Sharp Boundaries for the Swampland

w/ Caron-Huot, Mazac, Rastelli

$\boxed{\text{Q: Which EFT's can be UV-completed?}}$

Not everything goes:

- <u>w/out gravity</u>

$$\mathcal{L} \ni g_2 (\partial\phi)^4 + g_3 \partial^6 \phi^4 + \dots$$

$$\mathcal{M}_{low}(s,u) \ni g_2 (s^2+t^2+u^2) + g_3 stu + \dots$$

UV complete $\Rightarrow$ $g_2 \geq 0$ [Adams et. al. '06]

2 arguments:

- dispersion relation
- causality in nontrivial background

Recently: [Arkani-Hamed, Huang, Huang '20]
[Bellazzini, Miro, Rattazzi, Riembau, Riva '20]
[Tolley, Wang, Zhou '20]
[Caron-Huot, Van-Duong '20]
...

sharp bounds: $-\frac{\#}{M^2} \leq \frac{g_3}{g_2} \leq \frac{\#}{M^2}$

UV cutoff

Slogan: <u>dimensional analysis is a theorem</u>

- with gravity

[Camanho, Edelstein, Maldacena, Zhiboedov '14], ...

$$A_{ggg} = \sqrt{32\pi G}\left(A_R + \alpha_2 A_{R^2} + \alpha_4 A_{R^3}\right)$$

$$\alpha_2 \sim \frac{1}{M^2}, \quad \alpha_4 \sim \frac{1}{M^4}$$

also : nonzero $\alpha_2, \alpha_4 \Rightarrow \infty$ tower of massive spinning particles

- using causality
- only parametric bounds

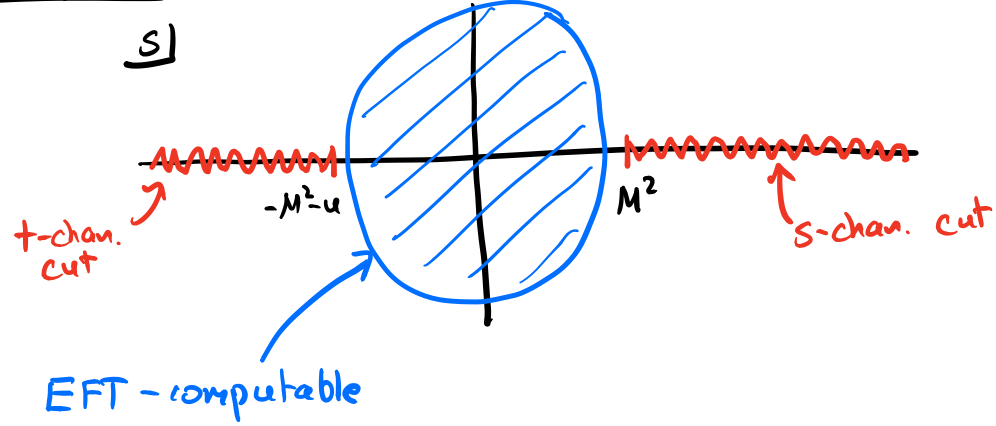
This talk: Sharp bounds with gravity

Key Assumptions + Dispersion Review

[1. Analyticity
2. Unitarity
3. Regge bound]

Consider $\phi\phi \to \phi\phi$ $\quad \mathcal{M}(s,u)$

1. Analyticity



simplicity : tree-level

$$\mathcal{M}_{low}(s,u) = 8\pi G\left(\frac{st}{u} + \frac{su}{t} + \frac{tu}{s}\right) - \lambda_3^2\left(\frac{1}{s} + \frac{1}{t} + \frac{1}{u}\right) - \lambda_4$$

$$+ g_2 (s^2+t^2+u^2) + g_3 stu + g_4 (s^2+t^2+u^2)^2$$

$$+ \ldots$$

- could include loops
  - nonlinear map between $\mathcal{L}$ and $\mathcal{M}_{low}$
  - we bound obs. linear in $\mathcal{M}_{low}$

2. Unitarity

partial wave decomp:

$$\mathcal{M}(s,u) = s^{\frac{4-D}{2}} \sum_{J \text{ even}} n_J \, c_J(s) \, P_J\left(1+\frac{2u}{s}\right)$$

P_J: Gegenbauer (Legendre in D=4)

$1+\frac{2u}{s}$: $\cos\theta$ scattering angle

$$SS^\dagger = \mathbb{1}, \quad S = \mathbb{1} + i\mathcal{M}$$

$$\Rightarrow \ |1+ic_J(s)|^2 \le 1$$

$$\rho_J(s) \equiv \operatorname{Im} c_J(s)$$

$$\Rightarrow \ \underbrace{0 \le \rho_J(s)}_{\text{this talk}} \le 2 \qquad (s>0,\ J \text{ even})$$

3. Bounded Regge Growth (spin 2)

$$\lim_{|s|\to\infty} \frac{\mathcal{M}(s,u)}{s^2} = 0 \qquad (u<0)$$

define spin-k-subtracted disp. relation

$$\mathcal{C}_{k,u}: \quad 0 = \oint_{\infty} \frac{ds}{s} \frac{\mathcal{M}(s,u)}{(s(s+u))^{k/2}}$$

boundedness assumption $\Rightarrow \mathcal{C}_{k,u}$ converges for $k = 2, 4, \ldots$

Comments

- Satisfied in weakly-coupled string theory

$$\mathcal{M} \sim s^{2+\alpha' u}$$

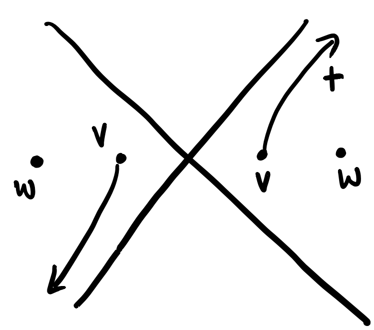
Reggeization of graviton

- $\mathcal{M}(s,u) \leq O(s^2)$ is the "Classical Regge Growth" conjecture

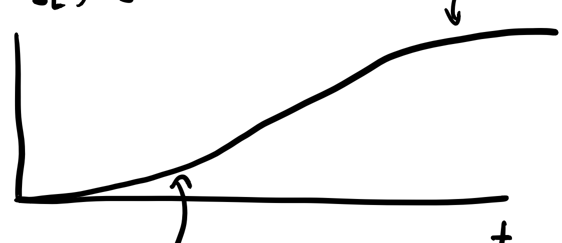
[Camanho, Edelstein, Maldacena, Zhiboedov '14]
[Maldacena, Shenker, Stanford '15]
[Chandorkar, Chowdhury, Kundu, Minwalla '21]

We actually need something stronger $\Rightarrow$ Rigorously provable in AdS

Aside: AdS/CFT + chaos



$\langle [V,W][V,W] \rangle \sim$



$\frac{1}{N^2} e^{(J_{eff}-1)t}$

chaos bound: $J_{eff} \leq 2$

spin-k dispersion relation

$$\sim \; (\ldots) = \int_0^\infty dt \, e^{-(k-1)t} \langle [V,W][V,W] \rangle$$

2 examples of convergence:

- $J_{eff} < 2 \Rightarrow \int_0^\infty dt \, e^{(J_{eff}-k)t} < \infty$

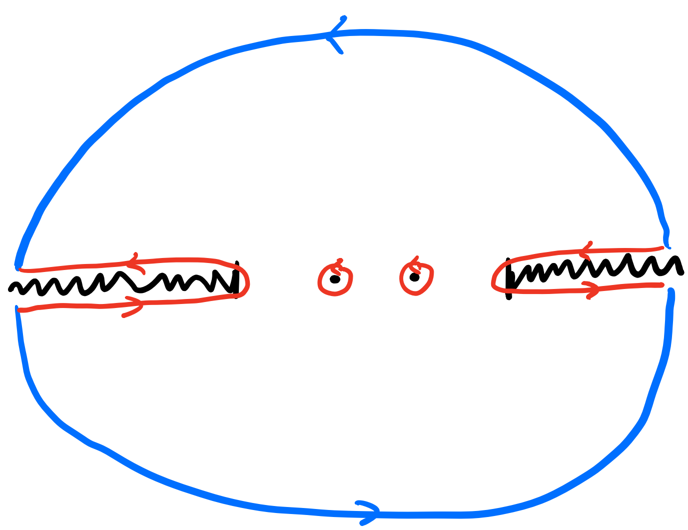
  (weakly-coupled string theory)

- saturation $\Rightarrow \int_0^\infty dt \, e^{(1-k)t} < \infty$

(Upcoming work [w/ Caron-Huot, Mazac, Rastelli]
flat-space limit of CFT dispersion relations
$\Rightarrow$ this talk)

Dispersion Relations: Applications

$0 =$



$$\mathcal{C}_{2,u}: \frac{8\pi G}{-u} + 2g_2 - g_3 u + 8 g_4 u^2 + \ldots = \left\langle \frac{(2m^2+u) P_J\left(1+\frac{2u}{m^2}\right)}{m^2(m^2+u)^2} \right\rangle$$

$$C_{4,u} : 4g_4 + \ldots \qquad = \left\langle \frac{(2m^2+u)\, P_J\left(1+\frac{2u}{m^2}\right)}{m^4\,(m^2+u)^3} \right\rangle$$

$$\text{where} \quad \langle \ldots \rangle = \frac{1}{\pi} \sum_J n_J \int_{M^2}^{\infty} \frac{dm^2}{m^2}\, m^{4-D}\, \rho_J(m^2)\, (\ldots)$$

Review: Adams et al. $\quad 8\pi G = 0$

$$C_{2,u=0} : 2g_2 = \left\langle \frac{2}{m^4} \right\rangle \geq 0$$

$\uparrow$ forward limit $\quad \cos\theta = 1 \quad P_J(1) = 1$

Now turn on G

problem: $\quad \left.\frac{8\pi G}{-u}\right|_{u=0} = \infty$

useful to view problem in impact param. space

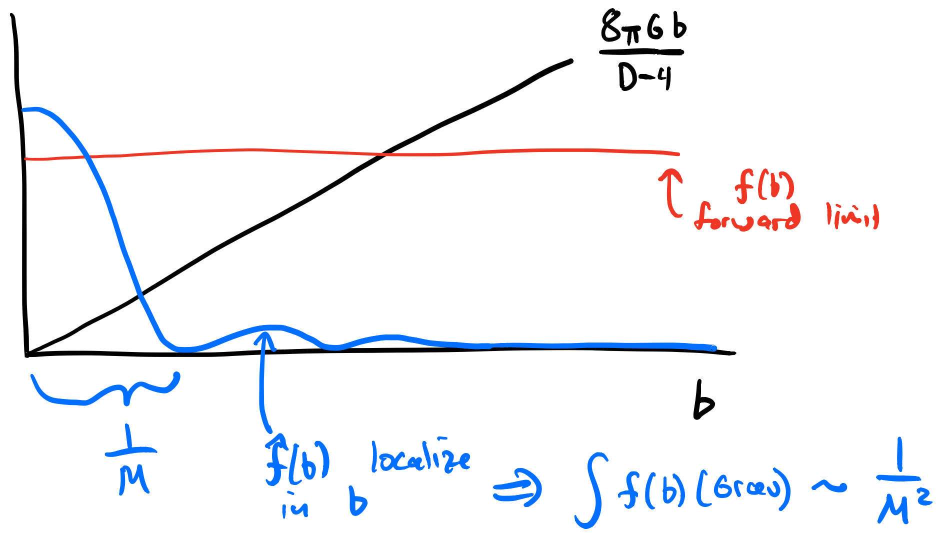
$u = -p^2 \qquad p = |\vec{p}| \qquad b = |\vec{b}|$

p: transv. momentum transfer $\qquad b$: impact param

$$\int_0^\infty dp\, f(p)\, \frac{8\pi G}{p^2} = \int_0^\infty db\, \hat{f}(b)\, \frac{8\pi G b}{D-4}$$

$$\hat{f}(b) \equiv \int d^{D-2}\vec{p}\; e^{i\vec{b}\cdot\vec{p}}\, \frac{f(p)}{p^{D-3}\,\mathrm{vol}\, S^{D-3}}$$

forward limit : $f(p) = \delta(p)$, $\hat{f}(b) = 1$



localize in b: need $|u| \sim M^2$

worry : EFT series breaks down

solution: "improved" sum rules

$$\mathcal{C}^{\text{improved}}_{2,u} = C_{2,u} - \big(\underset{\substack{\uparrow \\ \text{cancels } 8g_4u^2}}{2u^2 C_{4,0}} + \underset{\substack{\uparrow \\ \text{cancels } g_6}}{u^3 C'_{4,0}} + \ldots\big)$$

$$\mathcal{C}^{\text{imp.}}_{2,u}: \quad \frac{8\pi G}{-u} + 2g_2 - g_3 u = \left\langle \frac{(2m^2+u)\,P_J\!\left(1+\frac{2u}{m^2}\right)}{m^2(m^2+u)^2} - \frac{u^2}{m^6}\left(\frac{(4m^2+3u)\,P_J(1)}{(m^2+u)^2} + \frac{P_J'(1)}{mu}\right)\right\rangle$$

$$= \left\langle C^{\text{imp.}}_{2,u}[m^2, J] \right\rangle$$

valid for $u = -p^2$, $p \in (0, M)$

Now: linear programming bounds

search for $f(p)$ s.t.

$$\int_0^M dp\, f(p)\, \mathcal{C}^{\text{imp}}_{2,-p^2}[m^2, J] \geq 0 \quad \text{for all } m > M,\ J = 0, 2, 4, \ldots$$

$$\Rightarrow \int_0^M dp\, f(p) \left[\frac{8\pi G}{p^2} + 2g_2 + g_3 p^2 \right] \geq 0$$

(can choose $f(p)$ to optimize different things)

<u>Intuition</u> scaling limit $m \to \infty$ w/ $b_{\text{eff}} = \frac{2J}{m}$ held fixed

$$\int_0^M dp\, f(p)\, \mathcal{C}^{\text{imp}}_{2,-p^2}[m^2, J] \sim \frac{\#}{m^4} \hat{f}(b_{\text{eff}})$$

$\Rightarrow$ bounds from
- positive $\hat{f}(b)$
- cpct. support $f(p)$

$$\left[\text{example: } \int_0^M \frac{dp}{M} \left(1 - \frac{p}{M}\right) \cos(pb) = \frac{1 - \cos(Mb)}{(Mb)^2} \geq 0 \right]$$

<u>In practice</u>

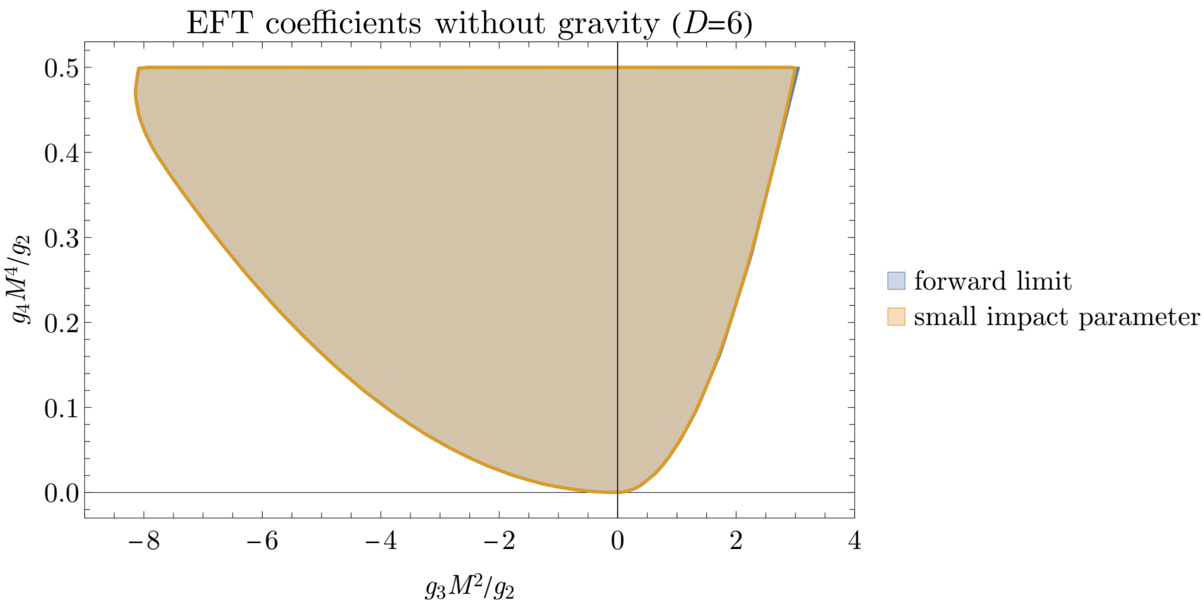
write: $f(p) = \sum_n a_n p^n$

impose positivity:
- for $J = 0, 2, \ldots J_{\max}$
- also in b-space

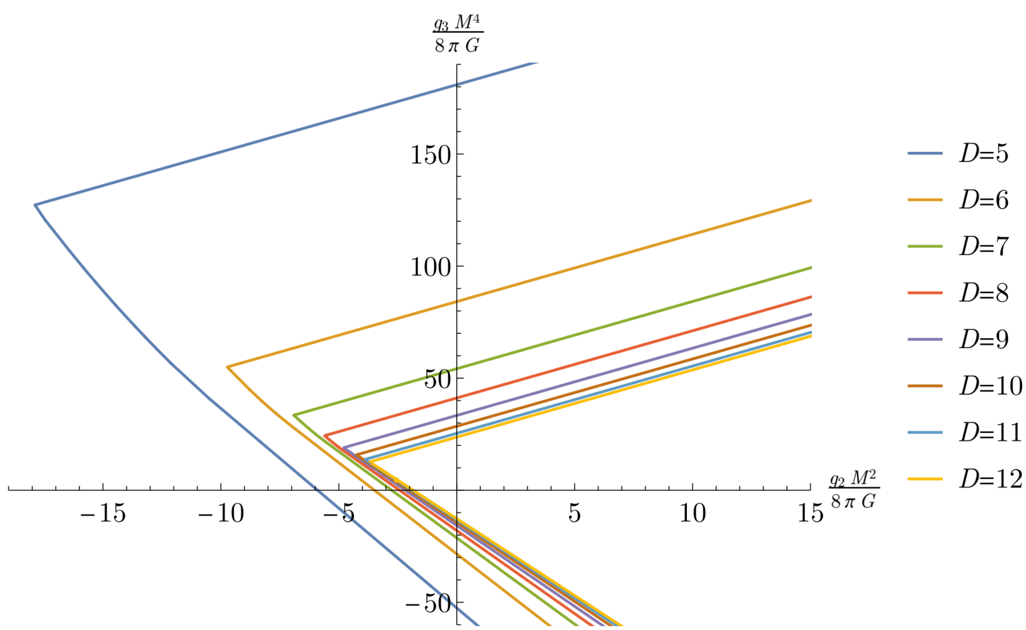
solve w/ SDPB [DSD '15], [Landry, DSD '19]

Results

no gravity: reproduces same results as forward limit



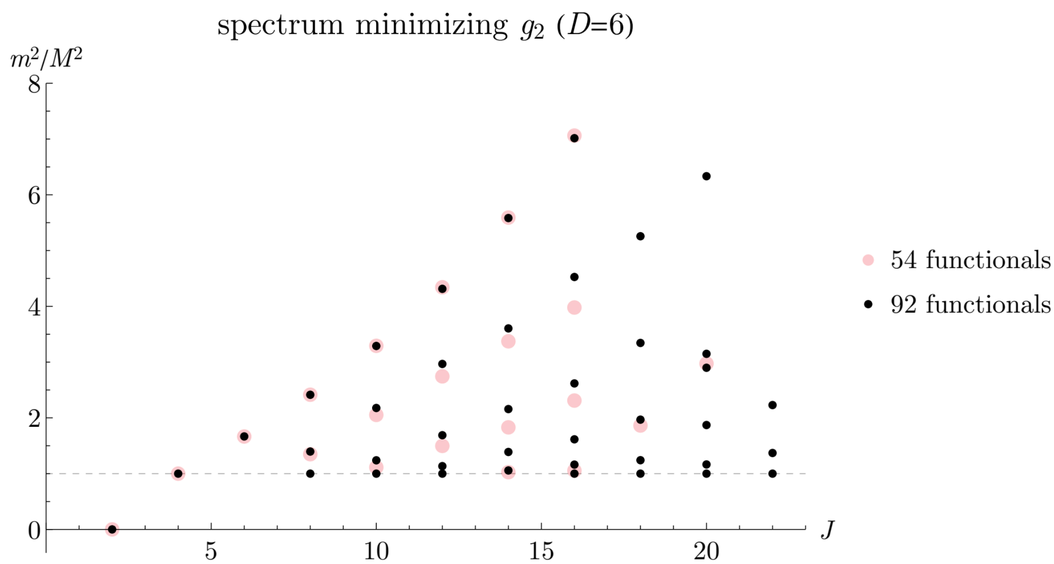
New: bounds with gravity



Note: g_2 can be negative, but has a lower bound.

(intuition: negative g_2 gives time advance, but this is ok because gravity gives a time delay)

"Extremal" spectrum



(does not look like string theory)

D=4 + IR divergences

$$\int db\, \hat{f}(b)\, \frac{8\pi G b}{D-4}$$

impact param amplitude for grav. diverges when $D=4$

$$g_2 \gtrsim -\frac{8\pi G}{M^2} \times 17 \log(1.7\, M\, b_{max}) \qquad (D=4)$$

IR cutoff