Use matrix-element corrections to replace parton-shower splitting kernel by full real-emission matrix element in first shower branching
- Multiply LO event weight by Born-local NLO K-factor (integrated over real corrections that can be mapped to Born according to parton-shower kinematics)

Both cases: Beware of sub-leading color terms and spin correlations!

ME+PS merging

- Separate phase space into "hard" and "soft" region
- Matrix elements populate hard domain
  Made exclusive by truncated vetoed parton shower Sudakov factors
- Parton shower populates soft domain
  Vetoed in hard domain to compute Sudakov factors (pseudo-shower)
- Need criterion to define "hard" & "soft"
  $\rightarrow$ jet measure Q and corresponding cut, $Q_{\rm cut}$



ME+PS merging

- LO schemes

Method	Shower Generator	Unitary	References
MLM	Herwig/Pythia	No	[Mangano,Moretti,Pittau] hep-ph/0108069 [Alwall et al.] arXiv:0706.2569
CKKW	Apacic	No	[Catani,Krauss,Kuhn,Webber] hep-ph/0109231
CKKW-L	Ariadne/Pythia	No	[Lönnblad] hep-ph/0112284 [Lönnblad,Prestel] arXiv:1109.4829
METS	Sherpa CSS	No	[Krauss,Schumann,Siegert,SH] arXiv:0903.1219
CKKW'	Herwig++	No	[Hamilton,Richardson,Tully] arXiv:0905.3072
UMEPS	Pythia/Herwig++	Yes	[Lönnblad,Prestel] arXiv:1211.4827 [Plätzer] arXiv:1211.5467

- NLO schemes

Method	Shower Generator	Unitary	References
NL^3	Ariadne/Pythia	No	[Lavesson,Lönnblad] arXiv:0811.2912
MEPS@NLO	Sherpa CSS	No	[Krauss,Schönherr,Siegert,SH] arXiv:1207.5030 [Gehrmann,Krauss,Schönherr,Siegert,SH] 5031
FxFx	Herwig(++)/Pythia	No	[Frederix,Frixione] arXiv:1209.6215
UNLOPS	Pythia/Herwig++	Yes	[Lönnblad,Prestel] arXiv:1211.7278

NNLOPS

[Hamilton,Nason,Zanderighi] arXiv:1212.4504
[Hamilton,Nason,Re,Zanderighi] arXiv:1309.0017

- Based on MINLO procedure
  [Hamilton,Nason,Zanderighi] arXiv:1206.3572
- Extended to NNLL resummation and reweighted to NNLO differentially in Born phase space

UN^{2}LOPS

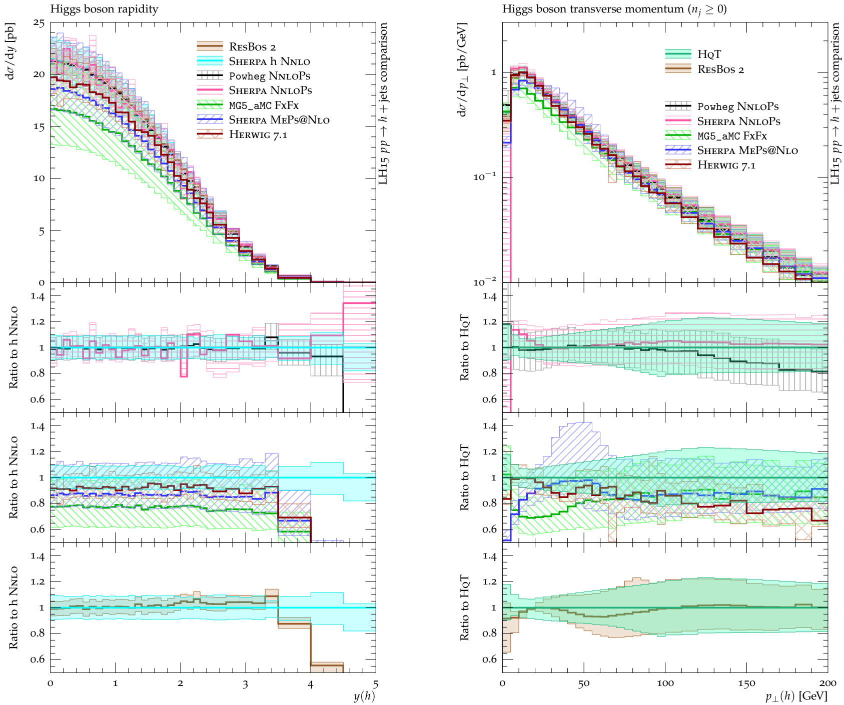
[Li,Prestel,SH] arXiv:1405.3607

- Based on UNLOPS merging
  [Lönnblad,Prestel] arXiv:1211.7278
- q_T-cutoff technique for NNLO, combined with subtracted MC@NLO for 1-jet contribution

Comparison of approaches to Higgs+jets simulation

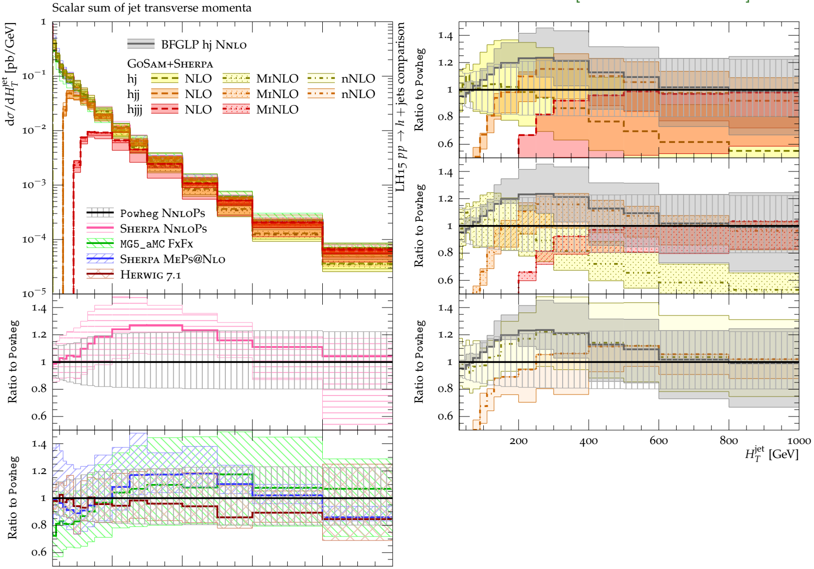
[Les Houches SM WG] arXiv:1605.04692

- Setup
  - Stable Higgs
  - anti-k_T jets, R =0.4, $p_{T,j}$ >30 GeV $|\eta_j|$ <4.4
- Calculations & tools in the comparison
  - Fixed-order NLO for $h+\leq 3$ jets, $H_T'/2$ & MINLO
  - LoopSim
  - NNLO for $pp \to h$ (Sherpa), $pp \to h+j$ (BFGLP)
  - Resummed h-p_T (HqT & ResBos)
  - Resummed jet veto cross section (STWZ)
  - NNLO+PS (POWHEG & Sherpa)
  - Multi-jet merging up to 2 jets at NLO (Madgraph5_aMC@NLO, Herwig 7.1)
  - Multi-jet merging up to 3 jets at NLO (Sherpa)
  - High-energy resummation (HEJ)


Higgs boson rapidity
dσ/dy [pb]
ResBos 2
Sherpa h Nnlo
Powheg NnloPs
Sherpa NnloPs
MG5_aMC FxFx
Sherpa MePs@Nlo
Herwig 7.1
LH15 pp → h + jets comparison
Ratio to h Nnlo
y(h)
Higgs boson transverse momentum ($n_j \geq 0$)
dσ/dp⊥ [pb/GeV]
HqT
Ratio to HqT
p⊥(h) [GeV]

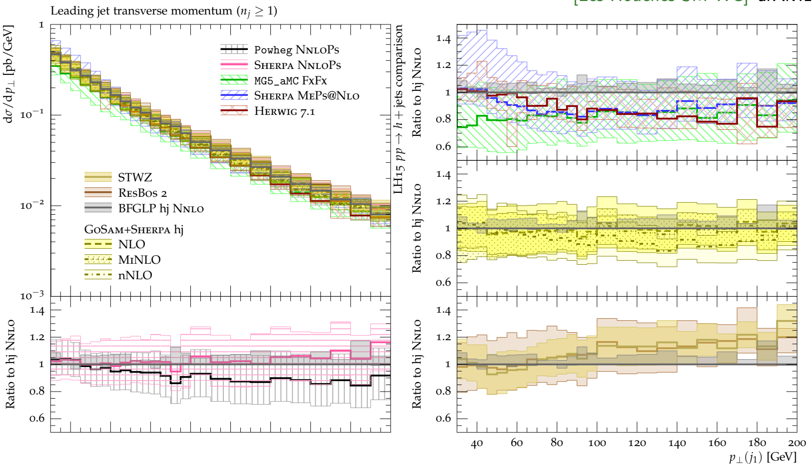
Comparison of approaches

[Les Houches SM WG] arXiv:1605.04692



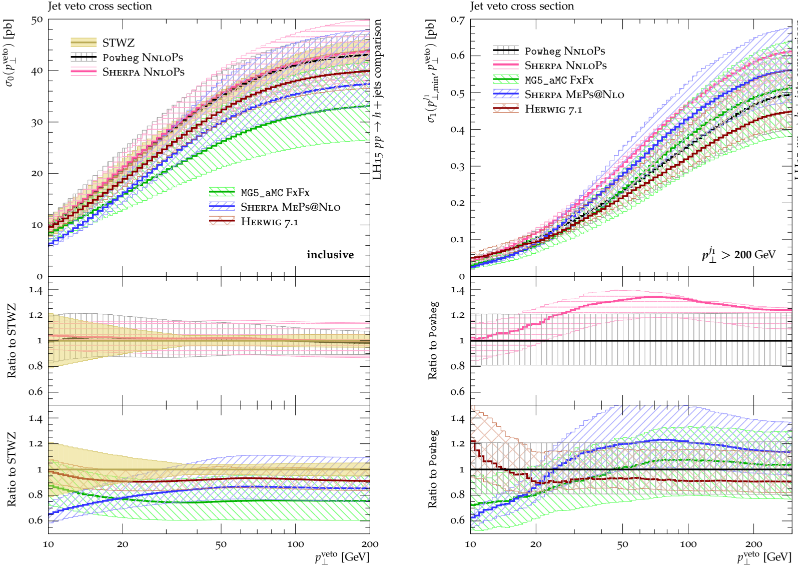
Comparison of approaches

[Les Houches SM WG] arXiv:1605.04692



Comparison of approaches

[Les Houches SM WG] arXiv:1605.04692



So far so good

On the upside

- NLO (and often NNLO) fixed-order calculations now standard
- Most simulators include NLO calculations fully automatically
- First NNLO calculations incorporated into simulations

On the downside

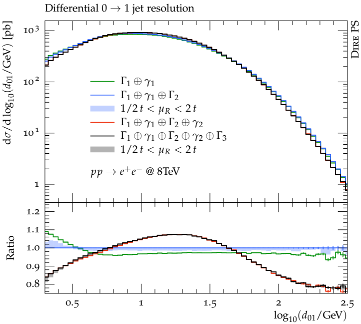
- Parton showers lack precision compared to analytical resummation
- Unfortunately, they are most important for getting jet shapes right

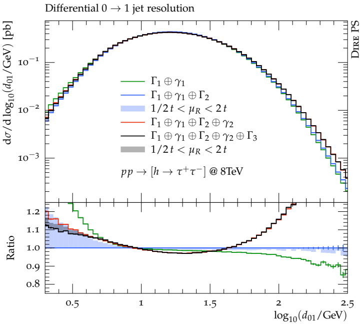
Precision calculations at fixed-order largely automated
Need to catch up on automating precision resummation

Towards higher logarithmic accuracy

[Catani,Krauss,Prestel,SH] any time soon

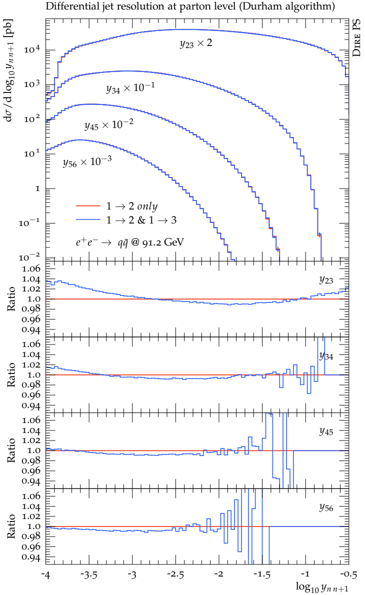
- Big drawback of parton showers is lack of higher-order kernels
- Start improving with spacelike NLO kernels
  [Curci,Furmanski,Petronzio] NPB175(1980)27, PLB97(1980)437
- 2-loop cusp term subtracted & combined with LO soft contribution (similar to CMW rescaling [Catani,Marchesini,Webber] NPB349(1991)635)
- Implemented using weighting algorithms [Schumann,Siegert,SH] arXiv:0912.3501

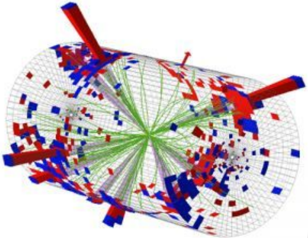


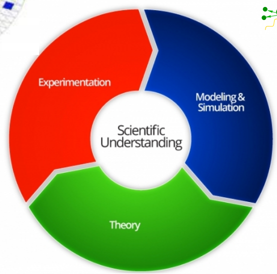


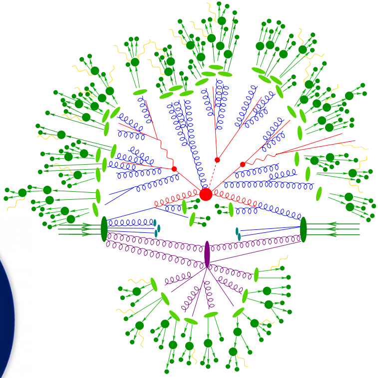
Towards higher logarithmic accuracy

- New topology at NLO from $q \to \bar{q}$ and $q \to q'$ splittings
- Generic $1 \to 3$ process in parton shower $2 \to 4$ process in dipole(-like) shower
- First branching treated as soft gluon radiation, second as collinear splitting (to match diagrammatic structure)
- FF, FI & II splittings complete and cross-checked (Pythia vs. Sherpa)
- IF dipoles to be validated (tricky kinematics!)









$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi}\not{D}\Psi + h.c.$$

- Simulations indispensable to extract theory parameters from LHC data
- New algorithms, improved theory & bigger computers pushing limits
- Hadron collider physics complicated, but reaching precision era!

Thank you for your attention!

Soft evolution with more color

- In soft limit real-emission amplitudes factorize as

$$|\mathcal{M}_0(1,\ldots,j,\ldots,n)|^2 \stackrel{j\to\text{soft}}{\longrightarrow} -\sum_{i,k\neq i}\frac{8\pi\mu^{2\varepsilon}\alpha_s}{p_ip_j}$$
$$\times\,\langle m_0(1,\ldots,i,\ldots,k,\ldots,n)|\,\frac{\mathbf{T}_i\cdot\mathbf{T}_k\;p_ip_k}{p_ip_j+p_kp_j}\,|m_0(1,\ldots,i,\ldots,k,\ldots,n)\rangle\;.$$

  $\mathbf{T}_i$ - color insertion operator for parton i
  $|m_0(1,\ldots,i,\ldots,k,\ldots,n)\rangle$ - Born amplitude

- Parton showers replace $\sum_{k\neq i}\mathbf{T}_i\cdot\mathbf{T}_k\to-\mathbf{T}_i^2$
- NLO Matched shower uses $\mathbf{T}_i\cdot\mathbf{T}_k$ in first emission
- Full matrix exponentiation in MC is work in progress
  Comparison to analytic resummation is a starting point

Soft evolution with more color

[Banfi,Salam,Zanderighi] hep-ph/0407286

- Generic NLL resummation framework exists (CAESAR)
- Observable dependence parametrized as

$$V(\{\tilde{p}\};k) = d_l \left(\frac{k_t^{(l)}}{Q}\right)^a e^{-b_l \eta^{(l)}} g_l\left(\phi^{(l)}\right)$$

- Resummed integrated spectrum for $V(\{\tilde{p}\};k) < v$ given by

$$\frac{1}{\sigma}\int_0^v \frac{d^2\sigma}{d\mathcal{B}dv'}dv' = \sum_{\delta\in\text{partonics}} \frac{d\sigma_0^{(\delta)}}{d\mathcal{B}} e^{Lg_1^{(\delta)}(\alpha_s L)+g_2^{(\delta,\mathcal{B})}(\alpha_s L)}\left[1+\mathcal{O}(\alpha_s)\right], \quad L = \log\frac{1}{v}$$

- LL / NLL coefficients g_1 and g_2 arise from 1- and 2-emission integrals
- g_2 depends on soft function $\mathcal{S}$ through

$$\log\mathcal{S}(T(L/a)), \qquad \text{where} \quad T(L) = \frac{1}{\pi\beta_0}\log\frac{1}{1-2\alpha_s\beta_0 L}$$

Soft evolution with more color

[Gerwick,Marzani,Schumann,SH] arXiv:1411.7325

- Soft function known analytically for low-multiplicity final states
- Generic structure in terms of anomalous dimension Γ is

$$\mathcal{S}(\xi) = \frac{\langle m_0|e^{-\frac{\xi}{2}\mathbf{\Gamma}^\dagger}e^{-\frac{\xi}{2}\mathbf{\Gamma}}|m_0\rangle}{\langle m_0|m_0\rangle}\;, \quad \mathbf{\Gamma} = -2\sum_{i<j}\mathbf{T}_i\cdot\mathbf{T}_j\,\log\frac{Q_{ij}}{Q_{12}} + i\pi\sum_{i,j=II,FF}\mathbf{T}_i\cdot\mathbf{T}_j$$

- Insertion of color projectors $|c_\alpha\rangle\langle c^\alpha|$ leads to matrix structure

$$\mathcal{S}(\xi) = \frac{c_{\alpha\beta}H^{\gamma\sigma}\mathcal{G}^\dagger_{\gamma\rho}c^{\rho\beta}c^{\alpha\delta}\mathcal{G}_{\delta\sigma}}{c_{\alpha\beta}H^{\alpha\beta}}\;, \qquad \mathcal{G}_{\alpha\beta}(\xi) = c_{\alpha\gamma}\exp\left(-\frac{\xi}{2}\,c^{\gamma\delta}\,\Gamma_{\delta\beta}\right)$$

  where $H^{\alpha\beta} = \langle m_0|c^\alpha\rangle\langle c^\beta|m_0\rangle$ and $\Gamma_{\alpha\beta} = \langle c_\alpha|\mathbf{\Gamma}|c_\beta\rangle$
- $c_{\alpha\beta} = \langle c_\alpha|c_\beta\rangle$ - color "metric", $H^{\alpha\beta}$ - hard matrix
- Much effort in the literature is spent on choosing orthogonal bases
  [Sjödahl] arXiv:0906.1121, [Keppeler,Sjödahl] arXiv:1207.0609

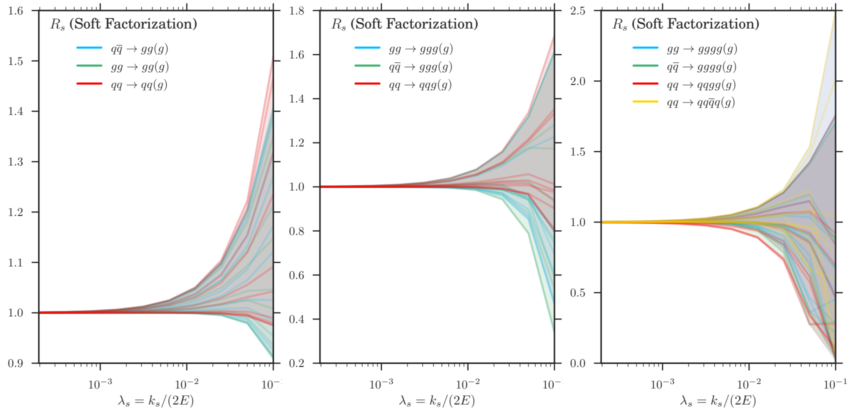
High-multiplicity NLL resummation & matching

[Gerwick,Marzani,Schumann,SH] arXiv:1411.7325

- Missing ingredients for resummation at higher multiplicity
  - Hard matrix → Matrix Element generator Comix
  - Soft anomalous dimension → Mathematica scripts
- Remaining problems
  - Non-orthogonality of color bases
    Solved by incorporation of inverse metric $c^{\alpha\beta} = (c_{\alpha\beta})^{-1}$
  - $N_c = 3$ pathologies in overcomplete color bases
    Solved by numeric matrix inversion at $N_c = 3 + \varepsilon$

High-multiplicity NLL resummation & matching

[Gerwick,Marzani,Schumann,SH] arXiv:1411.7325



- Ratio of sum-over-dipole dressed Born to exact matrix elements
- Checks correctness of soft anomalous dimension and color metric

High-multiplicity NLL resummation & matching

[Gerwick,Marzani,Schumann,SH] arXiv:1411.7325

- Expansion of resummation formula to NLO leads to LL and NLL coefficients

$$G_{12} = -\sum_{l=1}^{n} \frac{C_l}{a(a+b_l)}\,, \quad G_{11} = -\left[\sum_{l=1}^{n} C_l \left(\frac{B_l}{a+b_l} + \dots\right) + \frac{1}{a}\frac{\mathrm{Re}[\Gamma_{\alpha\beta}]\,H^{\alpha\beta}}{c_{\alpha\beta}H^{\alpha\beta}} + \dots\right]$$

  Combination with LO prediction $\to$ matching term

- Should be cast into fully differential procedure and automated
  Use Catani-Seymour dipole formula to generate coefficients

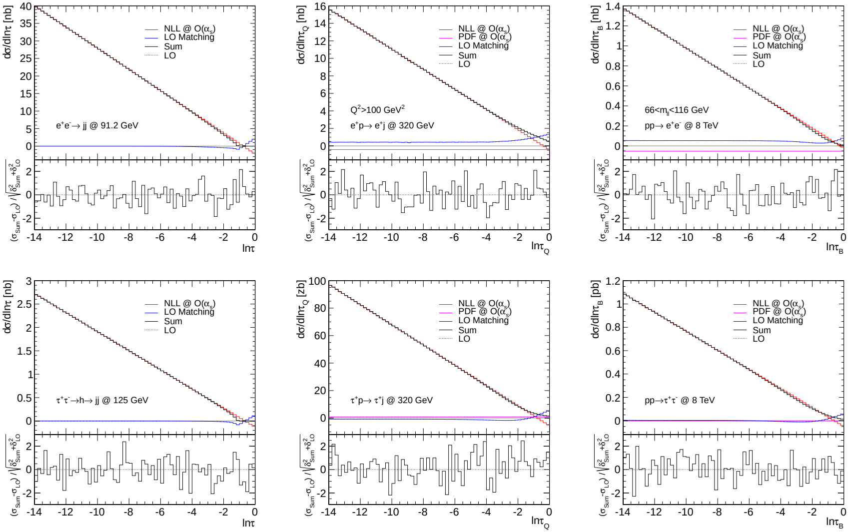
$$\mathcal{D}_{ij,k}(1,\dots,n) \;=\; -\frac{1}{2p_ip_j}\,\langle m_0|\frac{\mathbf{T}_i\cdot\mathbf{T}_k}{\mathbf{T}_{ij}^2}\,\hat{V}_{ij,k}(z,k_T,\varepsilon)\,|m_0\rangle$$

- Obtain $\mathrm{Re}[\Gamma_{\alpha\beta}]\,H^{\alpha\beta}/c_{\alpha\beta}H^{\alpha\beta}$ from replacement
  $\hat{V}_{ij,k}(z,k_T,\varepsilon) \to \log Q_{(ij)k}/Q_{12}$, and rescaling by $1/a$
- Obtain G_{12} and B_l-dependent term in G_{11} from replacement
  $\hat{V}_{ij,k} \to P_{ij,i}$, restricting LL terms to $z^a > v$, and rescaling by $1/(a+b_l)$
- Rescale integration region $\leftrightarrow$ momentum non-conservation in resummation

High-multiplicity NLL resummation & matching

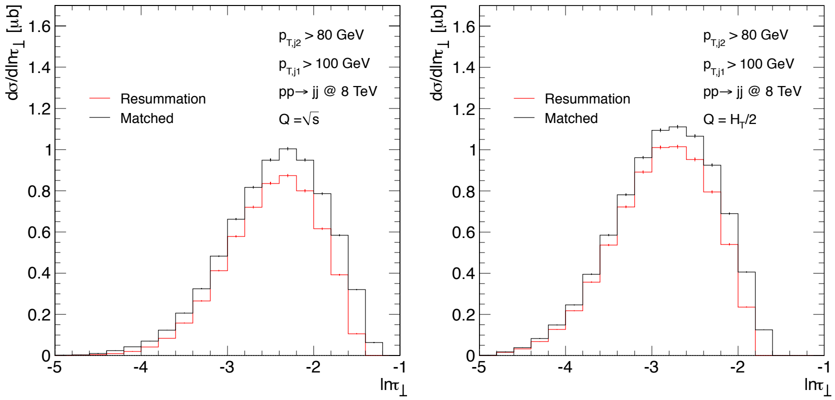
SLAC

[Gerwick,Marzani,Schumann,SH] arXiv:1411.7325



High-multiplicity NLL resummation & matching

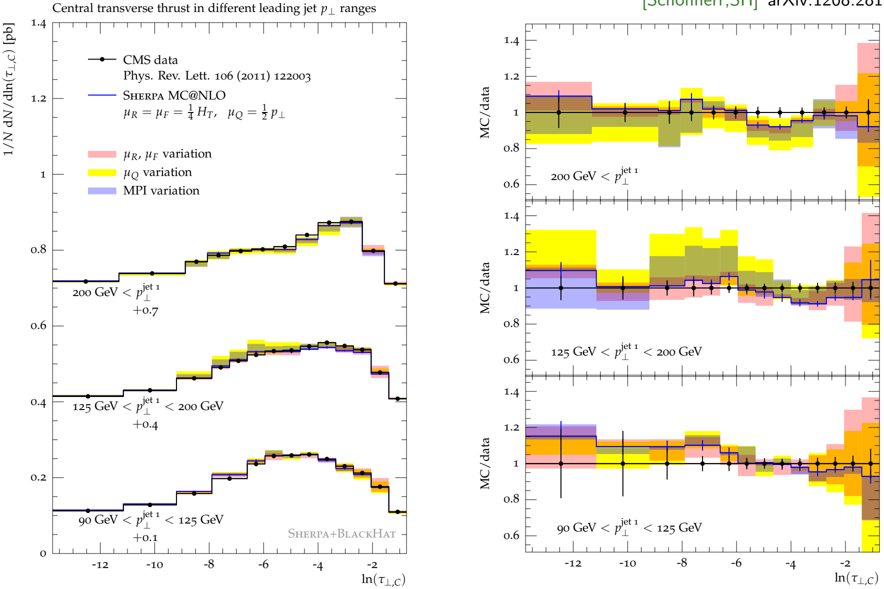
[Gerwick,Marzani,Schumann,SH] arXiv:1411.7325



- Full result (NLL resummed and matched) for transverse thrust in $pp \to jj$

Back to the parton shower

[Schönherr,SH] arXiv:1208.2815



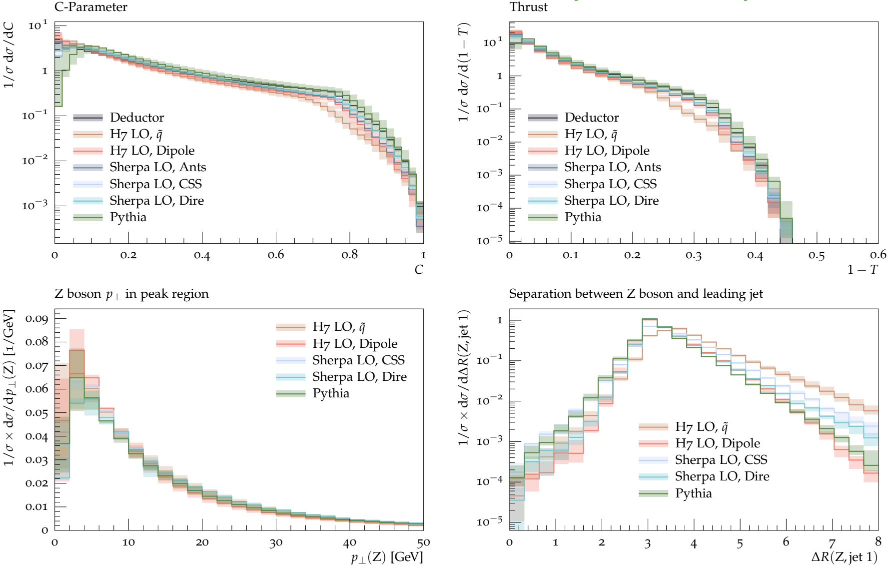
Parton-shower uncertainty estimates

[Les Houches SM WG] arXiv:1605.04692

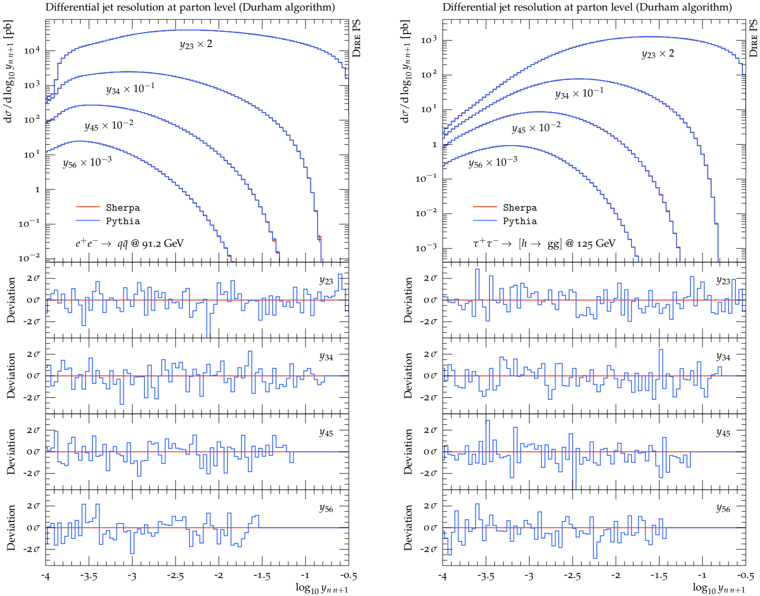
- Renormalization scale choice in parton showers
  - k_T [Amati,Bassetto,Ciafaloni,Marchesini,Veneziano] NPB173(1980)429
  - CMW rescaling [Catani,Marchesini,Webber] NPB349(1991)635
  - plus additional factor to be tuned to data (≈ 1)
- Scale variations typically not considered
  First attempt during LesHouches '15
- Participating projects
  - Deductor [Nagy,Soper] arXiv:1401.6364
  - Herwig [Bellm,Plätzer,Richardson,Siódmok,Webster] arXiv:1605.08256
    - $\tilde{q}$-shower [Gieseke,Stephens,Webber] hep-ph/0310083
    - Dipole shower [Plätzer,Gieseke] arXiv:0909.5593
  - Sherpa [Bothmann,Schönherr,Schumann] arXiv:1606.08753
    - Ants [Krauss,Zapp] in preparation
    - CSS [Schumann,Krauss] arXiv:0709.1027
    - Dire [Prestel,SH] arXiv:1506.05057
  - Pythia [Mrenna,Skands] arXiv:1605.08352

Parton-shower uncertainty estimates

[Les Houches SM WG] arXiv:1605.04692

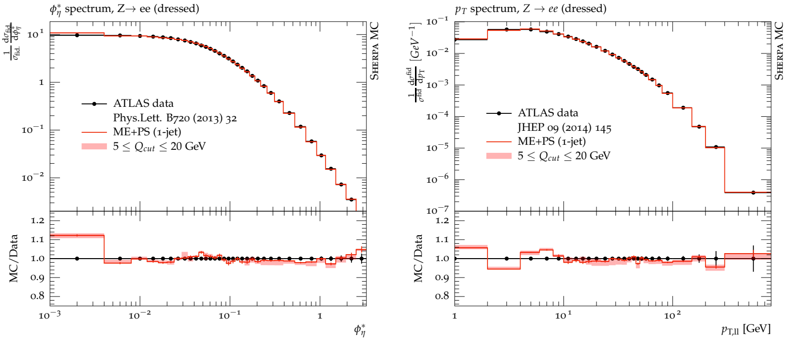


The midpoint between dipole and parton showers



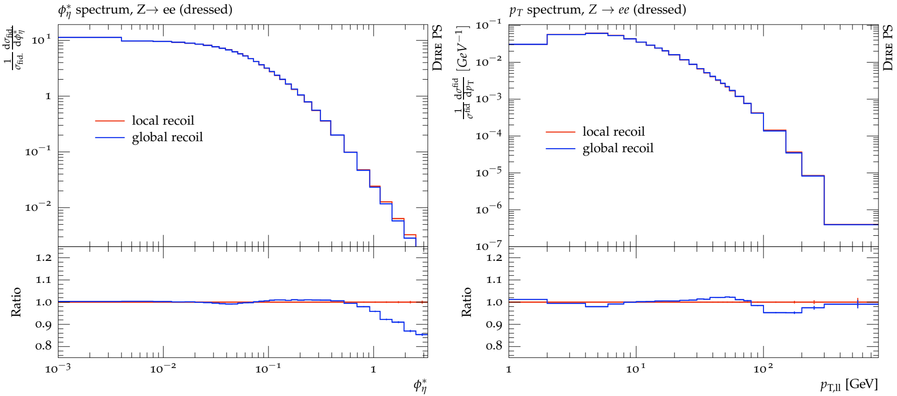
The midpoint between dipole and parton showers

[Prestel,SH] arXiv:1506.05057



- Parton shower merged with 1-jet tree-level ME using CKKW-L

Kinematics mapping in IF dipoles



- Two mapping schemes for IF dipoles → local [Catani,Seymour] hep-ph/9605323 and global [Plätzer,Gieseke] arXiv:0909.5593, [Schumann,Siegert,SH] arXiv:0912.3501
- Negligible impact on q_T-spectrum in low-q_T range (spectrum dominated by singlet evolution at LHC energies)

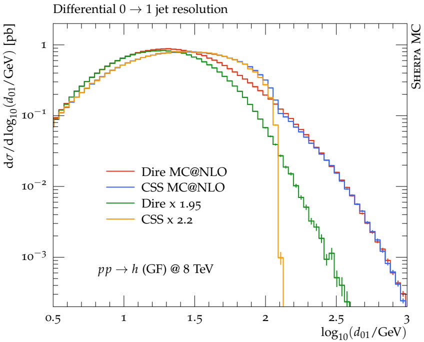
DiRe as a subtraction method and MC@NLO

[SH] in preparation

- Can view new shower as modified Catani-Seymour (CS) subtraction
- All counterterms computed and implemented in Sherpa generator

- Sherpa MC@NLO based on exponentiation of CS dipole subtraction terms

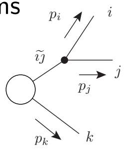
  [Krauss,Siegert,Schönherr,SH]
  arXiv:1111.1220, arXiv:1208.2815
- Modified CS subtraction automatically available for MC@NLO matching
- Important differences due to evolution variables and kernels



Matrix-element corrections

SLAC

[Sjöstrand] PLB185(1987)435



- Parton shower evolution kernels similar to NLO subtraction terms
  Equivalent at leading color and w/o spin correlations:

$$\mathrm{D}_{ij,k}(\Phi_\mathrm{R}) = \frac{8\pi\alpha_s}{2p_ip_j}\mathrm{B}(b_{ij,k}(\Phi_\mathrm{R}))\mathrm{P}_{ij,k}(t_{ij,k}, z_{ij,k}, \phi_{ij,k})$$

  $b_{ij,k}$ maps real kinematics to Born, Catani-Seymour style

- Can project real-emission term onto singular regions in PS
  $\rightarrow$ no unmatched singularities (full color & spin $\nearrow$ next slide)

$$\mathrm{R}_{ij,k}(\Phi_\mathrm{R}) = \rho_{ij,k}(\Phi_R)\mathrm{R}(\Phi_\mathrm{R})\;, \qquad \rho_{ij,k} = \frac{\mathrm{D}_{ij,k}(\Phi_\mathrm{R})}{\sum_{mn,l}\mathrm{D}_{mn,l}(\Phi_\mathrm{R})}$$

- Now replace PS kernels by full real-emission corrections using weight

$$w(\Phi_R) = \left[\sum_{mn,l}\frac{8\pi\alpha_s}{2p_np_m}\frac{\mathrm{B}(b_{mn,l}(\Phi_\mathrm{R}))\mathrm{P}_{mn,l}(t', z', \phi')}{\mathrm{R}(\phi_R)}\right]^{-1}$$

  $\rightarrow$ generic form of a matrix-element correction
  Note splitter-spectator independence, i.e. $w_{ij,k} = w$ for all ij, k

Interlude: How to turn P<0 into a probability

- Problem in sub-leading color terms and spin correlation terms lies in negative weights $\rightarrow$ no-emission probability *locally* exceeds unity
- Recall standard parton shower: $\mathcal{P}_{\rm no}(t,t') = \exp\{F(t) - F(t')\}$
  Exact MC solution $t = F^{-1}\big[\,F(t') + \log R\,\big]$, R – random number

  But don't want to compute $F(t) = -\int_t {\rm d}\bar{t} f(\bar{t})$, as $f(t) = \sum_b \int {\rm d}z \frac{\alpha_s}{2\pi\, t} {\rm P}_{ab}(t,z)$

Solution: Veto algorithm (hit-or-miss for Poisson distributions)

- Find simple function $g(t) > f(t)$ with simple integral $G(t)$
- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Interlude: How to turn P<0 into a probability

SLAC

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Probability for one acceptance

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} \mathrm{d}\bar{t}\, g(\bar{t})\right\}$$

Interlude: How to turn P<0 into a probability

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- Find simple function $g(t) > f(t)$ with simple integral $G(t)$
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Probability for one acceptance with one rejection

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} {\rm d}\bar{t}\, g(\bar{t})\right\} \left[\int_t^{t'} {\rm d}t_1 \left(1 - \frac{f(t_1)}{g(t_1)}\right) g(t_1)\, \exp\left\{-\int_{t_1}^{t'} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$

Interlude: How to turn P<0 into a probability

- Problem in sub-leading color terms and spin correlation terms lies in negative weights $\rightarrow$ no-emission probability *locally* exceeds unity
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  But don't want to compute $F(t) = -\int_t {\rm d}\bar{t} f(\bar{t})$, as $f(t) = \sum_b \int {\rm d}z \frac{\alpha_s}{2\pi\, t} {\rm P}_{ab}(t,z)$

Solution: Veto algorithm (hit-or-miss for Poisson distributions)

- Find simple function $g(t) > f(t)$ with simple integral $G(t)$
- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Probability for one acceptance with two rejections

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} {\rm d}\bar{t}\, g(\bar{t})\right\} \left[\int_t^{t'} {\rm d}t_1 \left(1 - \frac{f(t_1)}{g(t_1)}\right) g(t_1)\, \exp\left\{-\int_{t_1}^{t_2} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$
$$\times \left[\int_{t_1}^{t'} {\rm d}t_2 \left(1 - \frac{f(t_2)}{g(t_2)}\right) g(t_2)\, \exp\left\{-\int_{t_2}^{t'} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$

Interlude: How to turn P<0 into a probability

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  Exact MC solution $t = F^{-1}\big[\,F(t') + \log R\,\big]$, R – random number

  But don't want to compute $F(t) = -\int_t {\rm d}\bar{t} f(\bar{t})$, as $f(t) = \sum_b \int {\rm d}z \frac{\alpha_s}{2\pi\, t} {\rm P}_{ab}(t,z)$

Solution: Veto algorithm (hit-or-miss for Poisson distributions)

- Find simple function $g(t) > f(t)$ with simple integral $G(t)$
- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Probability for one acceptance with n rejections

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} {\rm d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} {\rm d}t_i \left(1 - \frac{f(t_i)}{g(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$

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Disentangle nested integrals:

$$f(t)\, \exp\left\{-\int_t^{t'} {\rm d}\bar{t}\, g(\bar{t})\right\} \frac{1}{n!} \left[\int_t^{t'} {\rm d}\bar{t}\, \big(g(\bar{t}) - f(\bar{t})\big)\right]^n$$

Interlude: How to turn P<0 into a probability

SLAC

- Problem in sub-leading color terms and spin correlation terms lies in negative weights $\to$ no-emission probability *locally* exceeds unity
- Recall standard parton shower: $\mathcal{P}_{\rm no}(t,t') = \exp\{F(t)-F(t')\}$
  Exact MC solution $t = F^{-1}\big[\,F(t')+\log R\,\big]$, R – random number

  But don't want to compute $F(t) = -\int_t {\rm d}\bar{t} f(\bar{t})$, as $f(t) = \sum_b \int {\rm d}z \frac{\alpha_s}{2\pi\, t} {\rm P}_{ab}(t,z)$

Solution: Veto algorithm (hit-or-miss for Poisson distributions)

- Find simple function $g(t) > f(t)$ with simple integral $G(t)$
- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Probability for one acceptance with n rejections

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} {\rm d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} {\rm d}t_i \left(1-\frac{f(t_i)}{g(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$

Disentangle nested integrals and sum over n:

$$f(t)\, \exp\left\{-\int_t^{t'} {\rm d}\bar{t}\, g(\bar{t})\right\} \frac{1}{n!} \left[\int_t^{t'} {\rm d}\bar{t}\, (g(\bar{t})-f(\bar{t}))\right]^n \to f(t)\, \exp\left\{-\int_t^{t'} {\rm d}\bar{t}\, f(\bar{t})\right\}$$

Interlude: How to turn P<0 into a probability

- Problem in sub-leading color terms and spin correlation terms lies in negative weights → no-emission probability *locally* exceeds unity
- Recall standard parton shower: $\mathcal{P}_{\rm no}(t,t') = \exp\{F(t)-F(t')\}$
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  But don't want to compute $F(t) = -\int_t {\rm d}\bar{t} f(\bar{t})$, as $f(t) = \sum_b \int {\rm d}z \frac{\alpha_s}{2\pi\, t} {\rm P}_{ab}(t,z)$

Solution: Veto algorithm (hit-or-miss for Poisson distributions)

- Find simple function $g(t) > f(t)$ with simple integral $G(t)$
- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Standard probability for one acceptance with n rejections

$$\frac{f(t)}{g(t)}\, g(t) \exp\left\{-\int_t^{t_1} {\rm d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} {\rm d}t_i \left(1-\frac{f(t_i)}{g(t_i)}\right) g(t_i) \exp\left\{-\int_{t_i}^{t_{i+1}} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$

Split weight into MC and analytic part using auxiliary function $h(t)$

$$\frac{f(t)}{h(t)}\, g(t) \exp\left\{-\int_t^{t_1} {\rm d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} {\rm d}t_i \left(1-\frac{f(t_i)}{h(t_i)}\right) g(t_i) \exp\left\{-\int_{t_i}^{t_{i+1}} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$

$$w(t,t_1,\ldots,t_n) = \frac{h(t)}{g(t)} \prod_{i=1}^{n} \frac{h(t_i)}{g(t_i)} \frac{g(t_i)-f(t_i)}{h(t_i)-f(t_i)}$$

Interlude: How to turn P<0 into a probability

Weighted veto algorithm

$$\frac{f(t)}{h(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} \mathrm{d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} \mathrm{d}t_i \left(1 - \frac{f(t_i)}{h(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} \mathrm{d}\bar{t}\, g(\bar{t})\right\}\right]$$

$$w(t, t_1, \ldots, t_n) = \frac{h(t)}{g(t)} \prod_{i=1}^{n} \frac{h(t_i)}{g(t_i)} \frac{g(t_i) - f(t_i)}{h(t_i) - f(t_i)}$$

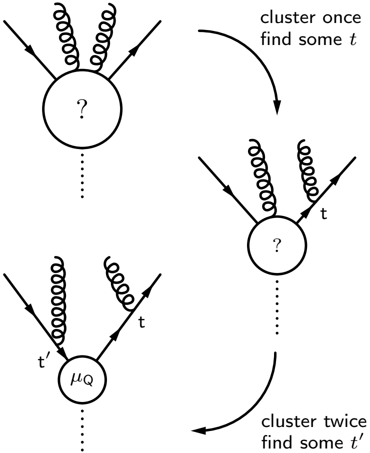
Looks trivial, surprisingly it's not: It allows to

- Resum sub-leading color terms in MC@NLO and POWHEG
  [Krauss,Schönherr,Siegert,SH] arXiv:1111.1220
- Implement higher-order splitting functions in parton showers
  [Catani,Krauss,Prestel,SH] in preparation, see later slides
- Use PDFs with negative values in parton showers
  [Prestel,SH] arXiv:1506.05057
- Enhance branching probabilities in parton showers
  [Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204
- Reweight parton showers [Bellm,Plätzer,Richardson,Siódmok,Webster] arXiv:1605.08256
  [Mrenna,Skands] arXiv:1605.08352, [Bothmann,Schönherr,Schumann] arXiv:1606.08753

Parton-shower histories

[André,Sjöstrand] hep-ph/9708390

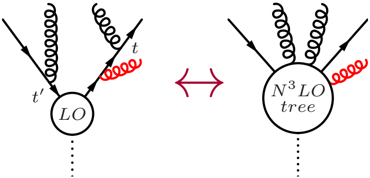
- Start with some "core" process
  for example $e^+e^- \to q\bar{q}$
- This process is considered inclusive
  It sets the resummation scale μ_Q^2
- Higher-multiplicity ME can be reduced to core by clustering
- Clustering algorithm uniquely defined by requiring exact correspondence between ME & PS
  - Identify most likely splitting according to PS emission probability
  - Combine partons into mother according to PS kinematics
  - Continue until no clustering possible



Truncated & vetoed parton showers

[Catani,Krauss,Kuhn,Webber] hep-ph/0109231
[Lönnblad] hep-ph/0112284, arXiv:1211.7204

- Higher-multiplicity MEs that can be reduced to core process are included in core's inclusive cross section (unitarity of PS)
- Sudakov suppression factors needed to make inclusive MEs exclusive
- Most efficiently computed with pseudo-showers
  - Start PS from core process
  - Evolve until predefined branching $\leftrightarrow$ truncated parton shower
  - Emissions producing additional hard jets lead to event veto/weight



$$\Delta(t; > Q_{\text{cut}}) = \exp\left\{ -\int_t \mathrm{d}t \int \mathrm{d}z \, \frac{\alpha_s}{2\pi t} P(t,z) \, \Theta(Q - Q_{cut}) \right\}$$

- At NLO, 1st order expansion of $\Delta(t; > Q_{\text{cut}})$ must be subtracted to avoid double-counting, N$^{>1}$LO terms vary by merging scheme

ME+PS merging – Unitarization

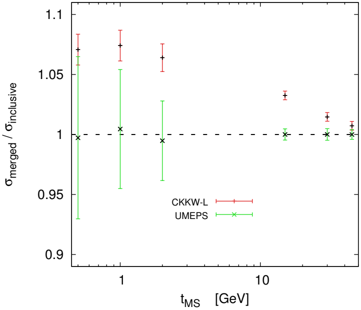
[Lönnblad,Prestel] arXiv:1211.4827, [Plätzer] arXiv:1211.5467

- Unitarity condition of PS:

$$1 = \Delta(t_c) + \int_{t_c} \mathrm{d}t \int \mathrm{d}z \, \frac{\alpha_s}{2\pi t} \mathrm{P}(t,z)\, \Delta(t)$$

- ME+PS(@NLO) violates PS unitarity as ME ratio replaces splitting kernels in emission terms, but not in Sudakovs

$$\frac{\alpha_s}{2\pi t} \mathrm{P}(t,z) \to \frac{\mathrm{R}(t,z,\Phi_B)}{\mathrm{B}(\Phi_B)}$$



- Can be corrected by explicit subtraction

$$1 = \underbrace{\left\{ \Delta(t_c) + \int_{t_c} \mathrm{d}t \int \mathrm{d}z \left[\frac{\alpha_s}{2\pi t} \mathrm{P}(t,z) - \frac{\mathrm{R}(t,z,\Phi_B)}{\mathrm{B}(\Phi_B)} \right] \Theta(Q - Q_{\mathrm{cut}})\, \Delta(t) \right\}}_{\text{unresolved emission / virtual correction}}$$

$$+ \underbrace{\int_{t_c} \mathrm{d}t \int \mathrm{d}z \left[\frac{\alpha_s}{2\pi t} \mathrm{P}(t,z) \Theta(Q_{\mathrm{cut}} - Q) + \frac{\mathrm{R}(t,z,\Phi_B)}{\mathrm{B}(\Phi_B)} \Theta(Q - Q_{\mathrm{cut}}) \right] \Delta(t)}_{\text{resolved emission}}$$