Pushing the precision frontier for Higgs production at the LHC

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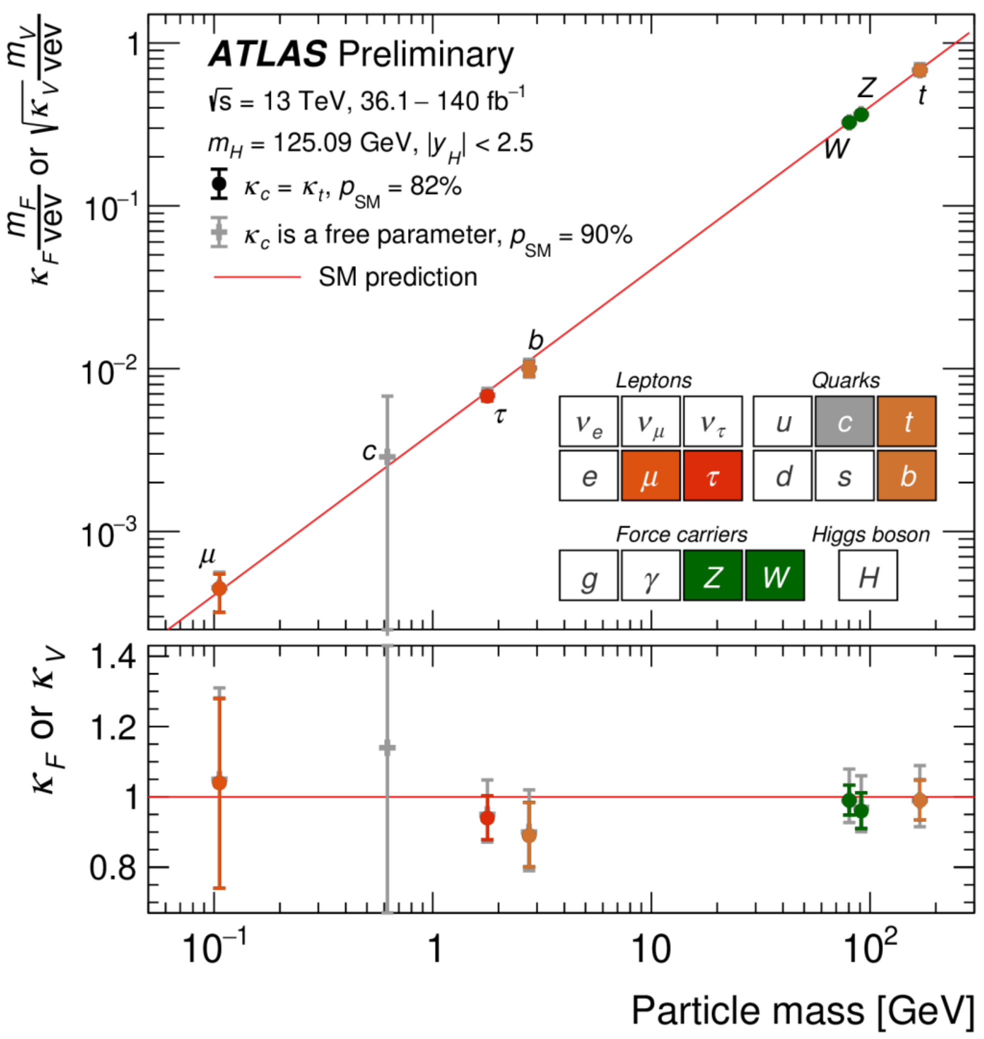
SLAC

Based on 2504.06490
with Xiang Chen and Bernhard Mistlberger

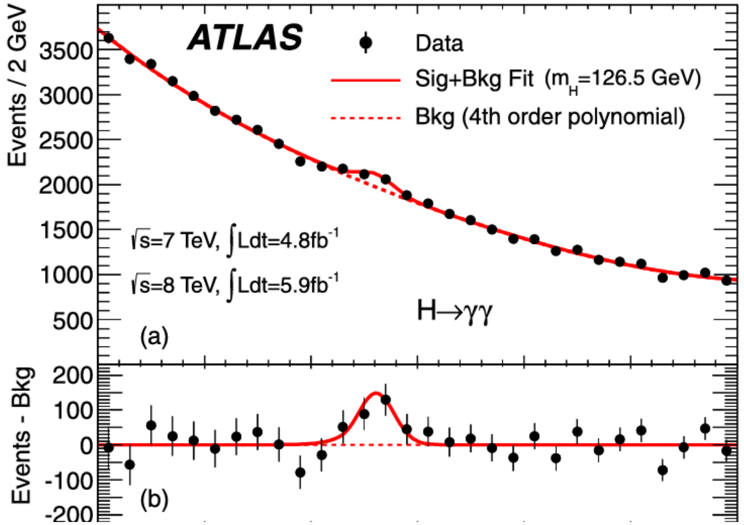
BAPTS
San Francisco State University, October 3, 2025

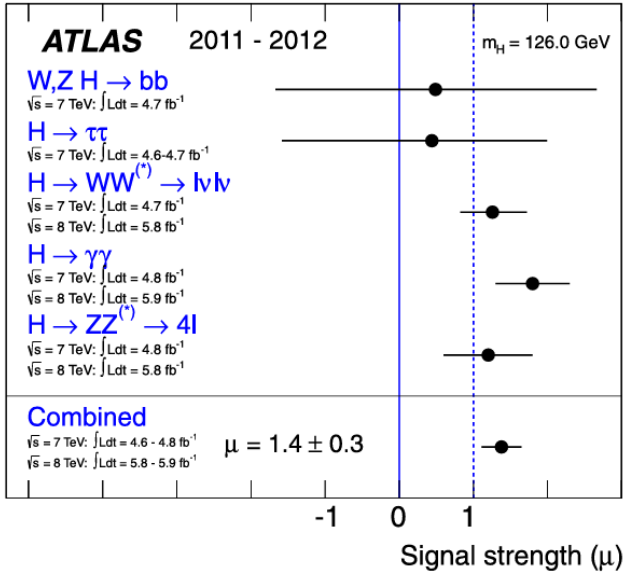
Why the Higgs boson

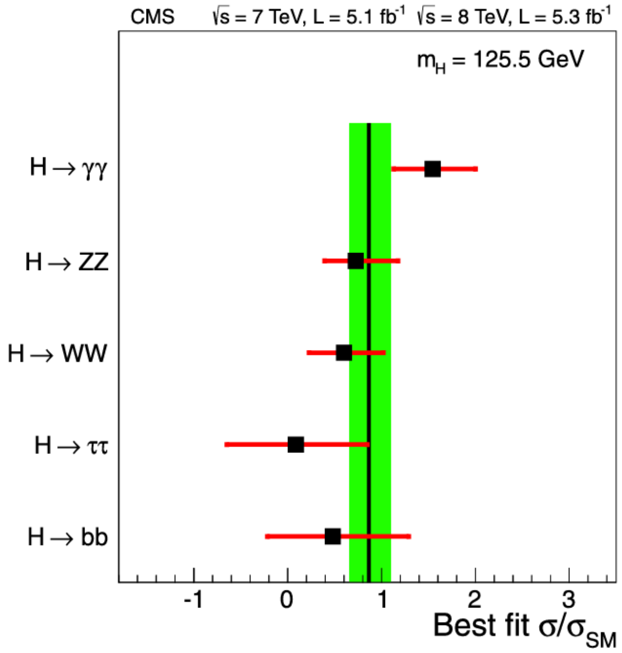
- Last missing piece of the Standard model
- Provides mass through electroweak symmetry breaking
- Coupling proportional to the mass (experimentally confirmed)
- Sensitive to new physics (heavy particle, mixing, …)



LHC: from discovery to precision







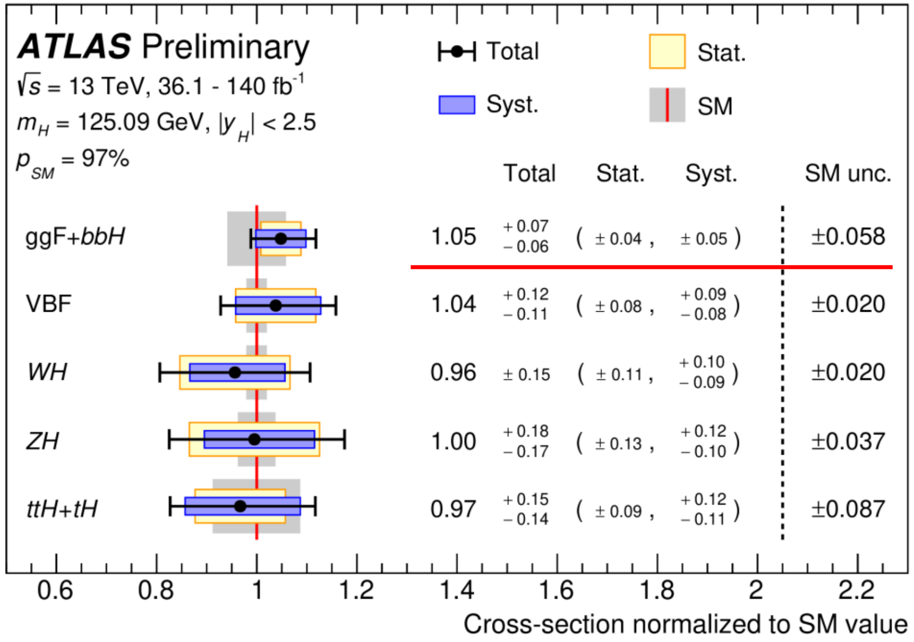
- 2012 discovery: 20-25% uncertainty

ATLAS: $\mu = \ 1.4 \pm 0.3$ ATLAS, 1207.7214
CMS: $\mu = \ 0.87 \pm 0.23$ CMS, 1207.7235

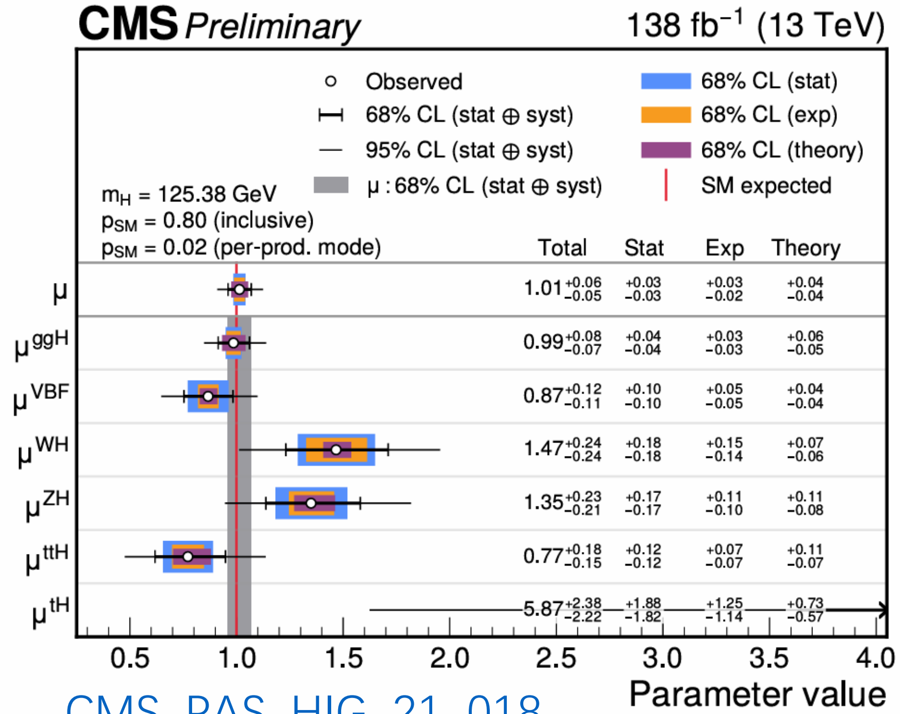
Consistency check of SM:

$$\mu = \sigma_{obs}/\sigma_{\mathrm{SM}}$$

LHC: from discovery to precision



[ATLAS-CONF-2025](#)



[CMS-PAS-HIG-21-018](#)

- Current measurements: ~5% uncertainty

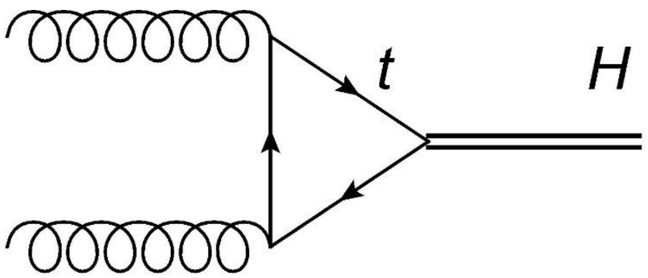
ATLAS: $1.023^{+0.056}_{-0.053}$

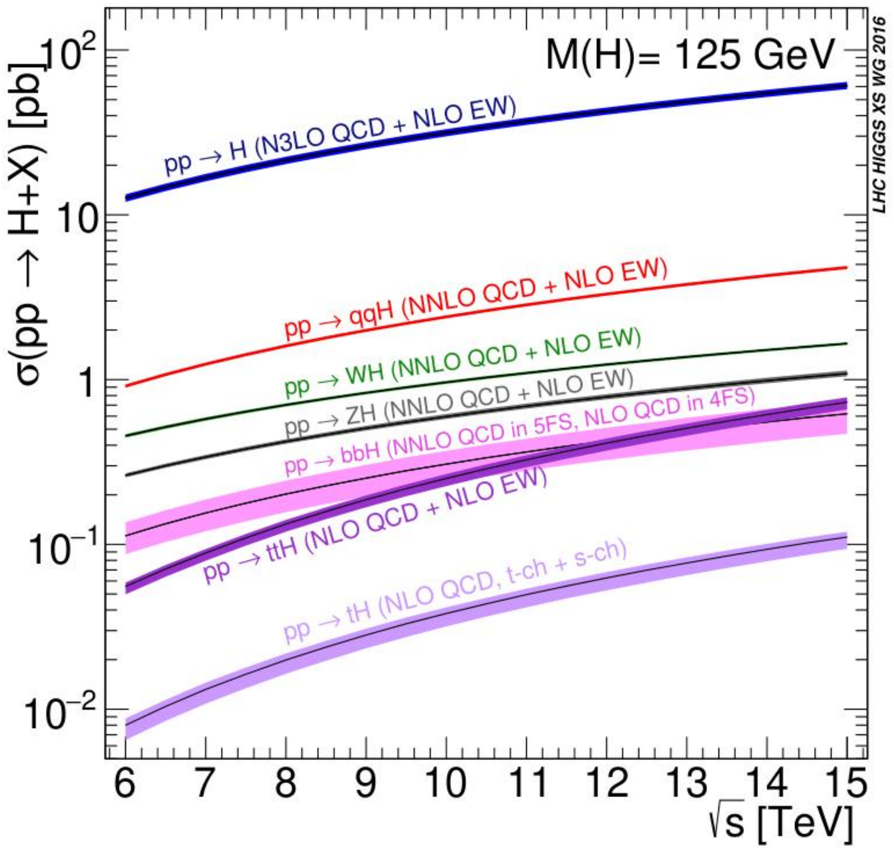
CMS: $1.014^{+0.055}_{-0.053}$

Inclusive Higgs production

➢ Gluon fusion dominates

- Gluon fusion (H) ~88%
- Vector-boson fusion (qqH) ~7%
- Associated production (ZH/WH) ~4%
- ttH ~1%





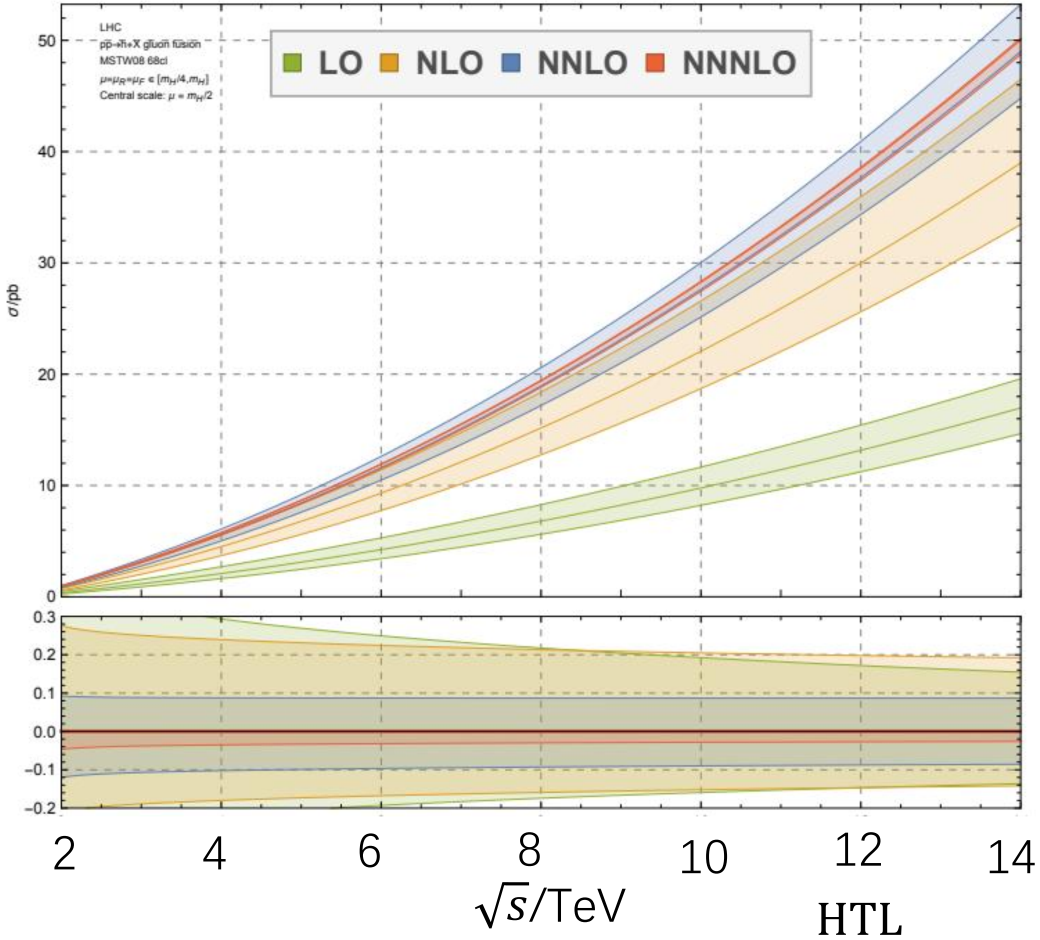
- Gluon PDFs: large corrections
- Loop induced: large theoretical uncertainties

Inclusive Higgs production frontier

- $\frac{\sigma_{NLO}}{\sigma_{LO}} \sim 2$: higher order corrections are essential
- $\mathrm{N}^3\mathrm{LO}$ within uncertainty band at NNLO
- scale uncertainty: ~3%
- HL-LHC will deliver measurements

at the percent level

- Aim for 1% -> $\mathrm{N}^4\mathrm{LO}$



Anastasiou et al., 1503.06056

Higgs + Jet frontier

- Differential distribution allows detailed comparison
- Broaden set of observables:

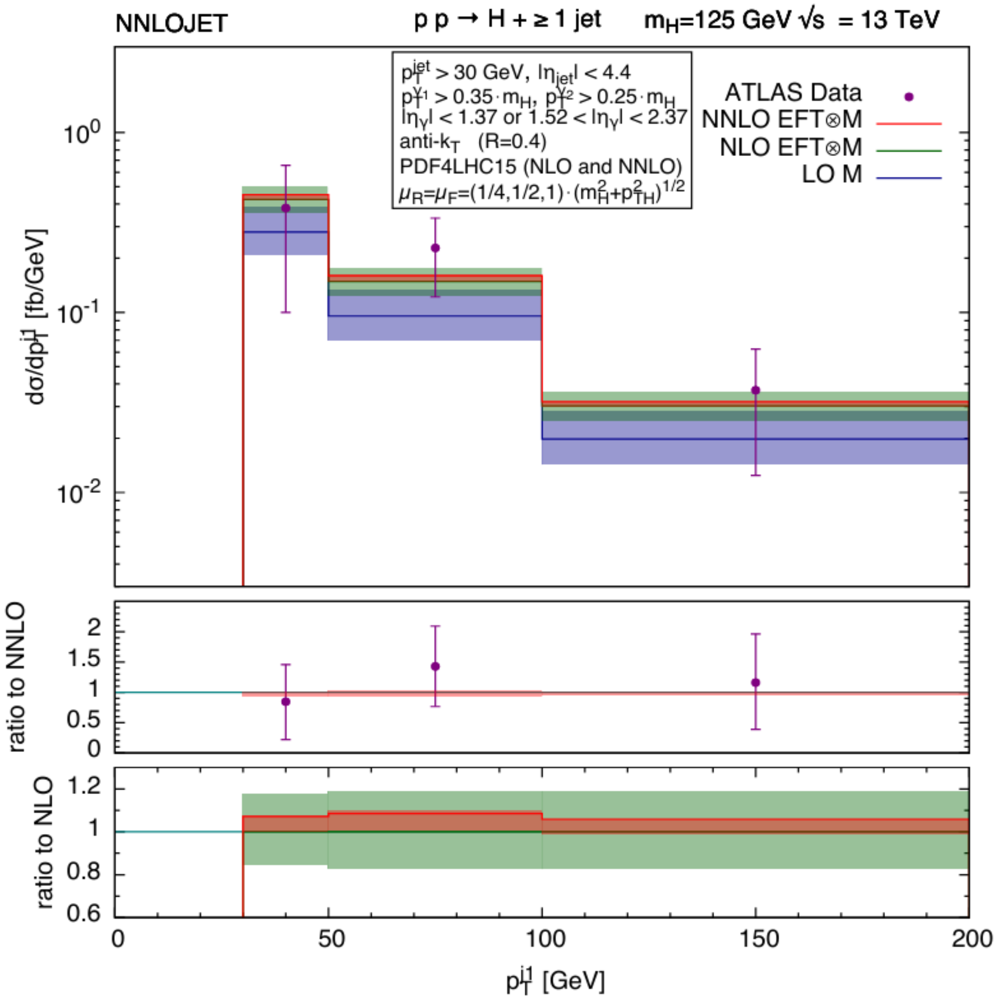
$P_T^H, P_T^{j_1}, y^{j_1}, m_{Hj}, \ldots$

- Sensitive to the top-quark loop at high P_T
- NNLO scale uncertainty: ~5%
- Current experimental uncertainty: 10-15%

ATLAS , 2004.03969; 2202.00487

- Expected (HL-LHC) uncertainty: <= 9.5%

N^3LO_{HTL} is crucial to rival the experimental precision!



X. Chen et al., 1607.08817

Path to the precision prediction

- PDFs x partonic cross section

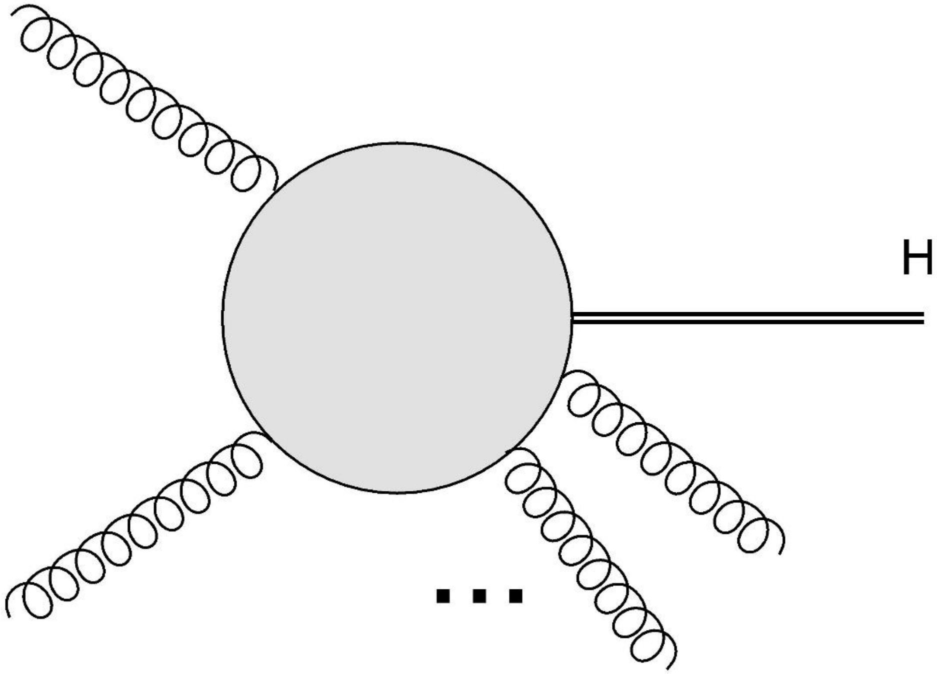
$$\sigma = \int dx\, dy\, f(x) f(y)\, \hat{\sigma} + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$

- Perturbative QCD input

$$\hat{\sigma} = \hat{\sigma}^{0} \quad + \alpha_s^1 \hat{\sigma}^{(1)} + \alpha_s^2 \hat{\sigma}^{(2)} + \alpha_s^3 \hat{\sigma}^{(3)} + \cdots$$

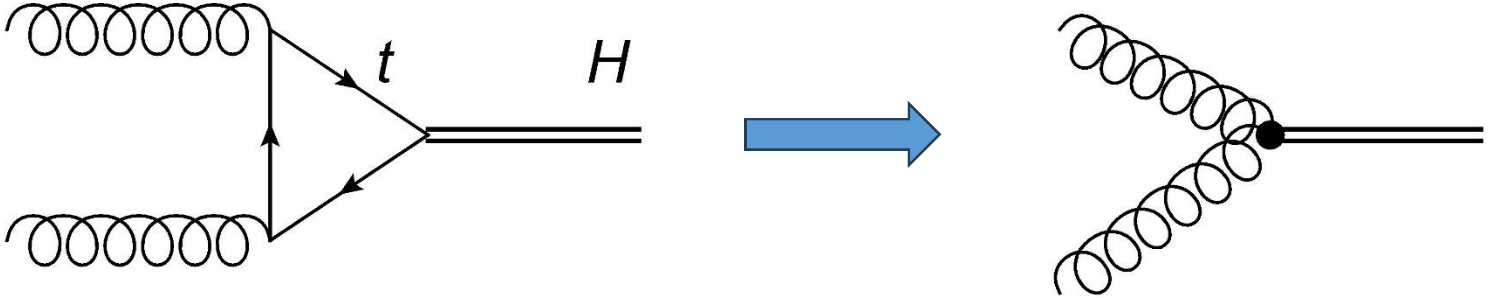
Loops & radiations

- KLN theorem: cancellations among loops and real radiations
- #Loops + #Emissions ~ α_S order



Heavy-Top effective theory

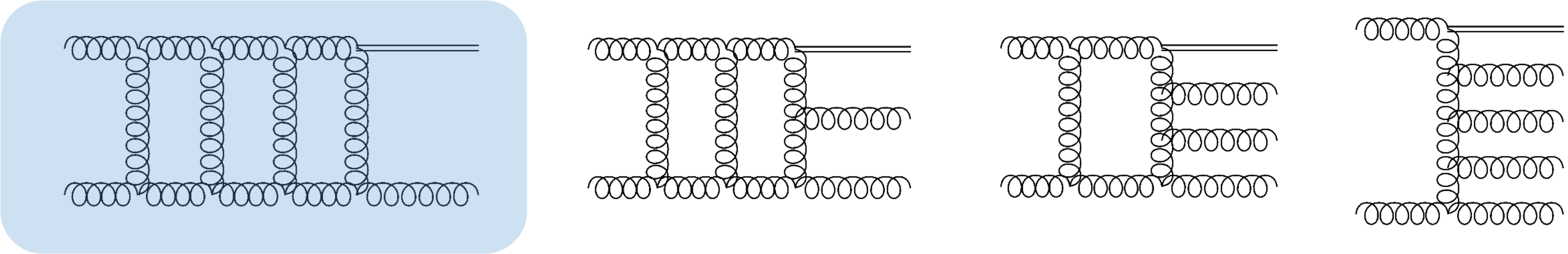
- Replace top loop by effective ggH vertex
- Greatly simplifies multi-loop calculations, ridiculous higher order
- Valid for Q $\ll 2\,m_t$ (a good approximation at LHC)



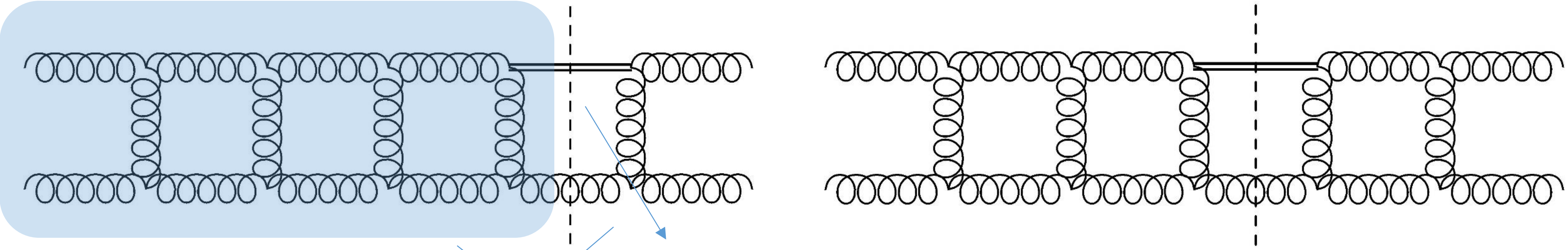
$$\sim H G_{\mu\nu,a} G^{\mu\nu,a}$$

Three loop Higgs+Jet

- Essential for $\mathrm{N^3LO}$ H+J



- Building block for $\mathrm{N^4LO}$ Higgs



RVVV TBD

Interference

Phase space integration

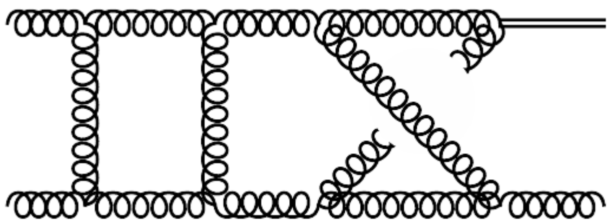
RVV x V complete

Mistlberger and Suresh 2504.10574

Calculation

Generate integrand

➢ Set up



- Dimensional regularization: $d = 4 - 2\epsilon$
- Heavy top limit
- Generalized leading color approximation: neglect $1/N_c^2$ suppressed terms

➢ Integrand

$$A = \sum_{i=1}^{\mathcal{O}(10^5)} f_i \times I_i$$

10k Feynman diagrams, 6M integrals → 46 topologies 200k integrals (symmetry)

Qgraf: Nogueira et al., Comput. Phyis. 1993

CalcLoop: Ma et al., https://gitlab.com/multiloop-pku/calcloop

IBP: linear relations among integrals

- Feynman integral

$$I(a_1, \dots a_n) = \int \frac{\mathrm{d}^d l}{i\,\pi^{d/2}} \frac{1}{D_1^{a_1} D_2^{a_2} \dots D_n^{a_n}}$$

- Integration-by-part identities

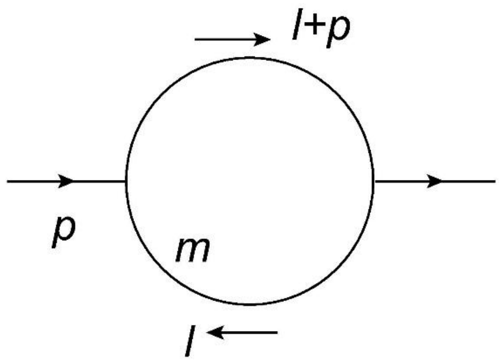
Chetyrkin et al., NPB(1981)

$$0 = \int \frac{\mathrm{d}^d l}{i\,\pi^{d/2}} \frac{\partial}{\partial l^\mu} \frac{v^\mu}{(q_1^2 - m_1^2)^{a_1} (q_2^2 - m_2^2)^{a_2} \dots (q_n^2 - m_n^2)^{a_n}}$$

- v^μ, q_i^μ: combinations of loop momenta and external momenta
- Rewrite the RHS in terms of combinations of $I(\vec{a})$
- Using IBP, integrals are reduced to a finite number of basis (master integrals)

Smirnov, Petukhov, 1004.4199

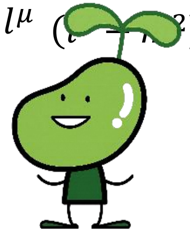
A toy example of IBP reduction



$$I(a_1, a_2)$$

$$0 = \int \frac{\mathrm{d}^d l}{i\,\pi^{d/2}} \frac{\partial}{\partial l^\mu} \frac{l^\mu}{(l^2 - m^2)^{a_1}\left((l+p)^2 - m^2\right)^{a_2}}$$

$$0 = \int \frac{\mathrm{d}^d l}{i\,\pi^{d/2}} \frac{\partial}{\partial l^\mu} \frac{p^\mu}{(l^2 - m^2)^{a_1}\left((l+p)^2 - m^2\right)^{a_2}}$$



- Target: $I(1, -2)$
- Sow seeds, reap equations

Seeds: I(1,0), I(1,-1), I(1,-2)

$$(2 - 2\epsilon)\, I[1, 0] - 2 m^2 I[2, 0] == 0$$
$$-I[1, 0] + I[2, -1] - s\, I[2, 0] == 0$$
$$(3 - 2\epsilon)\, I[1, -1] + (2 m^2 - s)\, I[1, 0] - 2 m^2 I[2, -1] == 0$$
$$-2 I[1, -1] - s\, I[1, 0] + I[2, -2] - s\, I[2, -1] == 0$$
$$(4 - 2\epsilon)\, I[1, -2] + (4 m^2 - 2 s)\, I[1, -1] - 2 m^2 I[2, -2] == 0$$
$$-3 I[1, -2] - 2 s\, I[1, -1] + I[2, -3] - s\, I[2, -2] == 0$$

- Gaussian elimination

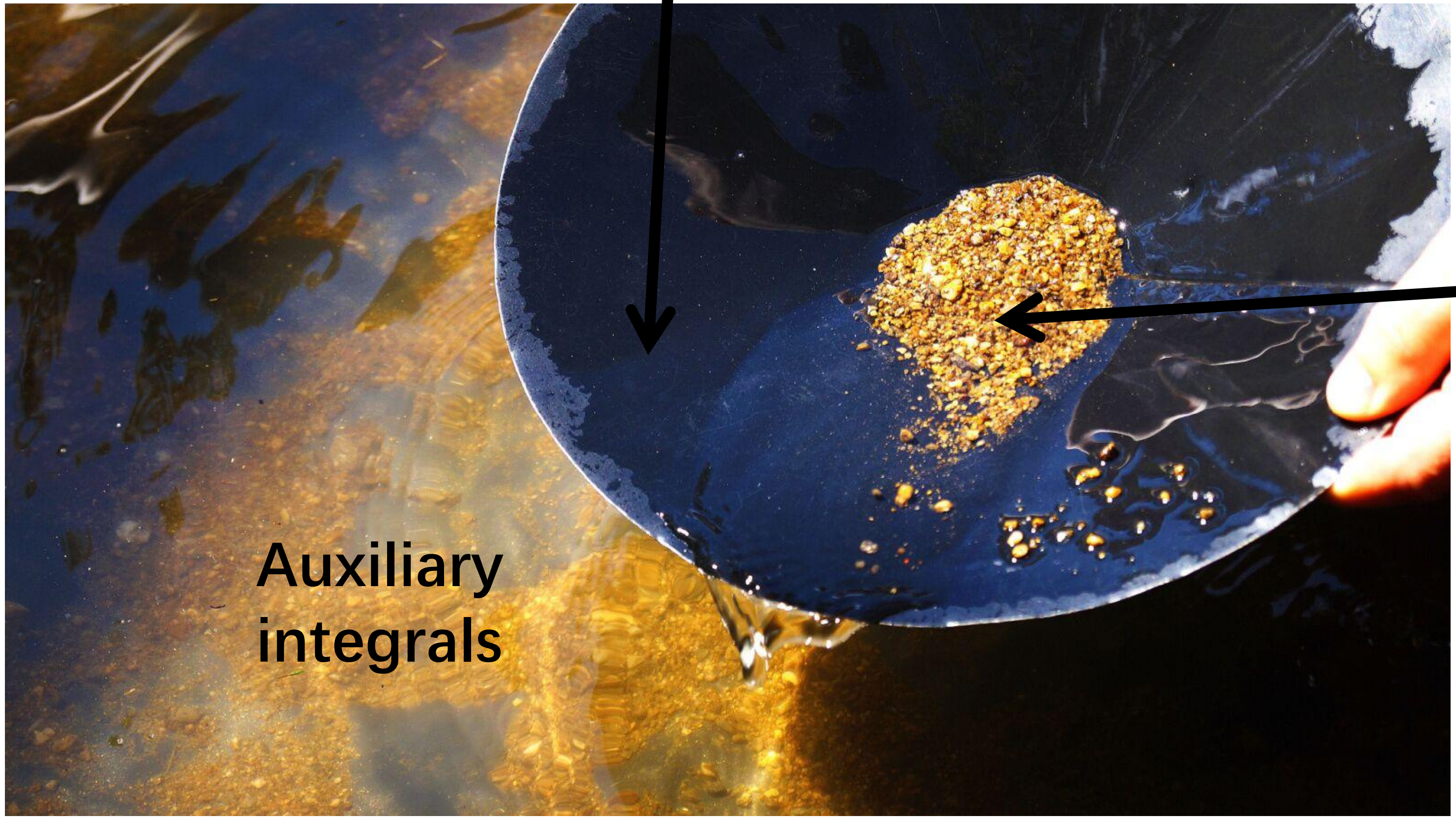
$$I[1, -2] \to \frac{s\,(-2 m^2 - 2 s + s\,\epsilon)\, I[1, 0]}{-2 + \epsilon}$$

Master integral

Simpler!

IBP Reduction = Panning for Gold

IBP reduction



$$(2-2\epsilon)\, I[1,0] - 2m^2\, I[2,0] = 0$$
$$-I[1,0] + I[2,-1] - s\, I[2,0] = 0$$
$$(3-2\epsilon)\, I[1,-1] + (2m^2 - s)\, I[1,0] - 2m^2\, I[2,-1] = 0$$
$$-2\, I[1,-1] - s\, I[1,0] + I[2,-2] - s\, I[2,-1] = 0$$
$$(4-2\epsilon)\, I[1,-2] + (4m^2 - 2s)\, I[1,-1] - 2m^2\, I[2,-2] = 0$$
$$-3\, I[1,-2] - 2s\, I[1,-1] + I[2,-3] - s\, I[2,-2] = 0$$

Target
integrals

$$I[1,-2] \to \frac{s\,(-2m^2 - 2s + s\epsilon)\, I[1,0]}{-2+\epsilon}$$

Analogy suggested by Lance

Challenge of IBP reduction

- Mathematically simple, computationally huge
  - High complexity of integrals: non-planar, rank 6, 305 master integrals in a family
  - Millions of equations
  - TB Memory, months runtime

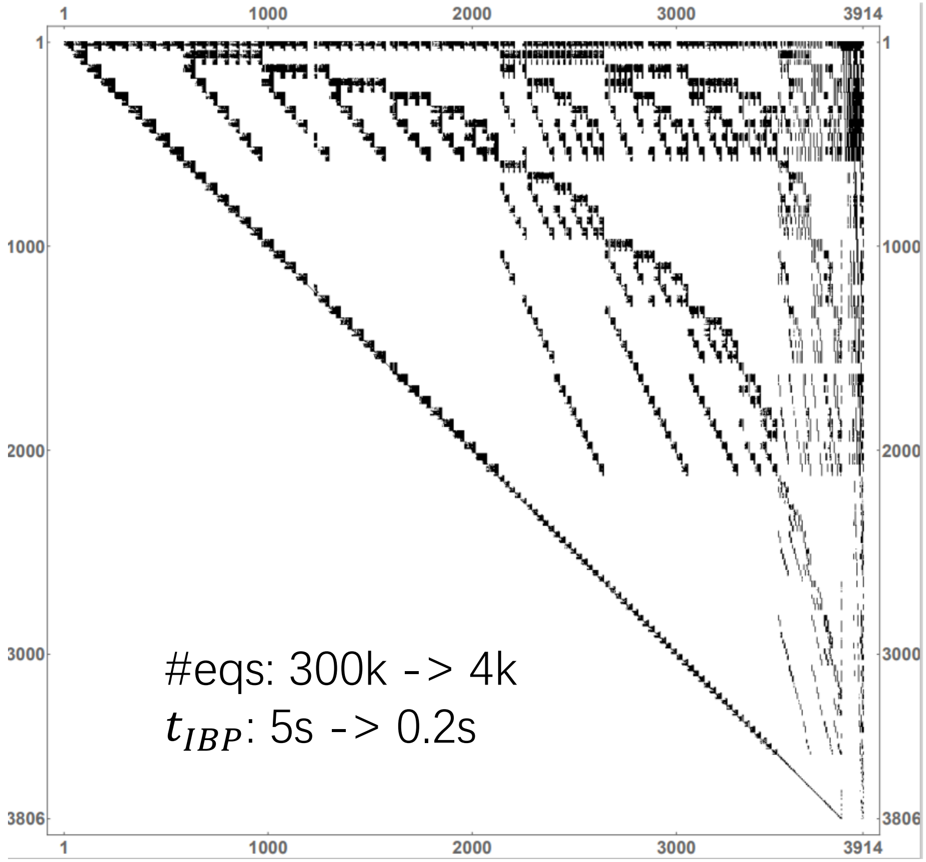
e.g. three loop W+J: rank 5 , planar , O(1TB) Memory

Mella, Pos LL2024

IBP technique: block-triangular form

- Compact and well-structured system
  - Search relations among a dedicated set of integrals
  - Order of magnitude fewer equations
  - Strictly solved block-by-block
- Cuts memory & runtime

Liu et al., 1801.10523
XG et al., 1912.09294



XG et al., 1912.09294

IBP summary

➢Memory and CPUh

FiniteField → **1TB? 10000? sec/pt** → Improved seeding + Spanning → **100GB 2000 sec/pt** → Block-triangular form → **<5 GB 20 sec/pt**

- 500k CPU*h
- 200k integrals -> 1509 master integrals

➢Blade package

B **Blade** Link: https://gitee.com/multiloop-pku/blade

Blade: XG et al., 2405.14621

项目信息

Block-triangular form improved Feynman integral decomposition .

Compute master integrals

➢ Differential equation method

- Derivatives with respect to kinematics relate integrals systematically

$$\frac{\partial M_i}{\partial x_j} = \sum_k c_k \, I_k = \sum_n a_n \, M_n$$

$$\frac{\partial}{\partial x_j} \vec{M} = A(\vec{x}, \epsilon) \vec{M}$$

Canonical differential equation

- ➢Canonical form

Henn, 1304.1806

$$\frac{\partial}{\partial x_j}\vec{M} = A(\vec{x},\epsilon)\vec{M}$$

$$\mathrm{d}\,\vec{J} = \epsilon \sum_k A_k\, \mathrm{dLog}(\alpha_k)\, \vec{J}$$

  - Algorithmic way to find basis $\vec{J}$

Lee, 1411.0911

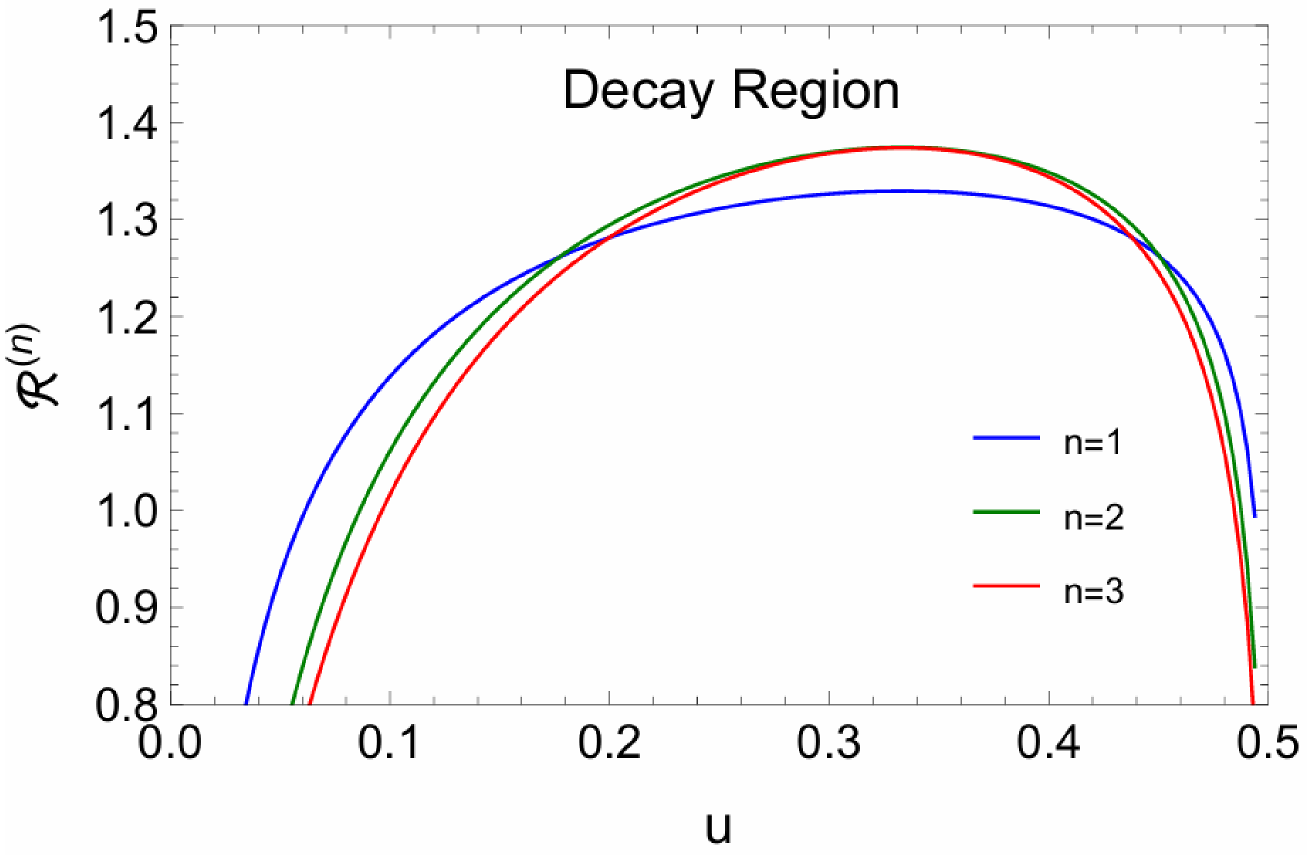
  - Iterative solution

- ➢Boundary condition

- ➢Three loop master integrals are solved in literature
  - Check the existing results

Gehrmann et al., 2410.19088
Di Vita et al., 1408.3107
Canko et al., 2112.14275
Henn et al., 2302.12776
Syrrakos et al., 2307.08432

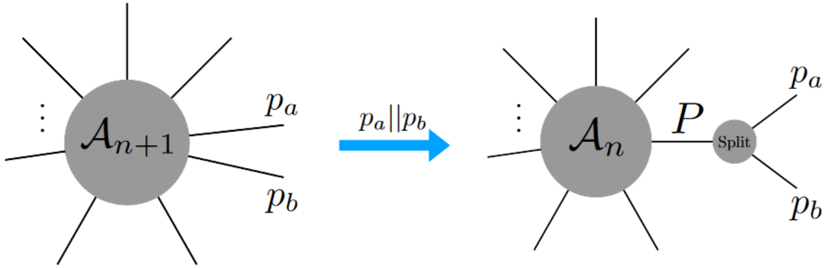
Results



- Finite remainder of interfered amplitude (normalized to leading order), along $t = u$
- The central region displays excellent perturbative stability

Checks

- Cancellation of UV & IR divergence
- Two-parton collinear limit -> universal splitting amplitudes



XG et al., 2408.03019

- One gluon soft limit -> universal soft current

Herzog et al., 2309.07884;
Chen et al., 2309.03832

- Leading transcendental principle

$$M^{(3)}_{H\to ggg} \sim tr(\phi^2) \text{ form factor in } N=4 \text{ super Yang-Mills theory}$$

Dixon et al., 2012.12286; 2204.11901
Lin et al., 2111.03021

- Numeric evaluation via AMFlow

AMFlow: Liu et al., 2201.11669

Conclusion

- Precision era of Higgs physics — the work continues
- Three loop HJ amplitude: new building block for precision
- Blade enables large-scale IBP reduction for multi-loop amplitudes

Thank you!

IBP technique: Finite field

➢Idea

Manteuffel et al., 1406.4513

- Work mod primes
- Functional reconstruction
- Rational reconstruction

$$f(t,u) = \frac{1+u^{100}}{1+t^{100}}$$

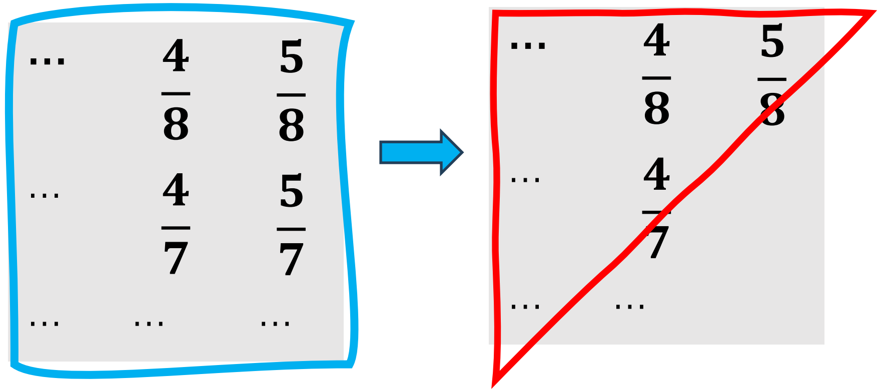
$$f(3,5) = 2 \bmod 7$$

➢ Advantage

- Fix length number -> cuts memory & runtime
- Easy to parallelize

IBP technique: improved seeding

- Smarter choice of input equations
- Avoids unnecessary/redundant relations
- Cuts memory & runtime



$$I(a_1, \ldots a_n)$$

IBP technique: spanning sector

- Divide the problem into smaller subsystems
- Solve spanning-sectors separately and merge results
- Cuts memory & runtime

System 1 $\quad I = c_1 M_1 + c_2 M_2$

System 2 $\quad I = c_2 M_2 + c_3 M_3$

$$I = c_1 M_1 + c_2 M_2 + c_3 M_3$$

(merge)