Understanding Chiral Gauge Theories using Extra Dimensions

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Work done with David B. Kaplan:
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Phys. Rev. D 94, 114504

Parity Violation

One of the great surprises of 20th century was discovery of parity violation: LH and RH fermions can carry different gauge charge!

Question of Parity Conservation in Weak Interactions*

T. D. Lee, *Columbia University, New York, New York*

AND

C. N. Yang,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

The question of parity conservation in β decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

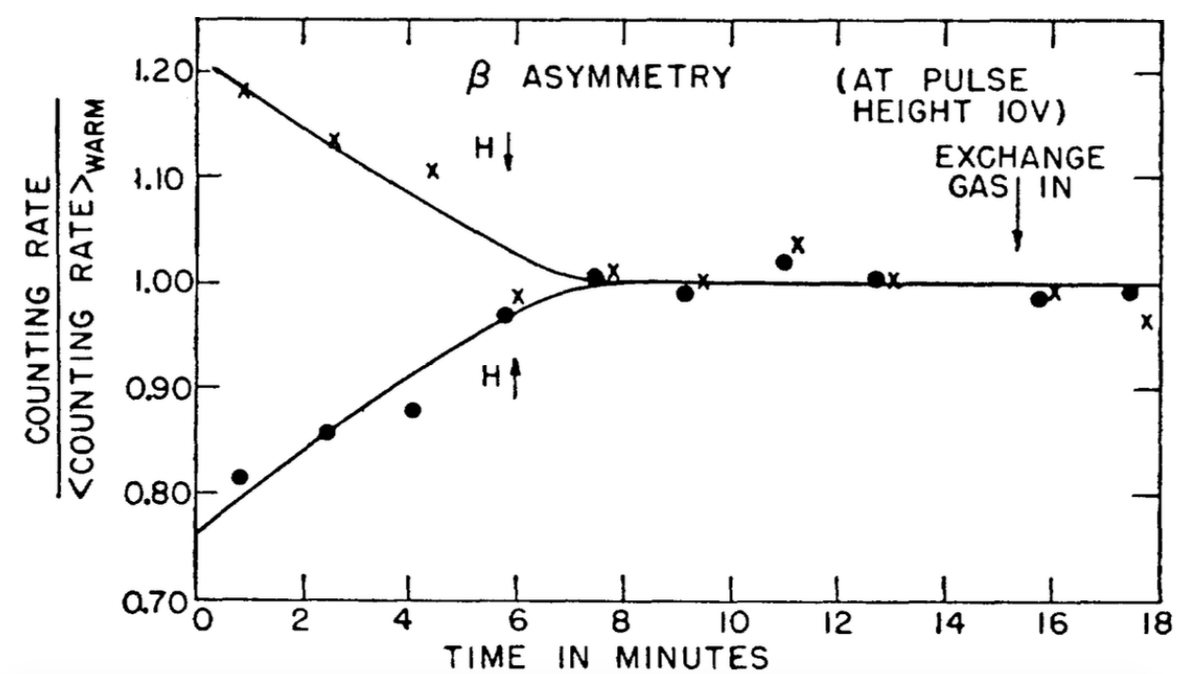
Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

AND

E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *National Bureau of Standards, Washington, D. C.*

(Received January 15, 1957)



Chiral Gauge Theories

We have seen exactly one chiral gauge theory: Standard Model

Extremely well-motivated

- Strong agreement between observations and predictions
- Ubiquitous in speculative models of BSM physics

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Could nonperturbative regulator lead to unexpected phenomena or address some outstanding puzzles?

Fermion Path Integrals

Need nonperturbative definition of fermion path integral

- Vector theory: fermions in real representations of gauge group

$$Z_V \equiv \int [DA]\; e^{-S_g(A)} \prod_{i=1}^{N_F} \det\left(\not{D} - m_i\right)$$

- Chiral theory: fermions in complex representations

$$Z_\chi \equiv \int [DA]\; e^{-S_g(A)} \Delta(A)$$

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Fermion path integral

Witten: 'We often call the fermion path integral a "determinant" or a "Pfaffian," but this is a term of art.'

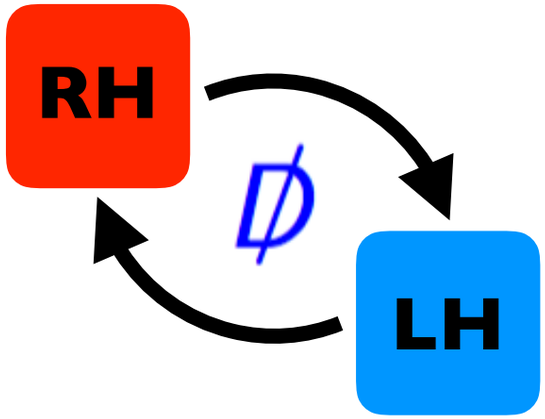
Δ(A) is really product of eigenvalues

The Problem

Need to find eigenvalues of fermion operator

- Vector theory:

$$\not{D}\psi = \begin{pmatrix} 0 & D_\mu \sigma_\mu \\ D_\mu \bar{\sigma}_\mu & 0 \end{pmatrix} \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} = \lambda \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$



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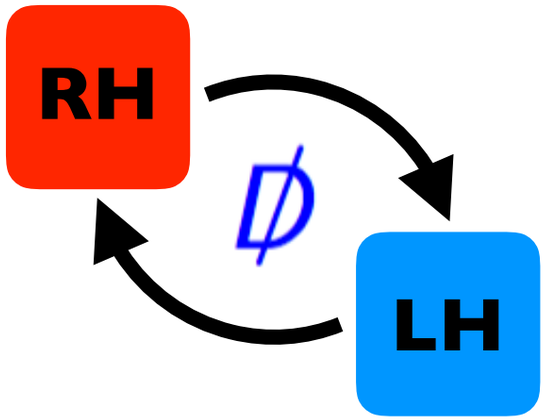

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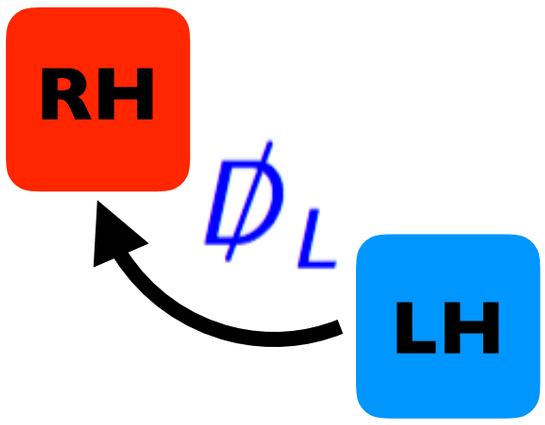

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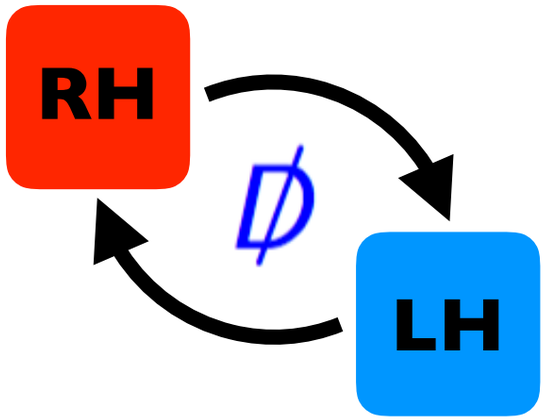

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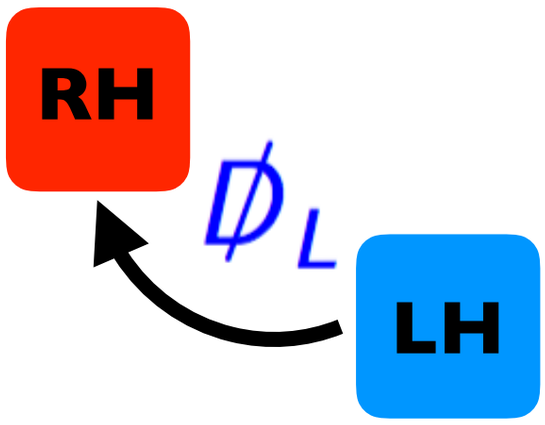

Ill-defined eigenvalue problem leads to phase ambiguity for product of the eigenvalues of chiral fermions

$$\Delta(A) = e^{i\delta(A)} \sqrt{|\det \not{D}|}$$

A (Perturbative) Proposal

Proposal: Introduce neutral RH spectator fermions*

$$\Delta(A) \sim \det \begin{pmatrix} 0 & D_\mu \sigma_\mu \\ \partial_\mu \bar{\sigma}_\mu & 0 \end{pmatrix}$$

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Neutral RH Weyl Fermion

- Well defined eigenvalue problem
- Overall phase of Δ(A) related to η-invariant*
- Uncertain if amenable to lattice regularization

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{ Are there other reasonable pert. limits for Δ(A)? }

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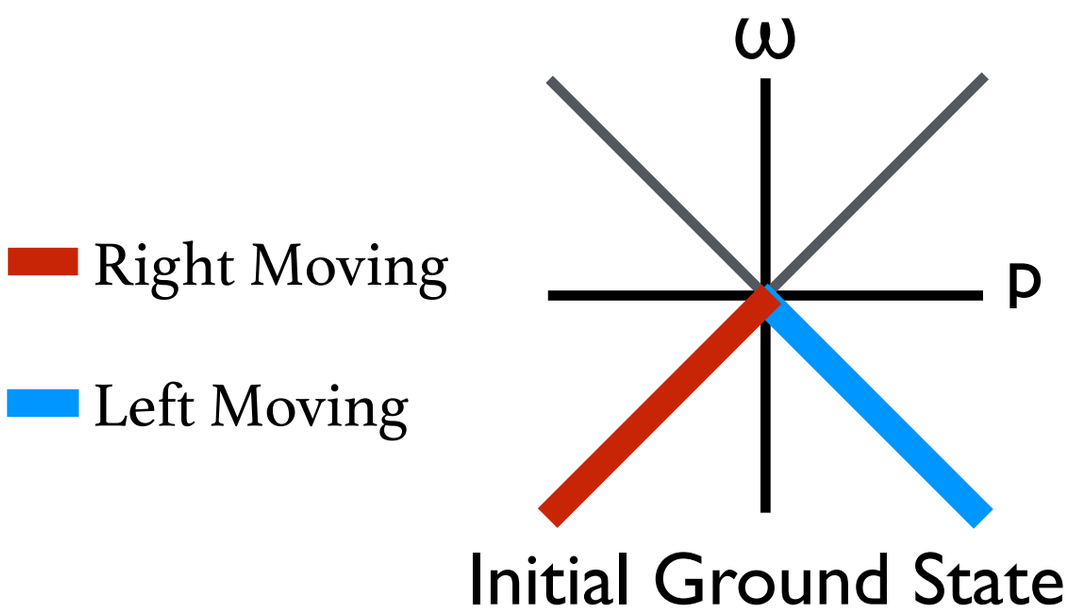
Anomalies

Classical symmetries violated by quantum effects

Ex: Massless electrons in two dimensions



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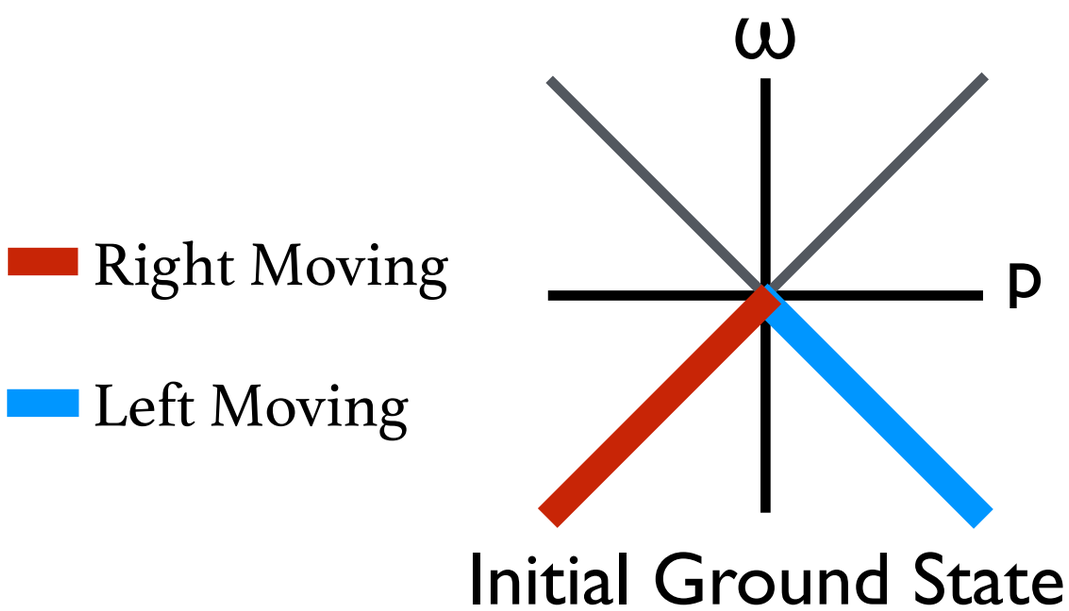

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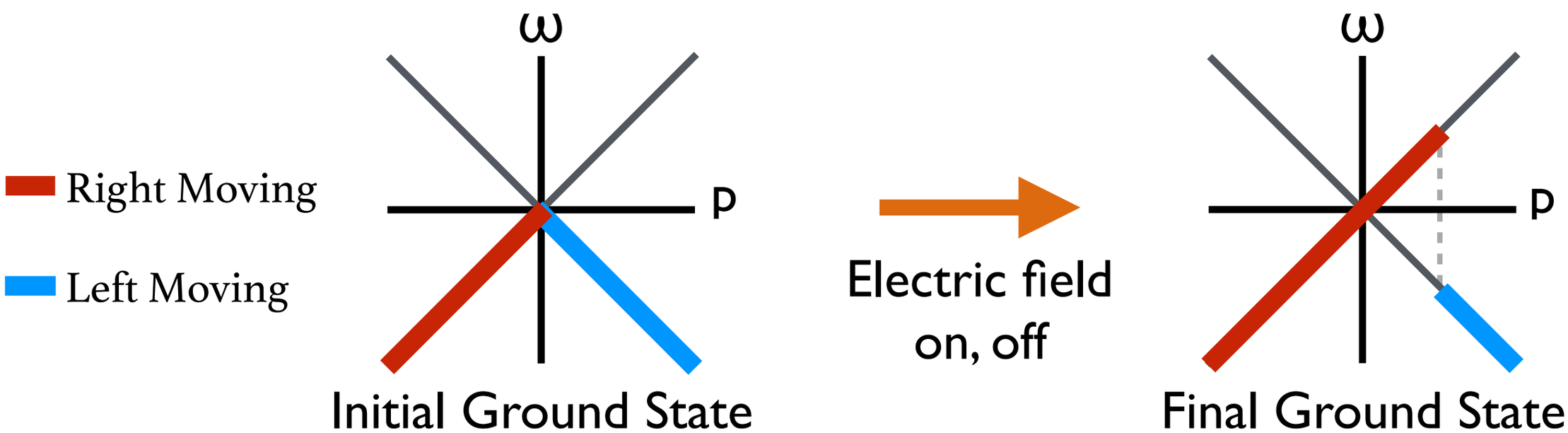

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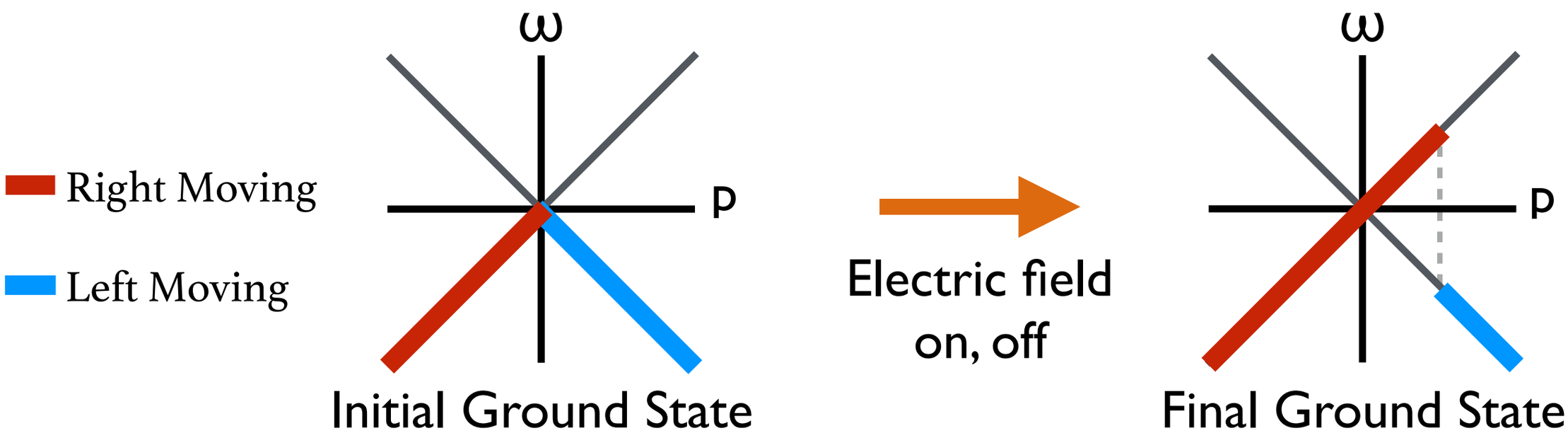

- Fermion charge **does not** change: $U(1)_V$ **preserved**
- Axial charge **does** change if Dirac sea *infinite*: $U(1)_A$ **violated**

Anomalies require infinite number of degrees of freedom

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Fundamental tension between taming UV behavior of chiral gauge theories and maintaining gauge invariance

No-Go Theorem*

No-Go Theorem: No lattice fermion operator can satisfy all four conditions simultaneously:

1. Periodic and analytic in momentum space
2. Reduces to Dirac operator in continuum limit
3. Invertible everywhere except at zero momentum
4. Anti-commutes with γ_5

*Nielsen & Ninomiya, '81

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Lattice regulated chiral fermions violate at least one condition

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Requirements

Basic building block is Dirac fermion

Lattice regulated chiral gauge theories **must** have:

- Global chiral symmetry with correct $U(1)_A$ anomaly
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Some Previous Proposals

Project out mirrors; construct measure {Lüscher}	Gauge fix; flow to correct continuum limit {Golterman, Shamir}	Give mass to mirrors {Golterman, Petcher, Smit, etc}

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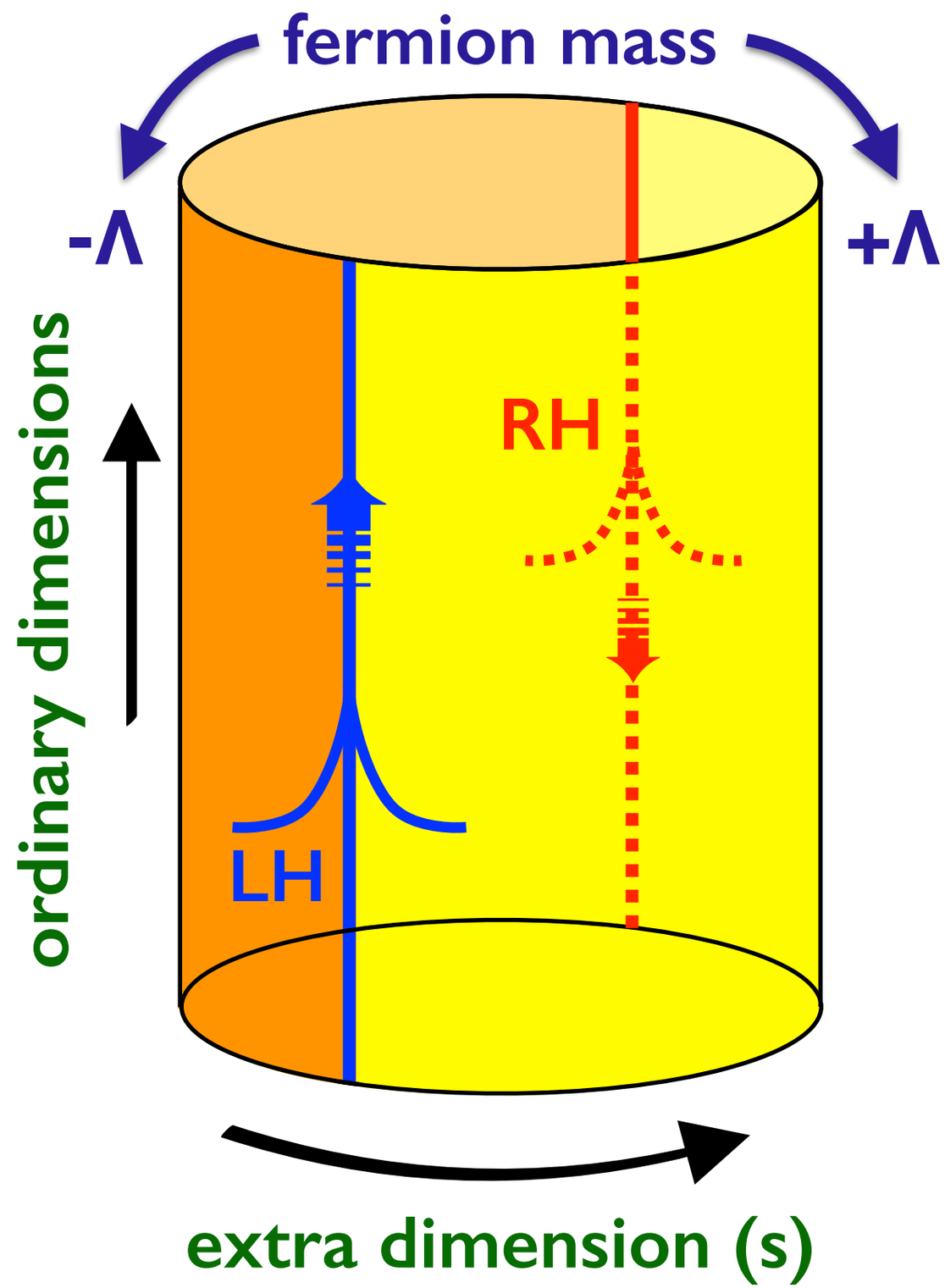
Our proposal is in a slightly different vein

Global Chiral Symmetries

Domain Wall Fermions*

- Introduce extra (compact) dimension, s
- Fermion mass depends on s
- Massless modes localized on mass defects
- Massive fermions delocalized into the bulk

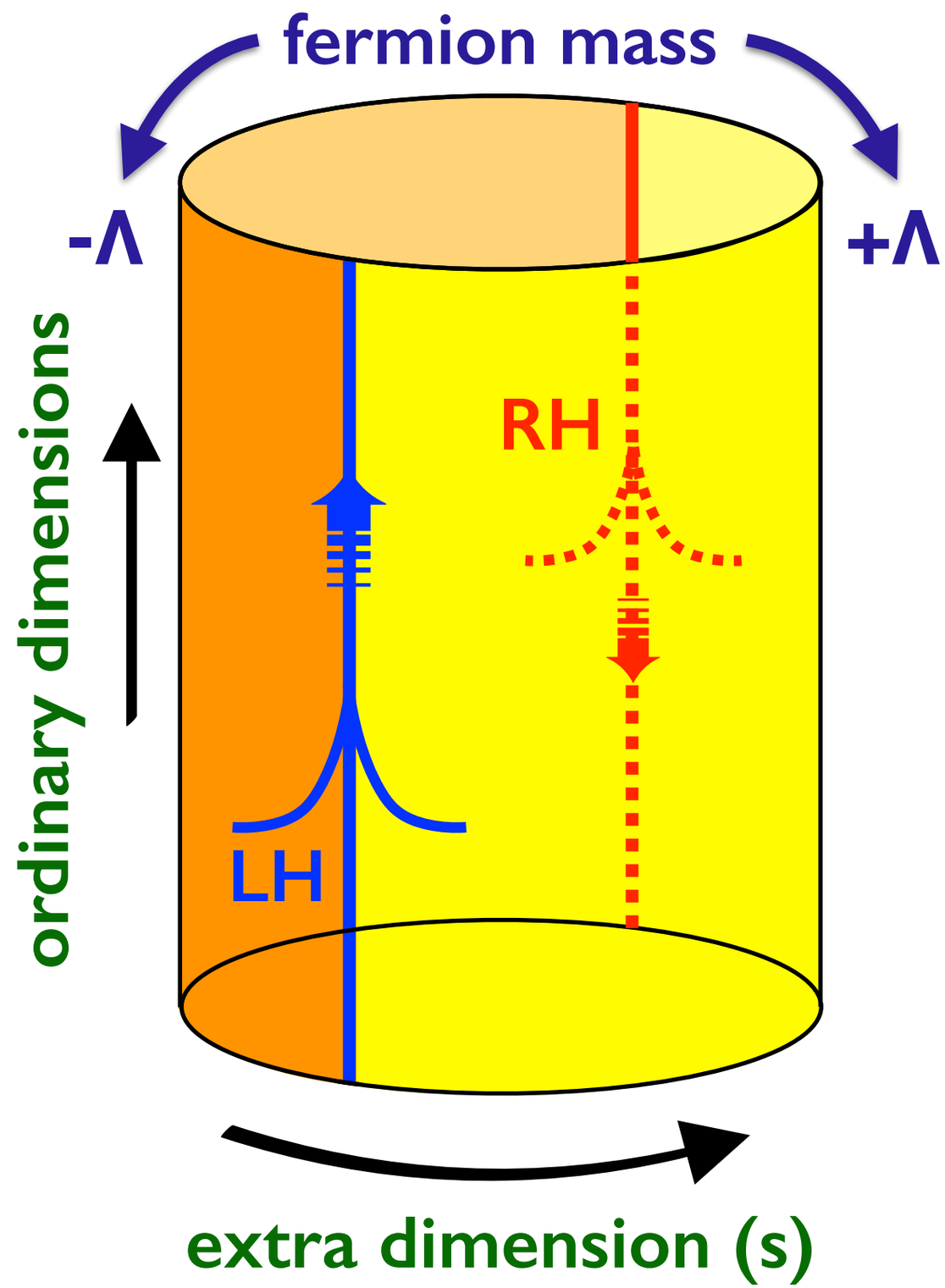


*Kaplan, '92

Callan-Harvey Mechanism*

$U(1)_A$ Anomaly

- Gauge fields independent of s
- Bulk fermions carrying charge between mass defects
- Boundary fermions see axial charge appearing/disappearing



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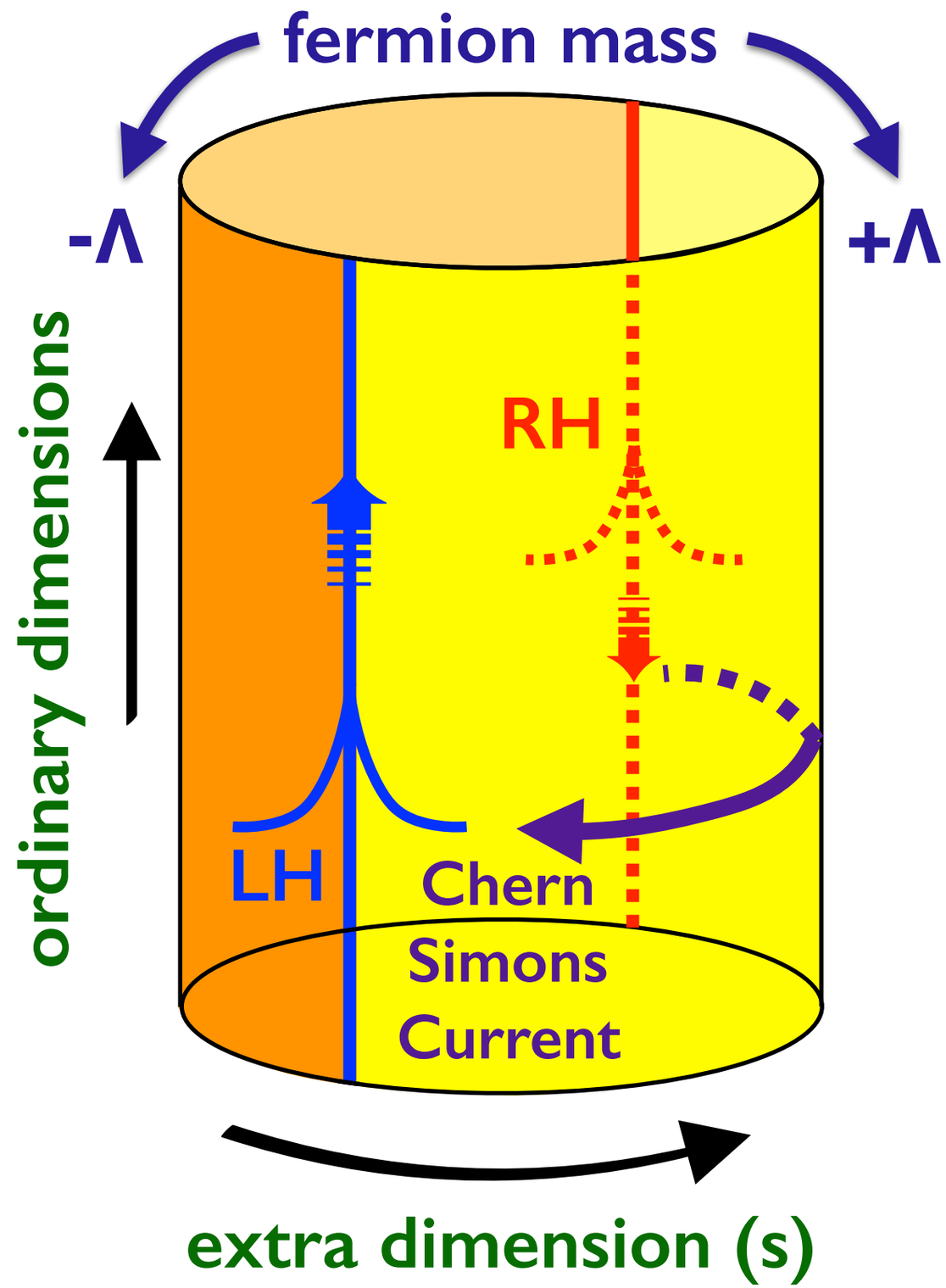
Anomaly: $U(1)_A$ explicitly violated

- Condensed matter physicists would call this a topological insulator



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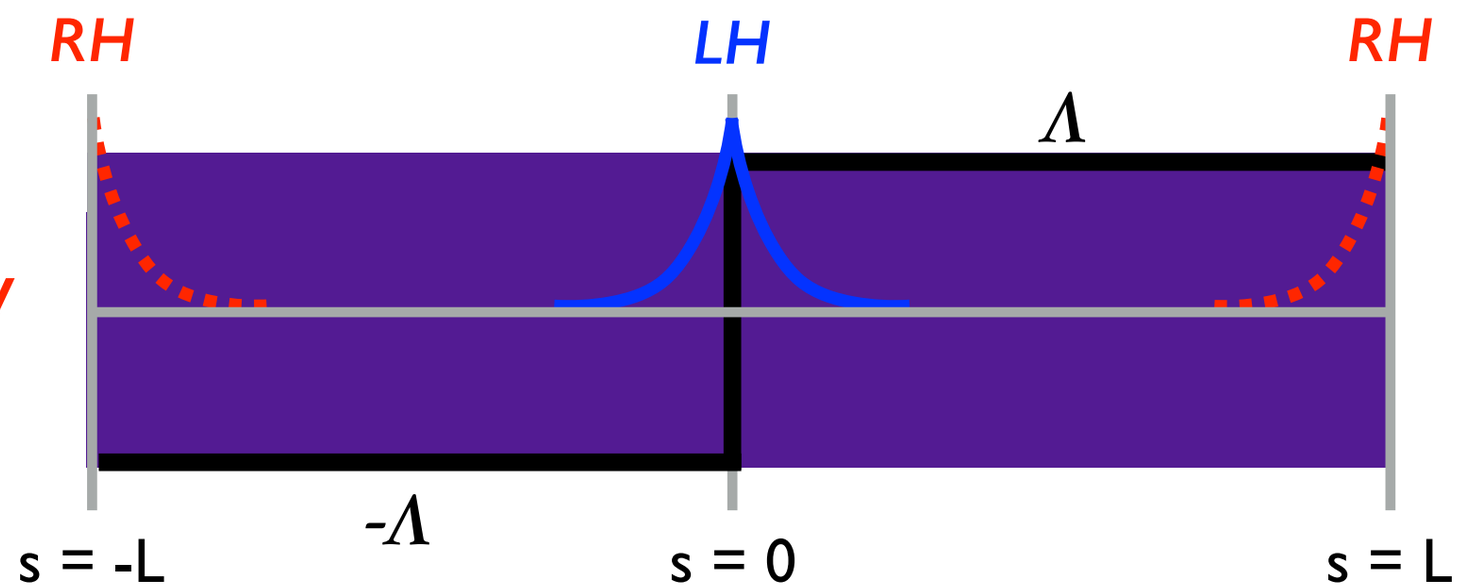
Fluffy Mirror Fermions*

Vector Theory



Idea: Localize gauge field around defect via gradient flow

*DMG & Kaplan '15

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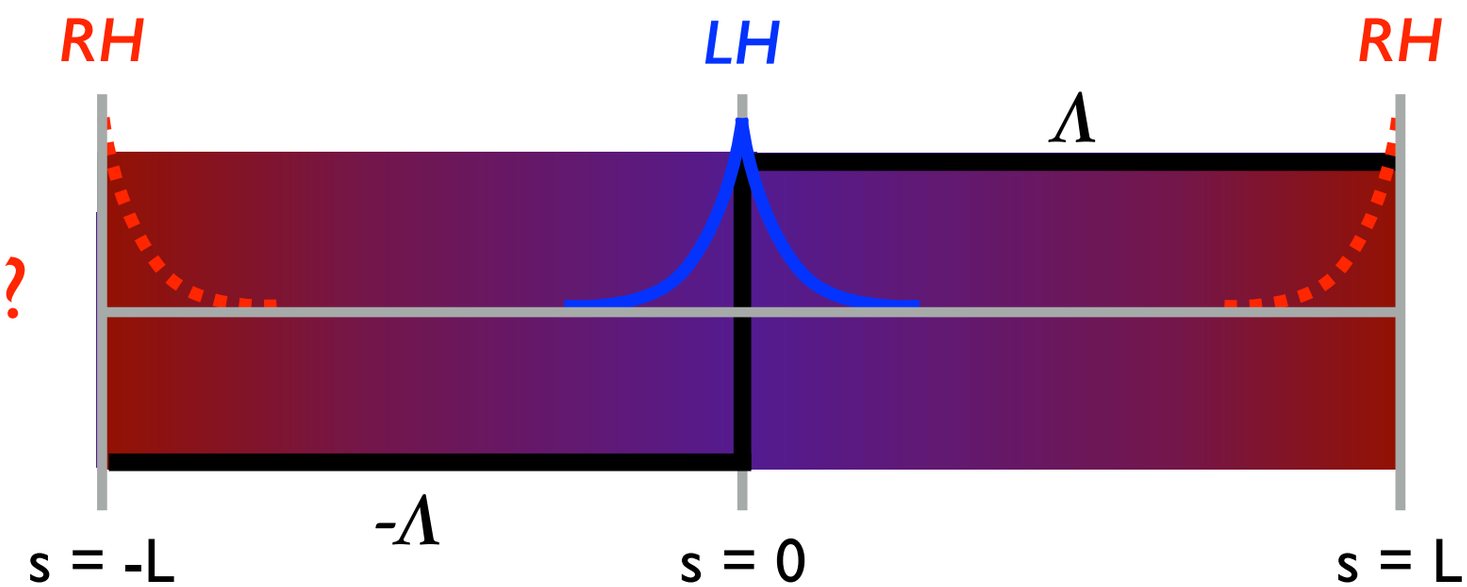

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Flow Eq: $\partial_s \mathcal{A}_\mu = \frac{\mathrm{sgn}(s)}{\Lambda} \mathcal{D}_\nu \mathcal{F}_{\nu\mu}$ *BC:* $\mathcal{A}_\mu(x,0) = A_\mu(x)$

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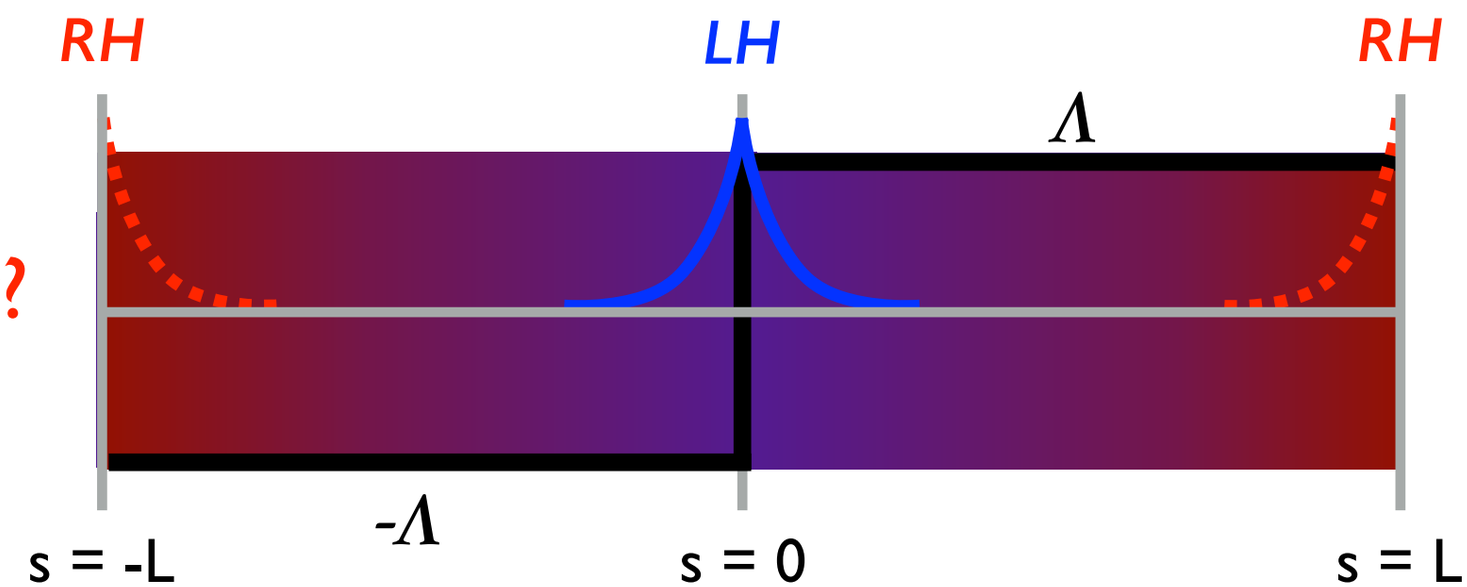

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Integration variable in path integral

Flow Eq: $\partial_s \mathcal{A}_\mu = \frac{\mathrm{sgn}(s)}{\Lambda} \mathcal{D}_\nu \mathcal{F}_{\nu\mu}$ *BC:* $\mathcal{A}_\mu(x,0) = A_\mu(x)$

- RH fermions couple to physical DoF with have soft form factor
- Both LH and RH fermions couple identically to gauge DoF

*DMG & Kaplan '15

Gradient Flow*

Ex: Two-dimensional QED

- Gauge field decomposes into gauge and physical DoF

$$\mathcal{A}_\mu = \partial_\mu \omega + \epsilon_{\mu\nu}\partial_\nu \lambda$$

*Used in LQCD (Luscher '10 etc)

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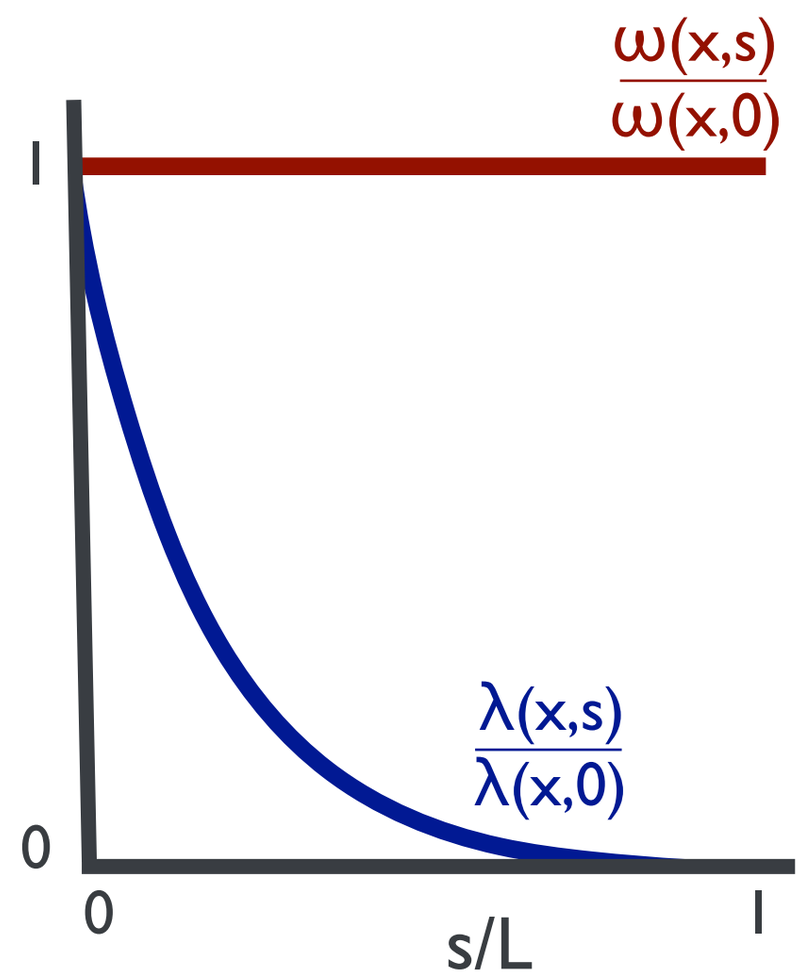
$$\partial_s \lambda = \frac{\mathrm{sgn}(s)}{\Lambda}\Box\lambda \qquad \partial_s \omega = 0$$

High momentum modes damped out

Gauge DoF unaffected



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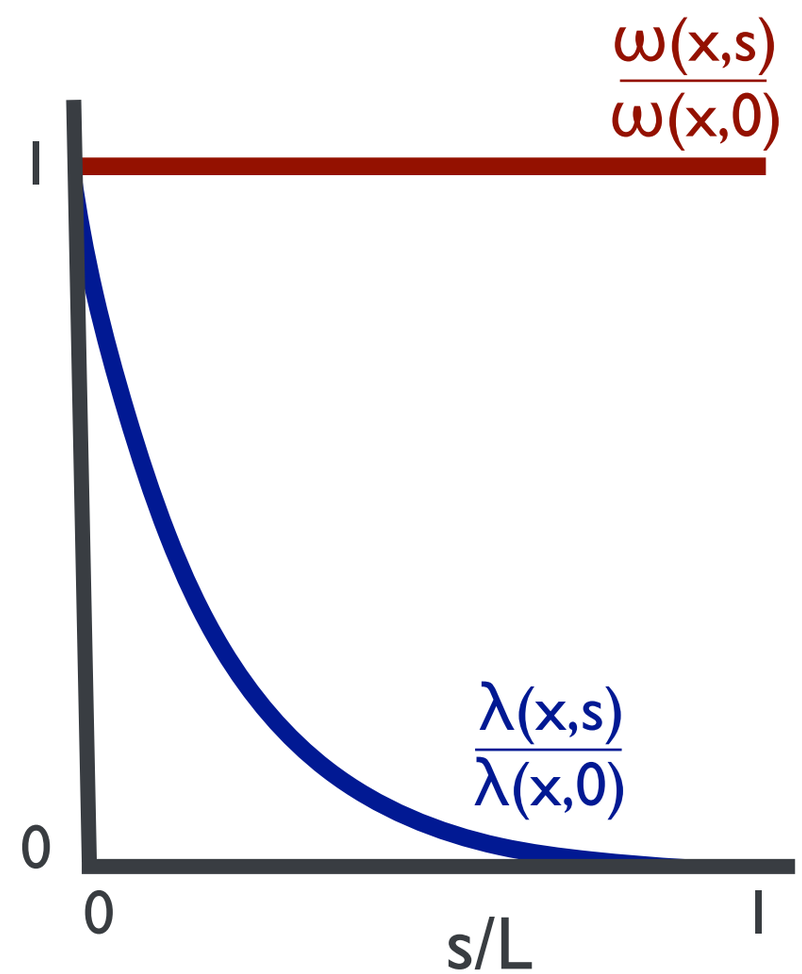

- RH couple with soft form factors

$$e^{-p^2 L/\Lambda}$$

Allows mirrors to decouple

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Callan-Harvey Mechanism Revisted

Bulk fermions **do not decouple** completely at low energy

- Generate Chern-Simons terms
- Same mechanism is responsible for $U(1)_A$ anomaly
- In 3 dimensions, the Chern Simons action is

$$S_3^{\text{bulk}} = c_3 \frac{\Lambda}{|\Lambda|} \int (\epsilon(s) - 1)\, \text{Tr}\left(\mathcal{F}\mathcal{A} - \tfrac{1}{3}\mathcal{A}^3\right)$$

CS only depends on sign of domain wall mass

Fermion Contribution

Pauli Villars Contribution

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Chern-Simons term is non-zero with flowed gauge fields

Ex: Two-dimensional QED

$$S_3^{\text{bulk}} = 2c_3 q^2 \frac{\Lambda}{|\Lambda|} \int dx^2 dy^2 \left(\frac{\partial_\mu \partial_\alpha}{\Box} A_\alpha(x) \right) \Gamma(x-y) \left(\frac{\partial_\mu \partial_\beta}{\Box} \epsilon_{\beta\gamma} A_\gamma(y) \right)$$

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- Effective two point function is nonlocal

$$\Gamma(r) = \left(\delta^2(r) - \frac{\mu^2}{4\pi} e^{-\mu^2 r^2/4} \right) \qquad \mu \equiv \sqrt{\frac{\Lambda}{\xi L}}$$

Determines speed of flow

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- When flow is turned off, Γ vanishes

Effective 2d theory is nonlocal due to Chern-Simons operator

Anomalies Cancellation

DWF with flowed gauge fields results in **nonlocal theory**

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- Multiple fermion fields give prefactor to Chern-Simons action

$$\sum_i q_i^2 \frac{\Lambda_i}{|\Lambda_i|}$$

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Prefactor depends on dimension

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Chiral fermion representations that satisfy this criteria are gauge anomaly free representations in continuum

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Are the mirror fermions truly decoupled???

Proposal

Recall: Defining chiral gauge theories requires defining $\Delta(A)$ in an unambiguous way

$$\Delta(A) = \prod_i \frac{\det\left[\not{D}(\mathcal{A}) - \Lambda_i \mathrm{sgn}(s)\right]}{\det\left[\not{D}(\mathcal{A}) - \Lambda_i\right]}$$

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Pauli Villars

Proposal

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Product over species

Proposal

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5d Dirac operator w/ flowed fields

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Decoupling the Fluff

Proposal: Decouple mirror fermions in gauge-invariant manner using soft form factors

- The Good: Theory can only be local for anomaly-free representations
- The Bad: Exponential form factors are problematic in Minkowski space
- The (Potentially) Ugly: Gradient flow does not damp out instanton configurations

Here Be Dragons

Decoupling Fluff: The Bad

Problem: Exponentially soft form factor violates unitarity under analytic continuation to Minkowski space

$$e^{-p^2 L/\Lambda}$$

Solution: Take extra dimension to be infinite*

- Gradient flow acts like a projector operator

$$P_{GF}[A^\mu(x)] = A^\mu_\star(x)$$

* Narayanan and Neuberger, '95

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Field LH Fermion Sees

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Field RH Fermion Sees

- Doing so results in the manifestly 2d or 4d overlap operator that is amenable to lattice simulations*

* DMG & Kaplan, '16
* Narayanan and Neuberger, '95

Decoupling Fluff: The (Potentially) Ugly

Continuum flow equation has **multiple** attractive fixed points, A_*

$$\partial_s \mathcal{A}_\mu = \frac{\mathrm{sgn}(s)}{\Lambda} \mathcal{D}_\nu \mathcal{F}_{\nu\mu}$$

- Gauge Degree of Freedom to maintain gauge invariance
- Topological configurations like instantons
- May result in non-extensive contributions to the action

Mirrors couple to topology if flow equation is continuous

What are the ramifications of these couplings?

Decoupling Fluff: The (Potentially) Ugly

If mirror fermions do not decouple, they are physical states and not just regularization artifacts

$$\Delta(A) \sim \begin{pmatrix} 0 & D(A)_\mu \sigma_\mu \\ D(A_\star)_\mu \bar{\sigma}_\mu & 0 \end{pmatrix}$$

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LH fermion's kinetic term

RH fermion kinetic term

- Strong CP problem: massless mirrors make θ unphysical
- Similarly nonstandard interactions with gravity (Ricci flow)

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Fundamentally different continuum limit than one would expect from perturbative chiral gauge theories

Decoupling Fluff: The Good

"Road to failure" for anomalous theories is based on non-locality

- Extra dim. theory is gauge invariant for all representations
- Bulk fermions **do not completely decouple** at low energy scales, resulting in (nonlocal) Cher-Simons current

Question 1: Does the theory correctly reproduce fermion number violating processes?

Question 2: Can this setup be used as a toolkit for the behavior of nonlocal quantum field theories?

Summary

Chiral gauge theories are extremely well-motived but on poor theoretical footing

New proposal combines domain wall fermions with gradient flow

- Extra dimension allows for naturally light fermions
- Gradient flow decouples mirrors with soft form factors
- Road to failure for anomalous representations is non-locality

Summary

Chiral gauge theories are extremely well-motived but on poor theoretical footing

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- Extra dimension allows for naturally light fermions
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Many questions, both formal and phenomenological, remain