From Gluon Scattering to Black Hole Orbits

Clifford Cheung

CC, Rothstein, Solon (1808.02489, *PRL* 121 251101)
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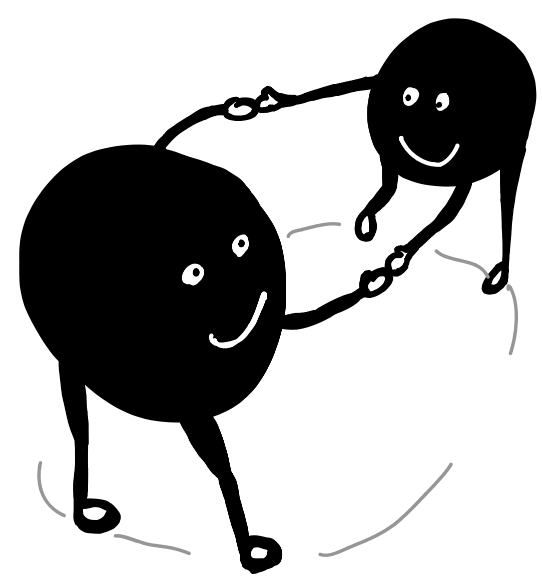

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scattering amplitudes in a nutshell

principles → action → observables

Poincare invariance, spacetime locality, etc.

↓

parameters not fixed by symmetry are couplings

↓

calculate via Feynman diagrams

scattering amplitudes in a nutshell

principles

~~action~~

observables

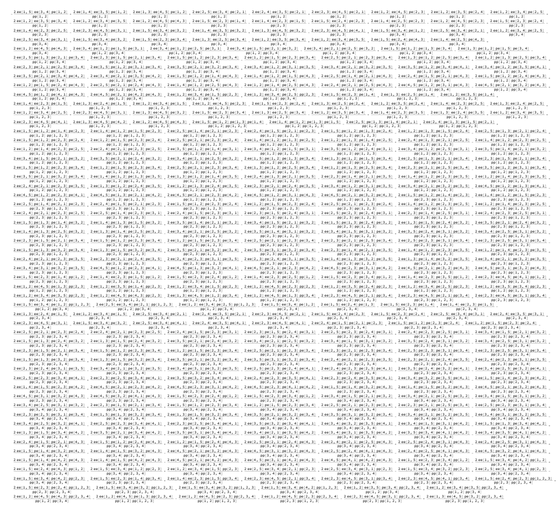
modern scattering amplitudes program

Poincare invariance, spacetime locality, etc.

parameters not fixed by symmetry are couplings

calculate via Feynman diagrams

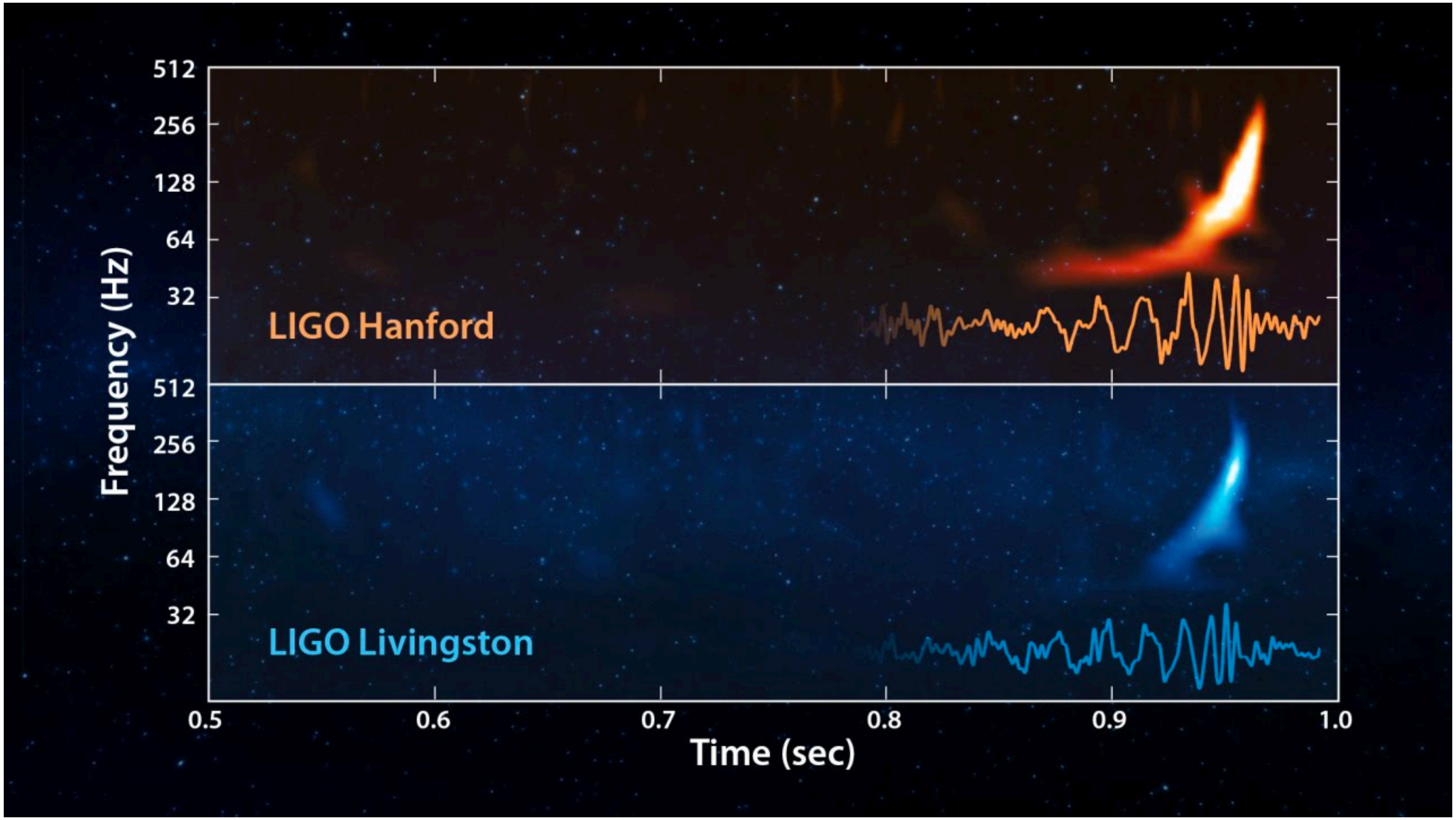
Amplitudes have revealed hidden simplicity

$A_{\mathrm{YM}} =$  $=$ 

and hidden structure linking gauge + gravity.

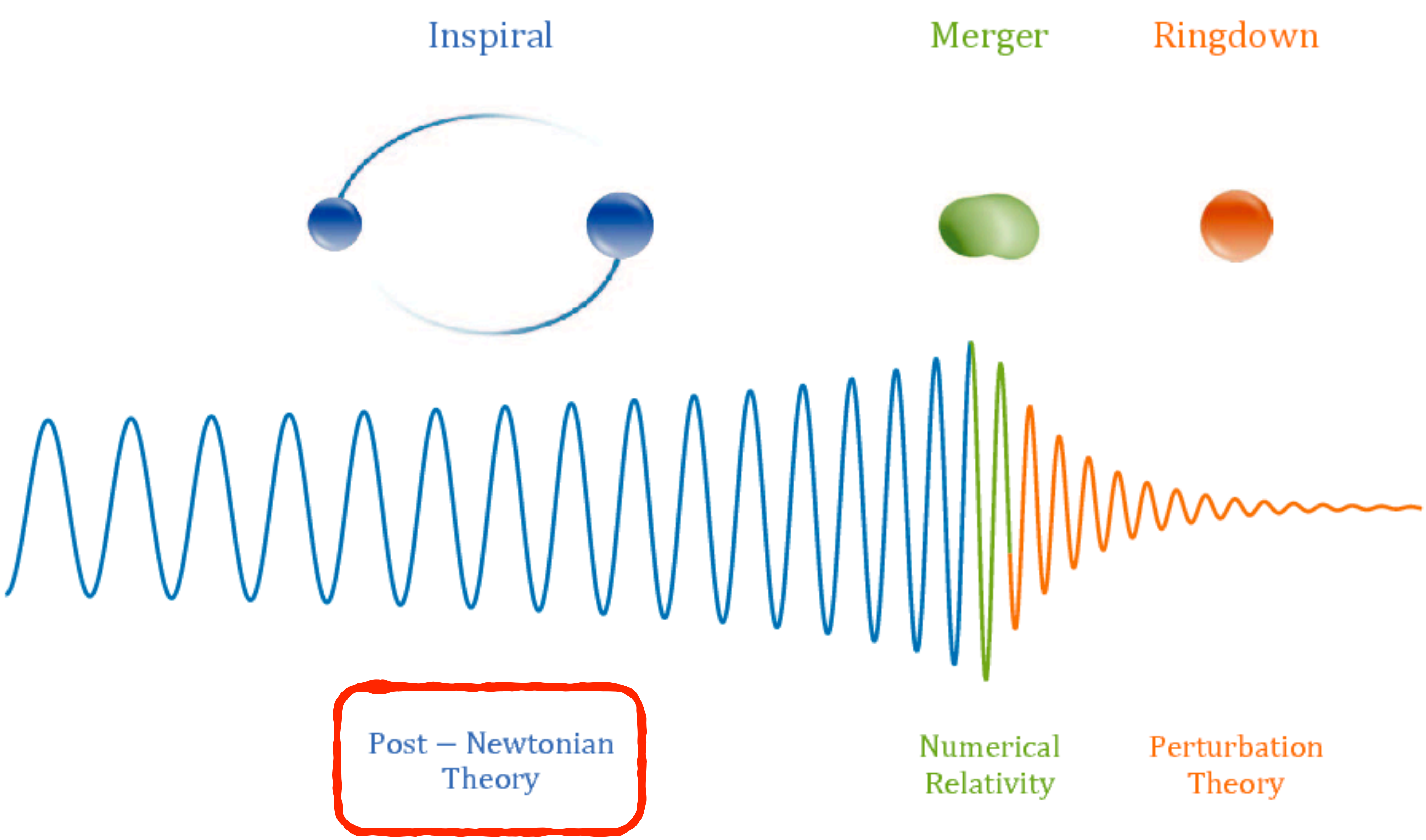
$$A_{\mathrm{YM}} \otimes A_{\mathrm{YM}} = A_{\mathrm{GR}}$$

Can we leverage these “miracles” for LIGO?



Yes! After ~1 yr we have new results for GR.

A black hole binary merger has three phases.



I will focus on the conservative potential

(figure from 1610.03567)

The post-Newtonian (PN) approximation is an expansion in powers of

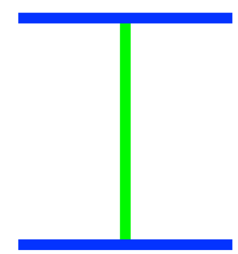
virial theorem

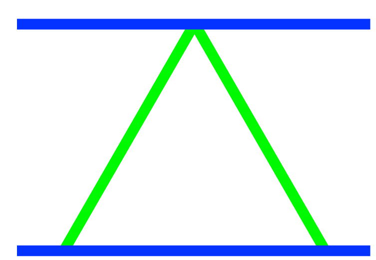
$$v^2 \sim \frac{GM}{r} \ll 1$$

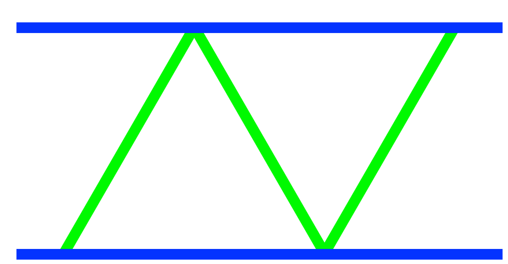
which are tiny and perturbatively calculable during the inspiral phase.

The post-Minkowskian (PM) expansion is an expansion purely in powers of G.

Here n-PN requires n-loops since $GM^2 \gg 1$.

$A(p,q) \sim \Big\{$  $\Big\}$ $G(1+v^2+\cdots)$

$+ \Big\{$  $+\cdots \Big\}$ $G^2(1+v^2+\cdots)$

$+ \Big\{$  $+\cdots \Big\}$ $G^3(1+v^2+\cdots)$

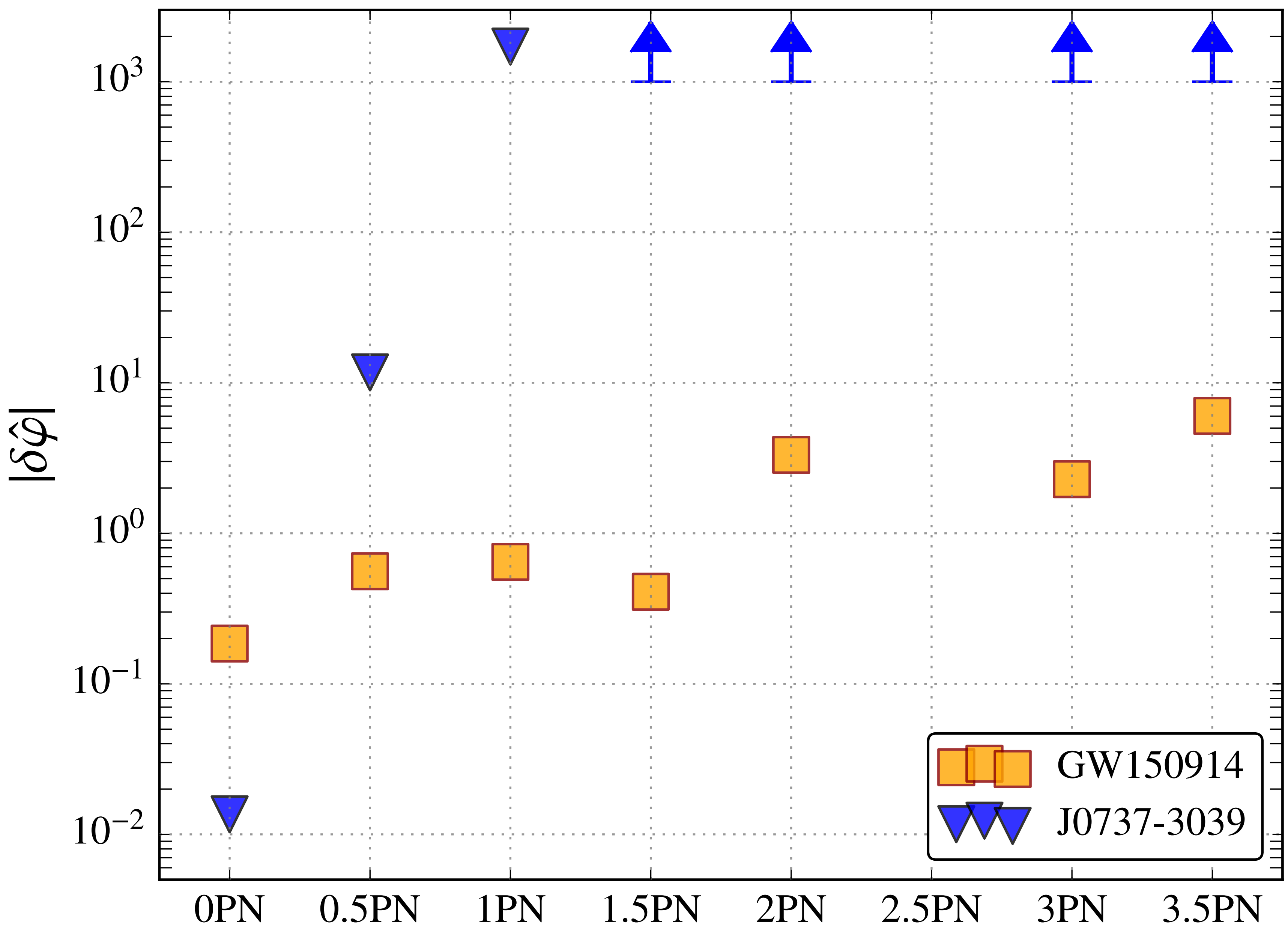
Perturbativity holds since $GMq \sim GM/r \ll 1$.

map of perturbation theory

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$\big(1$	$+\, v^2$	$+\, v^4$	$+\, v^6$	$+\, v^8$	$+\, v^{10}$	$+\, v^{12}$	$+\, v^{14}$	$+\cdots\big)\, G$
2PM		$\big(1$	$+\, v^2$	$+\, v^4$	$+\, v^6$	$+\, v^8$	$+\, v^{10}$	$+\, v^{12}$	$+\cdots\big)\, G^2$
3PM			$\big(1$	$+\, v^2$	$+\, v^4$	$+\, v^6$	$+\, v^8$	$+\, v^{10}$	$+\cdots\big)\, G^3$
4PM				$\big(1$	$+\, v^2$	$+\, v^4$	$+\, v^6$	$+\, v^8$	$+\cdots\big)\, G^4$
5PM					$\big(1$	$+\, v^2$	$+\, v^4$	$+\, v^6$	$+\cdots\big)\, G^5$

$$\vdots$$

LIGO will continue to test PN corrections.



"Tests of general relativity with GW150914" (1602.03841)

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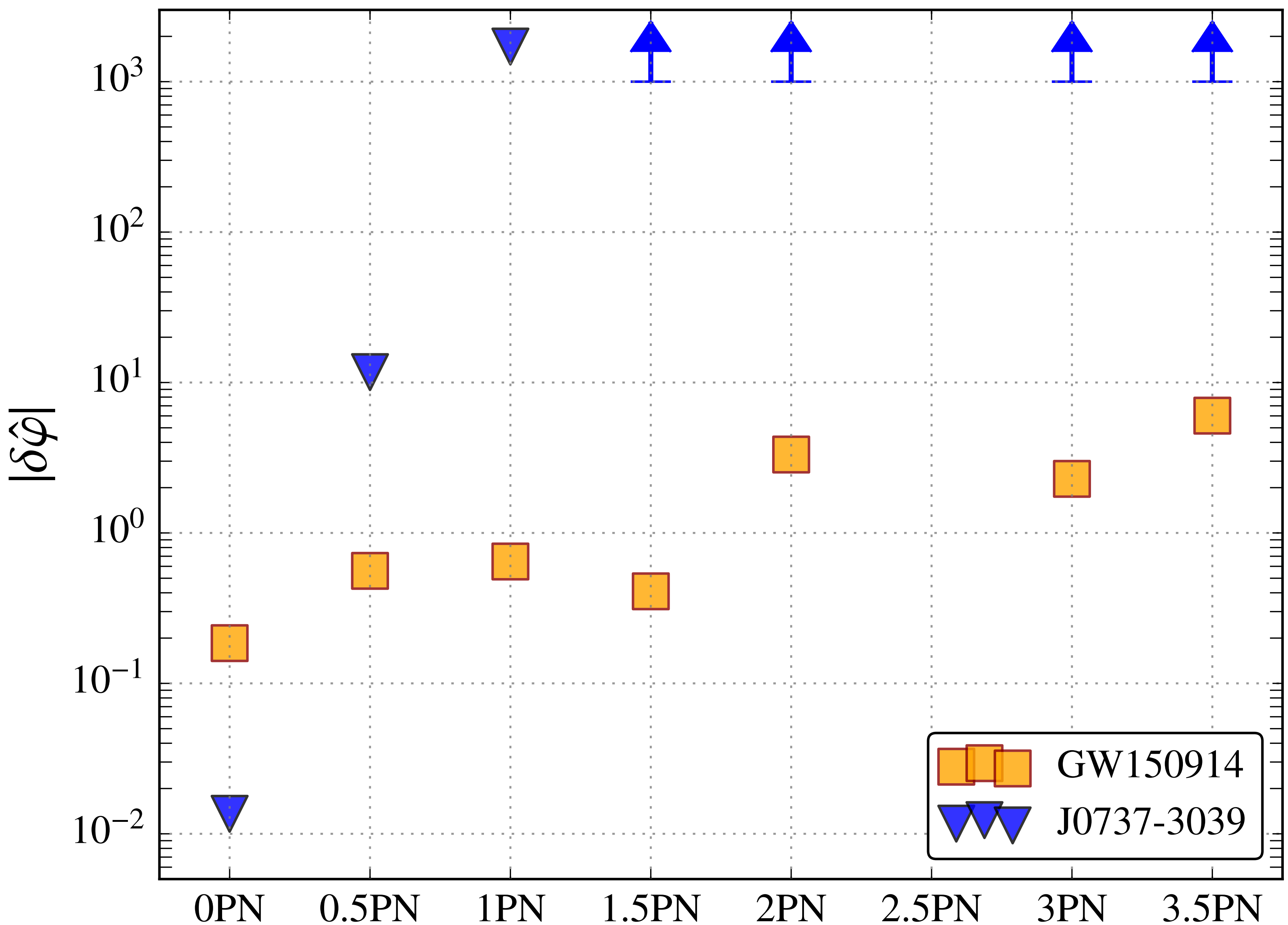

Newton (1686)

Einstein (1938)

Ohta (1974) Damour (1981)

Blanchet (2000) Damour (2000)

Blanchet (2015) Damour (2014)

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							$\vdots$		

map of perturbation theory

0PN 1PN 2PN 3PN 4PN 5PN 6PN 7PN

1PM $\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + v^{14} + \cdots\right) G$

2PM $\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots\right) G^2$

3PM $\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots\right) G^3$

4PM $\left(1 + v^2 + v^4 + v^6 + v^8 + \cdots\right) G^4$

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- evaluating high loop integrals = hard

  → but potential is *simple* so the integrals should be too!

full theory effective theory

full theory | effective theory

amplitudes methods ↓

BH / graviton tree amplitudes: A_{tree}

generalized unitarity ↓

integral representation: $$A = \sum_i d^{(i)} I^{(i)}$$

multi-loop integration ↓

full loop amplitude: $A(p, q)$

full theory		effective theory	
	amplitudes methods ↓	build ansatz ↓	
BH / graviton tree amplitudes	A_{tree}	$V(p,q)$	effective BH Lagrangian
	generalized unitarity ↓	Feynman diagrams ↓	
integral representation	$A = \sum_i d^{(i)} I^{(i)}$	$A_{\text{EFT}} = \sum_i d^{(i)}_{\text{EFT}} I^{(i)}$	integral representation
	multi-loop integration ↓	multi-loop integration ↓	
full loop amplitude	$A(p,q)$	$A_{\text{EFT}}(p,q)$	EFT loop amplitude

full theory	effective theory
amplitudes methods ↓	build ansatz ↓
BH / graviton tree amplitudes: A_{tree}	effective BH Lagrangian: $V(p,q)$
generalized unitarity ↓	Feynman diagrams ↓
integral representation: $A = \sum_i d^{(i)} I^{(i)}$	integral representation: $A_{\text{EFT}} = \sum_i d^{(i)}_{\text{EFT}} I^{(i)}$
multi-loop integration ↓	multi-loop integration ↓
full loop amplitude: $A(p,q)$	EFT loop amplitude: $A_{\text{EFT}}(p,q)$

identical physics

$$A(p,q) = A_{\text{EFT}}(p,q)$$

I will present our results at 2PM and 3PM, focusing on certain aspects of:

- calculating the scattering amplitudes

  (methods for integrands and integration)

- extracting the conservative potential

  (EFT matching at all orders in velocity)

- analyzing the results for consistency

  (old and new diagnostics on the final answer)

map of perturbation theory

0PN 1PN 2PN 3PN 4PN 5PN 6PN 7PN

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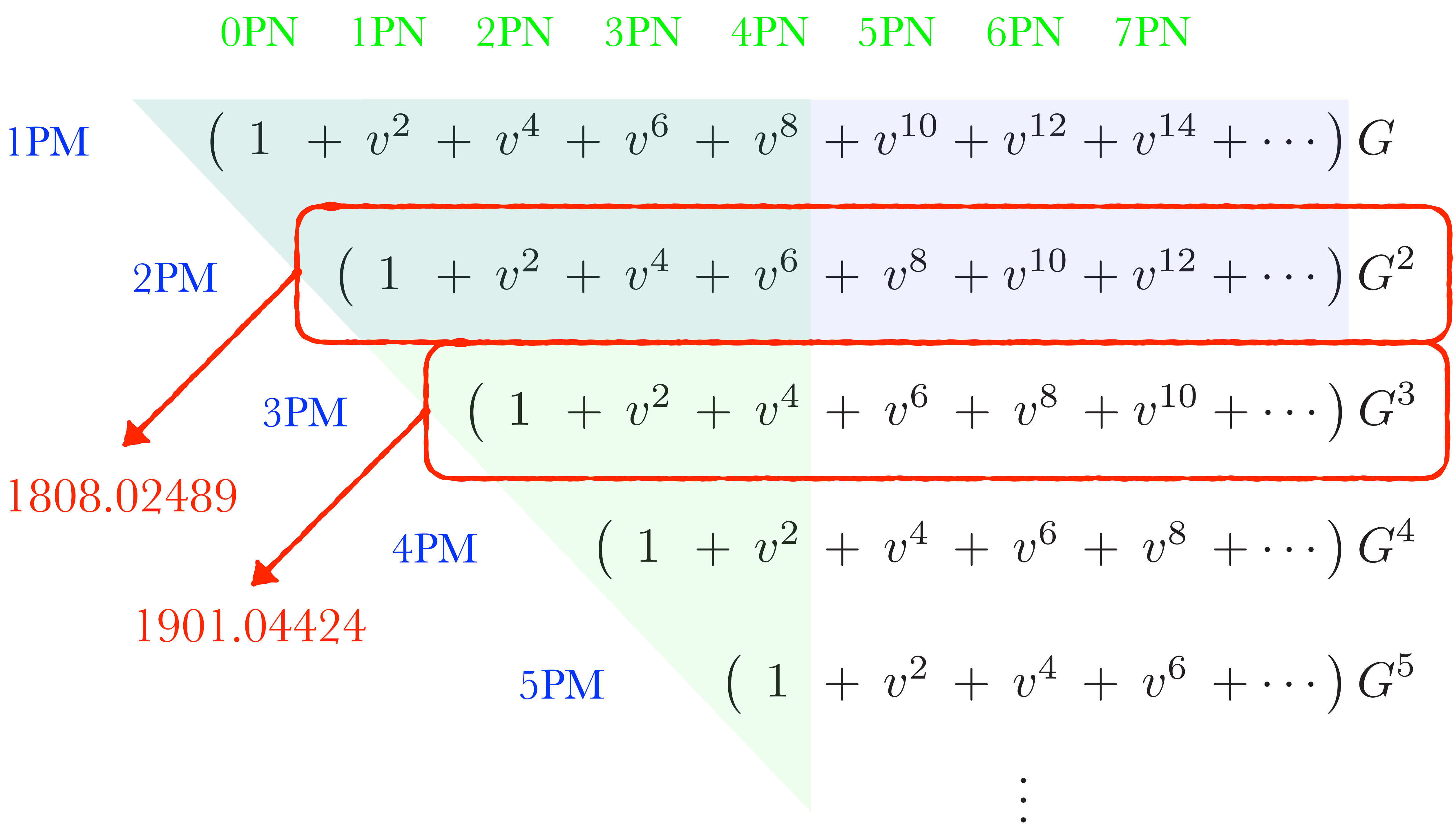
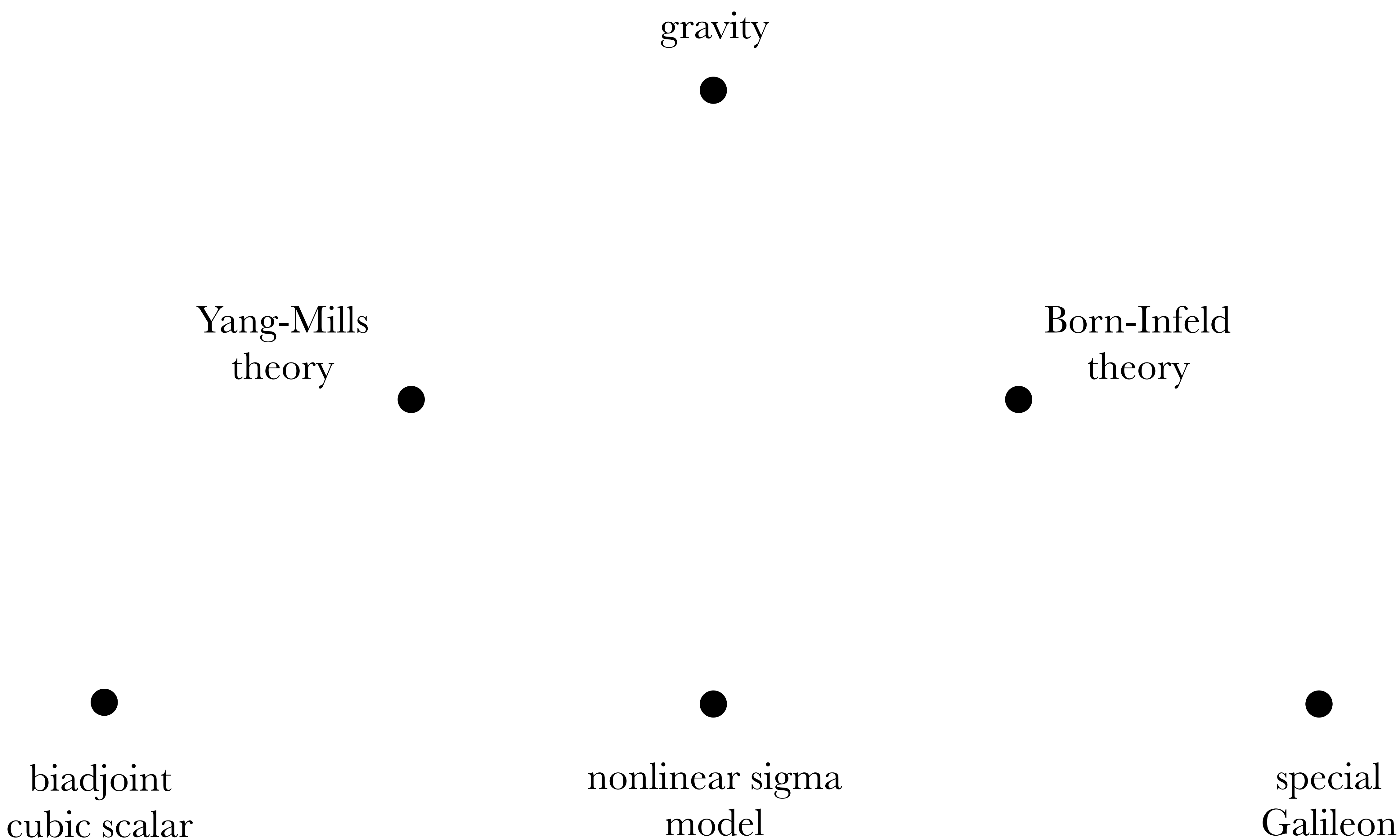
map of perturbation theory



i) calculating the amplitude

Amplitudes link and unify disparate theories.



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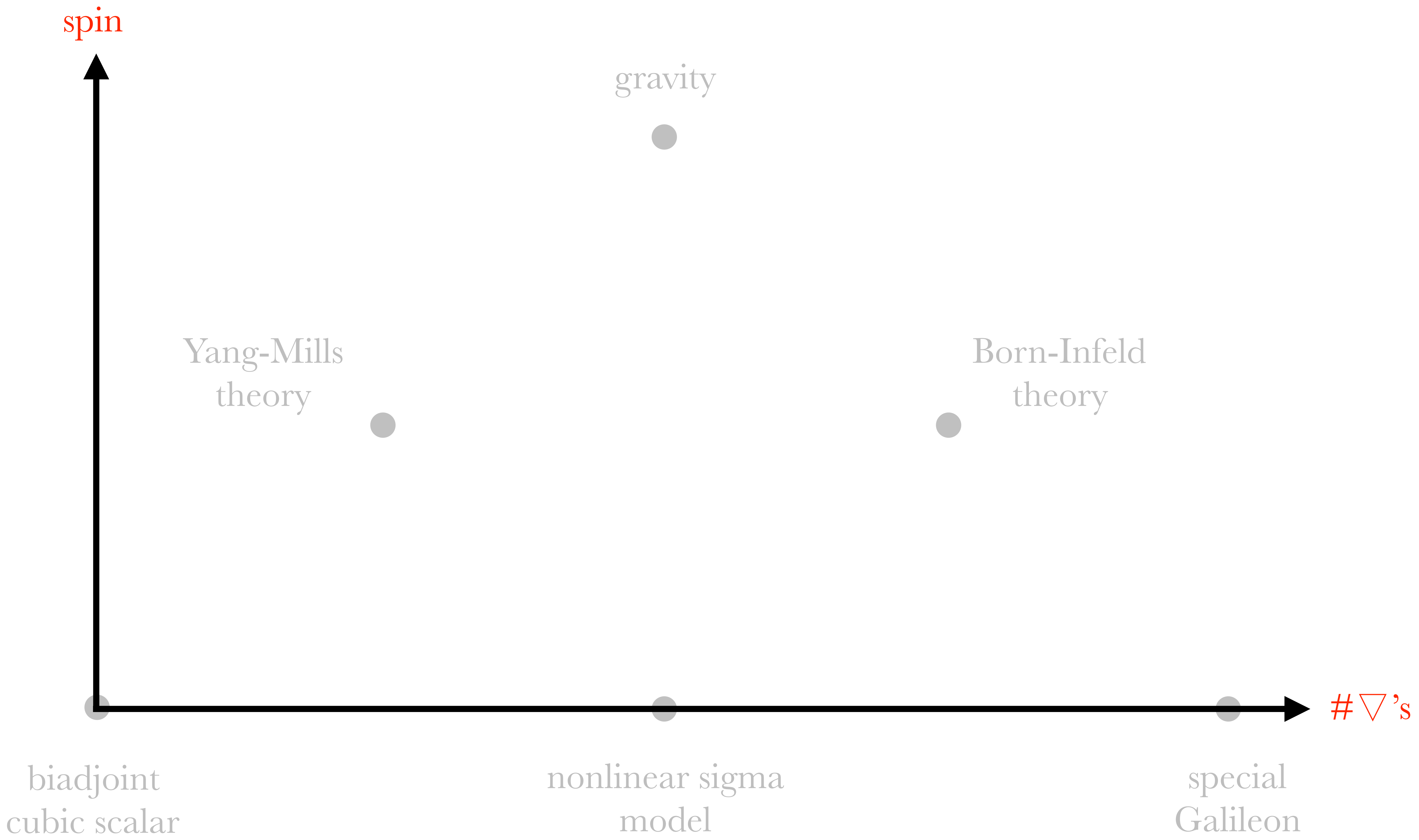

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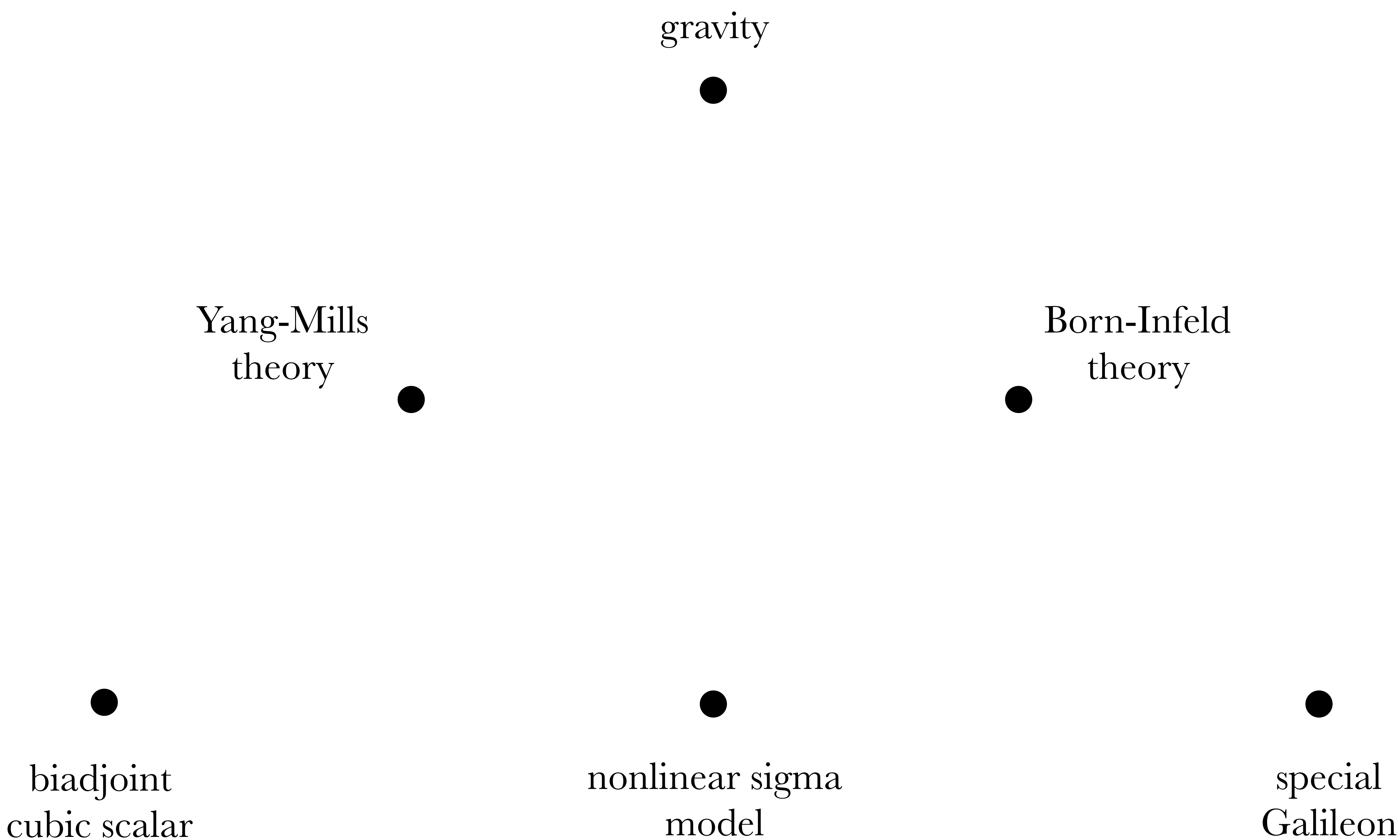

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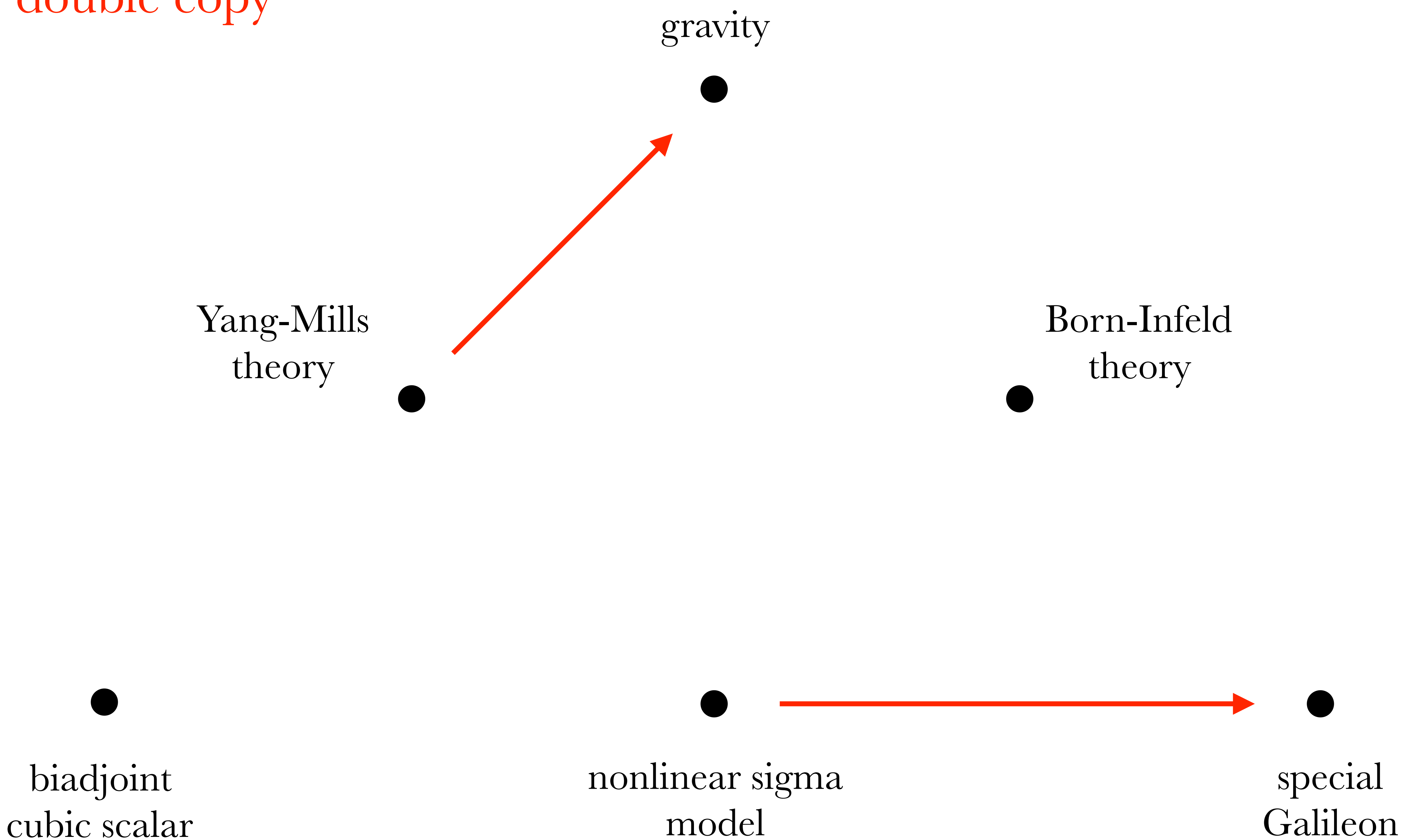
i) double copy



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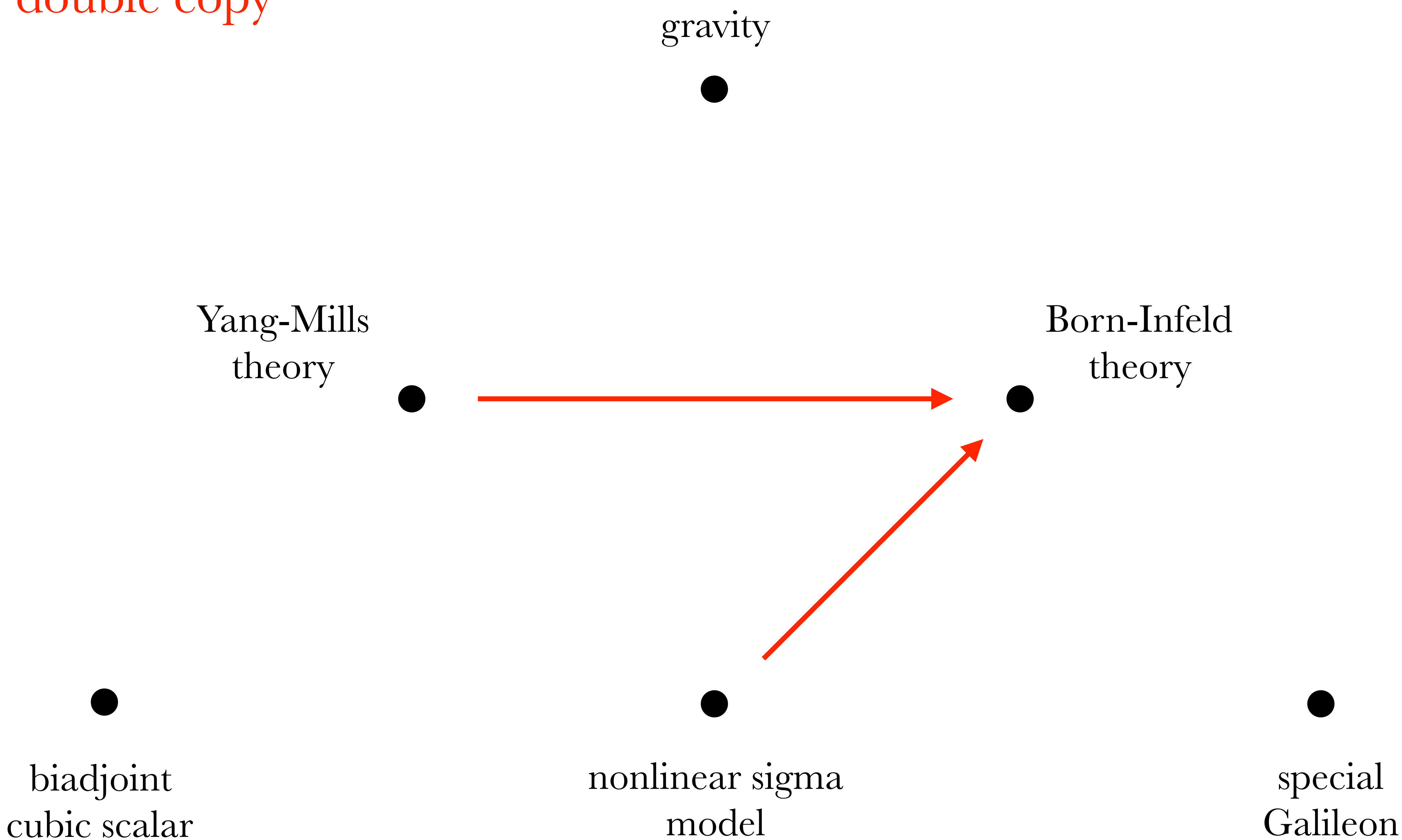

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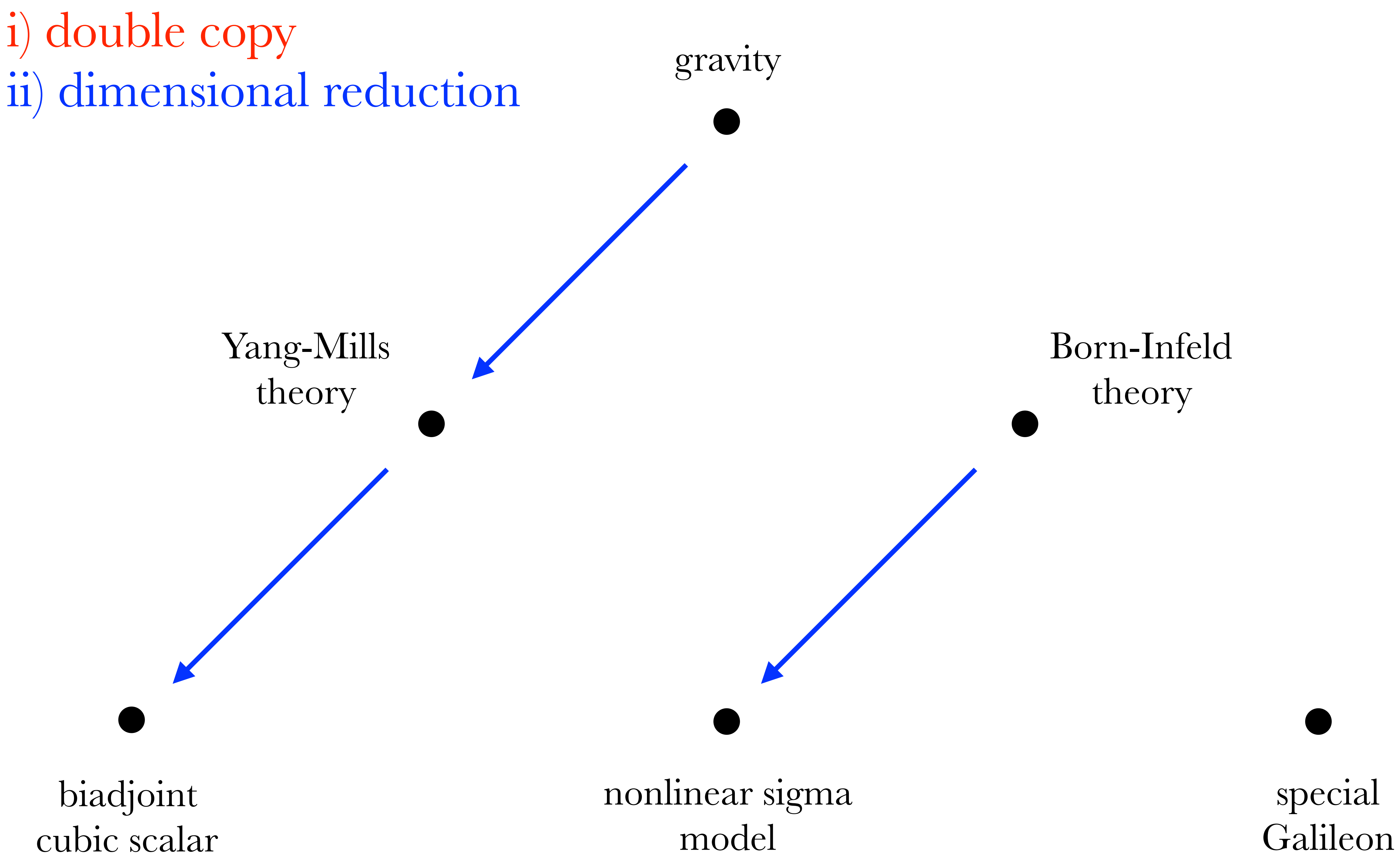

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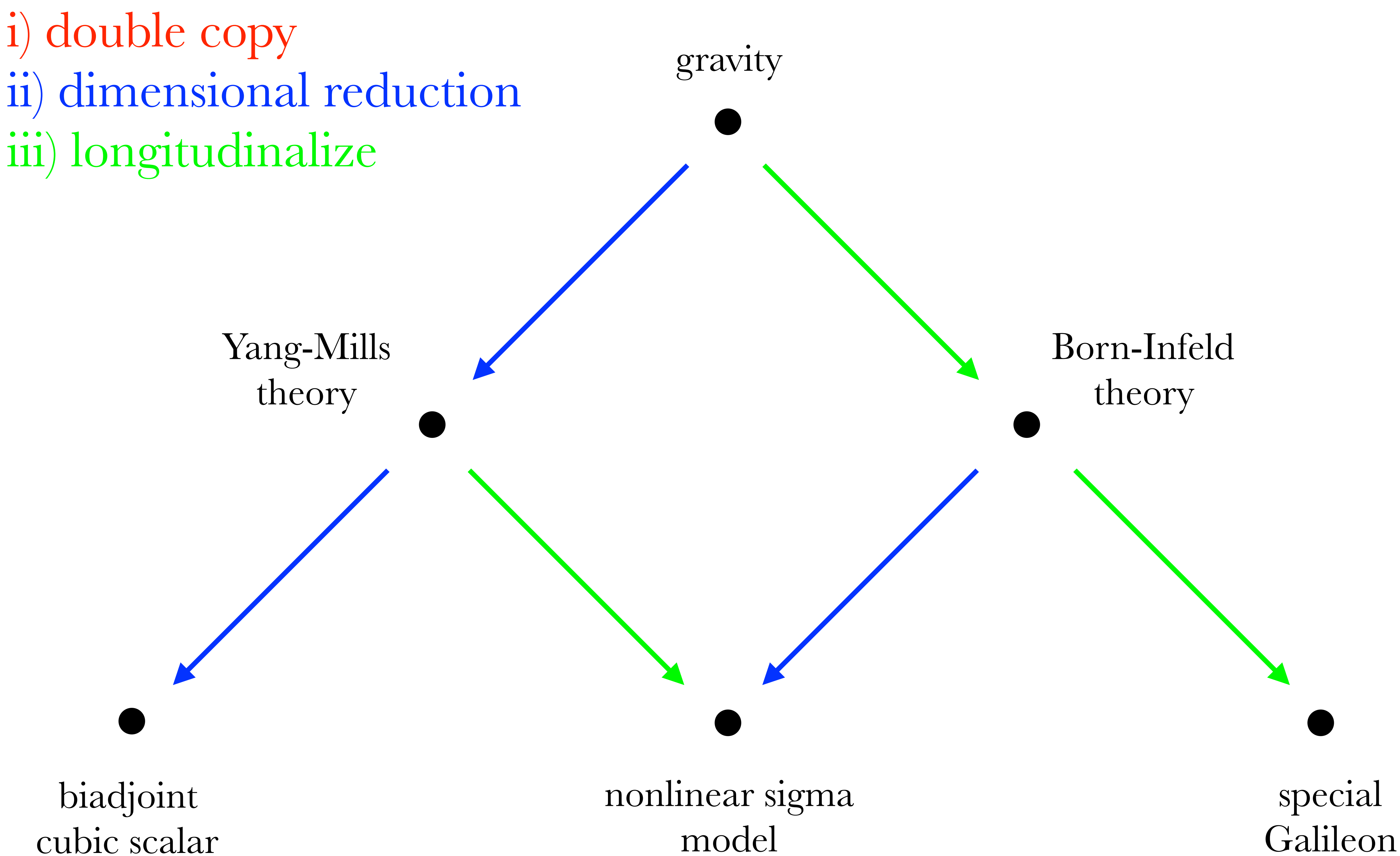


First, recast gluon tree amplitudes as abstract functions of kinematic invariants.

$$\begin{aligned} A &= e_1^{\mu_1} e_2^{\mu_2} \cdots e_n^{\mu_n} A_{\mu_1 \mu_2 \cdots \mu_n} \\ &= \text{scalar function of } p_i p_j \,,\, p_i e_j \,,\, e_i e_j \end{aligned}$$

Crucially, we maintain on-shell conditions.

massless helicity basis

$$p_i p_i = p_i e_i = e_i e_i = 0$$

transverse

We define a set of simple transmutation operators which convert between species:

$$\mathcal{T}_{ij} = \partial_{e_i e_j}$$ 2 gluon → 2 scalar

$$\mathcal{T}_{ijk} = \partial_{p_i e_j} - \partial_{p_k e_j}$$ 1 gluon → 1 scalar

$$\mathcal{L}_i = \sum_j p_i p_j \partial_{p_j e_i}$$ 1 gluon → 1 pion

These operations have been proven via on-shell recursion and checked up to 8pt.

gluons

↓ $A_{\rm YM} \otimes A_{\rm YM} = A_{\rm GR}$

(double copy)

gravitons

↓ $\partial_{e_1 e_4} \partial_{e_2 e_3} A_{\rm GR}$

(dimensional reduction)

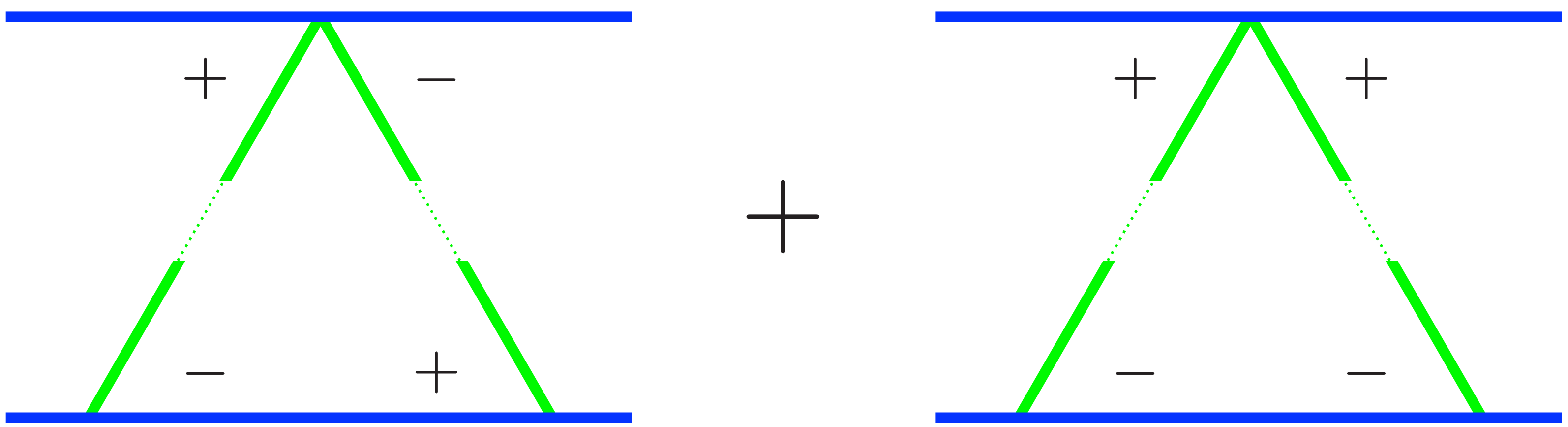
massless scalars + gravitons

↓ $p_1 p_2 \to p_1 p_2 - m_1 m_2$

(lift to massive)

massive scalars + gravitons

The loop integrands of amplitudes are built by matching singularities to tree amplitudes,



which is called generalized unitarity.

Integration is more involved, and we apply both relativistic and nonrelativistic methods.

$$A = \int d^4\ell_1 d^4\ell_2 \, \mathcal{I}_{\text{4d}}$$

↓ evaluate energy integrals via residues

$$= \int d^3\ell_1 d^3\ell_2 \, \mathcal{I}_{\text{3d}}$$

↓ expand in large mass

$$= \int d^3\ell_1 d^3\ell_2 \, \left(\mathcal{I}_{\text{3d}}^{(0)} + \mathcal{I}_{\text{3d}}^{(1)} + \mathcal{I}_{\text{3d}}^{(2)} + \cdots \right)$$

↓ evaluate 3d integrals

the answer

After 3d reduction, the vast majority of terms are iterated bubble integrals of the form

$$\int d^3\ell \, \frac{\ell_{i_1} \ell_{i_2} \cdots \ell_{i_m}}{\ell^\alpha (\ell + q)^\beta} \quad = \quad \text{(A.10) of Smirnov's "Feynman Integral Calculus"}$$

The 3d integrals which appear exactly mirror the simplicity of those in NRGR.

We also applied several highly nontrivial 4d checks using IBPs and differential equations.

For the nonrelativistic scattering amplitude for spinless particles at O(G^3) we obtain

$$\mathcal{M}_1 = -\frac{4\pi G\nu^2 m^2}{\gamma^2\xi\boldsymbol{q}^2}(1-2\sigma^2)\,,$$

$$\mathcal{M}_2 = -\frac{3\pi^2 G^2\nu^2 m^3}{2\gamma^2\xi|\boldsymbol{q}|}(1-5\sigma^2) + \frac{32\pi^2 G^2\nu^4 m^5(1-2\sigma^2)^2}{\gamma^3\xi}\int\frac{d^{D-1}\boldsymbol{\ell}_1}{(2\pi)^{D-1}}\frac{1}{\boldsymbol{\ell}^2(\boldsymbol{\ell}+\boldsymbol{q})^2(\boldsymbol{\ell}^2+2\boldsymbol{p\ell})}\,,$$

$$\begin{aligned}\mathcal{M}_3 = \frac{\pi G^3\nu^2 m^4\ln\boldsymbol{q}^2}{6\gamma^2\xi}\Bigg[&3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3\\ &-\frac{48\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}-\frac{18\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{(1+\gamma)\left(1+\sigma\right)}\Bigg]\\ &+\frac{8\pi^3 G^3\nu^4 m^6}{\gamma^4\xi}\Bigg[3\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)\int\frac{d^{D-1}\boldsymbol{\ell}}{(2\pi)^{D-1}}\frac{1}{\boldsymbol{\ell}^2|\boldsymbol{\ell}+\boldsymbol{q}|(\boldsymbol{\ell}^2+2\boldsymbol{p\ell})}\\ &-32m^2\nu^2\left(1-2\sigma^2\right)^3\int\frac{d^{D-1}\boldsymbol{\ell}_1}{(2\pi)^{D-1}}\frac{d^{D-1}\boldsymbol{\ell}_2}{(2\pi)^{D-1}}\frac{1}{\boldsymbol{\ell}_1^2(\boldsymbol{\ell}_2-\boldsymbol{\ell}_1)^2(\boldsymbol{\ell}_2+\boldsymbol{q})^2(\boldsymbol{\ell}_1^2+2\boldsymbol{p\ell}_1)(\boldsymbol{\ell}_2^2+2\boldsymbol{p\ell}_2)}\Bigg]\end{aligned}$$

We have resummed the series expansion into an analytic all orders in velocity expression.

ii) extracting the potential

Define a general Lagrangian for the EFT,

$$L_{\text{kin}} = \int_p A^\dagger(-p)\left(i\partial_t - \sqrt{p^2 + m_A^2}\right)A(p)$$

$$+ \int_p B^\dagger(-p)\left(i\partial_t - \sqrt{p^2 + m_B^2}\right)B(p)$$

$$L_{\text{int}} = -\int_{p,p'} V_{\text{int}}(p,p')\, A^\dagger(p')A(p)B^\dagger(-p')B(-p)$$

which describes the dynamics of two spinless, nonrelativistic particles denoted A and B.

potential
(position space)

$$V(p,r) = \sum_{i=1}^{\infty} c_i(p^2)\left(\frac{\kappa}{4\pi r}\right)^i$$

↓ Fourier transform

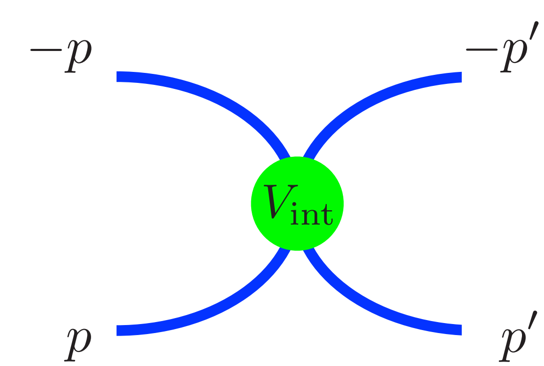
potential
(momentum space)

$$\widetilde{V}(p,q) = \frac{\kappa c_1(p^2)}{q^2} + \frac{\kappa^2 c_2(p^2)}{8q} - \frac{\kappa^3 c_3(p^2)\log q}{16\pi^2} + \cdots$$

↓ off-shell continuation

interaction vertex

$$V_{\text{int}}(p,p') = \widetilde{V}\left(\sqrt{\frac{p^2+p'^2}{2}},\, |p-p'|\right) =$$

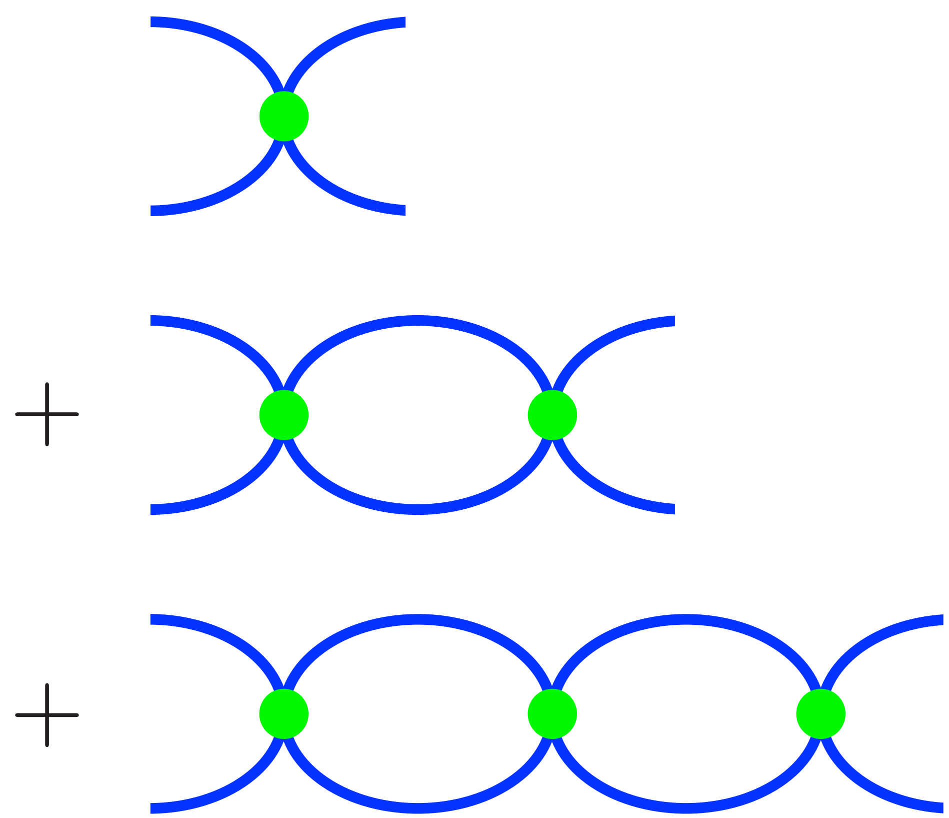


Scattering amplitudes in the EFT trivial to compute using Feynman diagrams.

$$A_{\rm EFT} = \sum_{i=1}^{\infty} A_{\rm EFT}^{(i)}$$

PM expansion

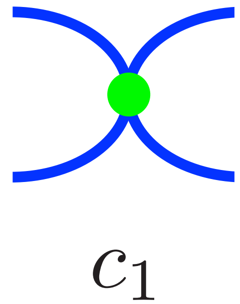
$=$



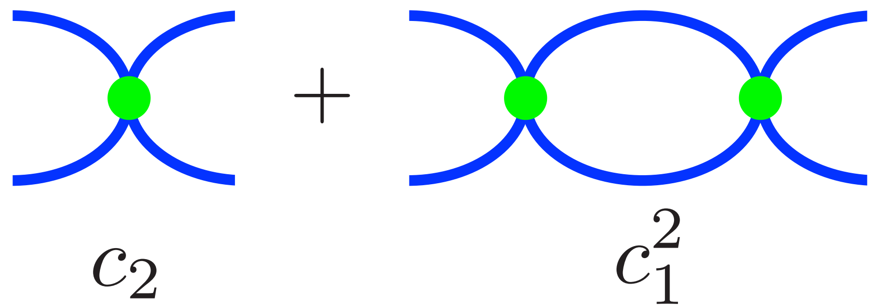
$+ \cdots$

Each PM order is not homogenous in loops.

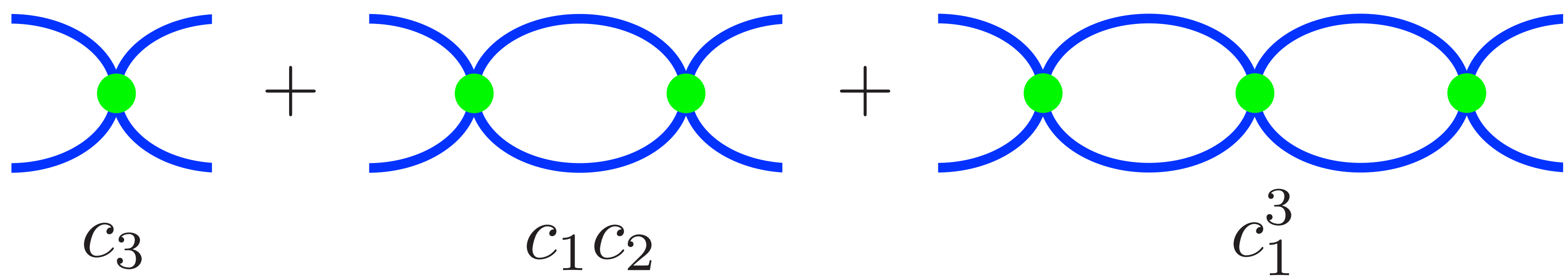
$A^{(1)}_{\text{EFT}} =$



$A^{(2)}_{\text{EFT}} =$



$A^{(3)}_{\text{EFT}} =$



Note: arbitrarily high loop diagrams can actually be computed algebraically!

We solve for the potential by setting the full amplitude equal to the EFT amplitude:

we have this at 3PM $\longrightarrow$ $$A = A_{\rm EFT}$$ $\longleftarrow$ we have this at all PM

Key point: even though both the full and EFT amplitudes are IR divergent, we can compute their IR finite difference.

$$A^{(2)}_{\rm EFT} - A^{(2)} = 0 = \kappa^2 \left[-\frac{c_2(p^2)}{q} + \int d^3\ell \, (\mathcal{I}_{\rm EFT} - \mathcal{I}) \right]$$

separately non-singular in velocity

From the one- and two-loop amplitude, we obtain the $O(G^3)$ potential,

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi} \left(1 - 2\sigma^2\right) ,$$

$$c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi} \left[\frac{3}{4} \left(1 - 5\sigma^2\right) - \frac{4\nu\sigma \left(1 - 2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2 (1-\xi) \left(1 - 2\sigma^2\right)^2}{2\gamma^3 \xi^2}\right] ,$$

$$c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi} \left[\frac{1}{12}\left(3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3\right) - \frac{4\nu \left(3 + 12\sigma^2 - 4\sigma^4\right) \operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} \right.$$
$$- \frac{3\nu\gamma \left(1 - 2\sigma^2\right) \left(1 - 5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma \left(7 - 20\sigma^2\right)}{2\gamma\xi} + \frac{2\nu^3 (3 - 4\xi)\sigma \left(1 - 2\sigma^2\right)^2}{\gamma^4 \xi^3}$$
$$\left. - \frac{\nu^2 \left(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2\right) \left(1 - 2\sigma^2\right)}{4\gamma^3 \xi^2} + \frac{\nu^4 (1 - 2\xi) \left(1 - 2\sigma^2\right)^3}{2\gamma^6 \xi^4}\right]$$

which contains arbitrarily high order PN data in the velocity expansion.

iii) checking the answer

Check #1: In the probe limit, $m_A \ll m_B$, this describes a particle in a black hole geometry.

$$\frac{V_{\rm 3PM}}{\mu} = \frac{1}{\rho^3}\left\{\left(-\frac{1}{4}-\frac{3\nu}{2}\right)+u^2\left(-\frac{25}{8}-\frac{65\nu}{4}-\frac{107\nu^2}{8}\right)+u^4\left(\frac{105}{32}-\frac{4049\nu}{160}-\frac{2589\nu^2}{32}-\frac{487\nu^3}{16}\right)\right.$$
$$+u^6\left(-\frac{273}{64}+\frac{178553\nu}{4480}-\frac{30993\nu^2}{640}-\frac{1527\nu^3}{8}-\frac{6607\nu^4}{128}\right)$$
$$+u^8\left(\frac{2805}{512}-\frac{1947527\nu}{32256}+\frac{3093791\nu^2}{17920}+\frac{5787\nu^3}{320}-\frac{168131\nu^4}{512}-\frac{19425\nu^5}{256}\right)$$
$$\left.+u^{10}\left(-\frac{7007}{1024}+\frac{2354633\nu}{26880}-\frac{23190389\nu^2}{64512}+\frac{3013571\nu^3}{7168}+\frac{340279\nu^4}{1024}-\frac{237639\nu^5}{512}-\frac{104655\nu^6}{1024}\right)+\cdots\right\}$$

Our result agrees with Schwarzschild metric.

$$\frac{H_{\rm Sch}}{\mu} = \left(1-\frac{1}{2\rho}\right)\left(1+\frac{1}{2\rho}\right)^{-1}\sqrt{1+\left(1+\frac{1}{2\rho}\right)^{-4}u^2}-1$$
$$= \cdots + \frac{1}{\rho^3}\left\{-\frac{1}{4}-\frac{25u^2}{8}+\frac{105u^4}{32}-\frac{273u^6}{64}+\frac{2805u^8}{512}-\frac{7007u^{10}}{1024}+\cdots\right\}$$

Check #2: Physically equivalent potentials generate the same energy for a circular orbit,

angular frequency

$$\varepsilon(x) = -\frac{x}{2}\left(1 - \frac{x}{12}(9+\nu) - \frac{x^2}{24}(81 - 57\nu + \nu^2) + \cdots\right)$$

energy per reduced mass

written in terms of the appropriate quantities.

We find agreement at the highest order for which our results apply to a virialized system, which is $O(G^3 v^0)$, *i.e.* 2PN.

Check #3: Physically equivalent potentials are related by a canonical transformation.

our potential

state of the art
4PN potential
from literature

$$V_1 \longleftrightarrow V_2$$

construct explicit
coordinate transform

$$A_1 = A_2$$

We found agreement at $O(G^3 v^4)$, *i.e.* 4PN.

LIGO theorists have run these results through their pipeline, comparing against NR, etc.

Energetics of two-body Hamiltonians in post-Minkowskian gravity

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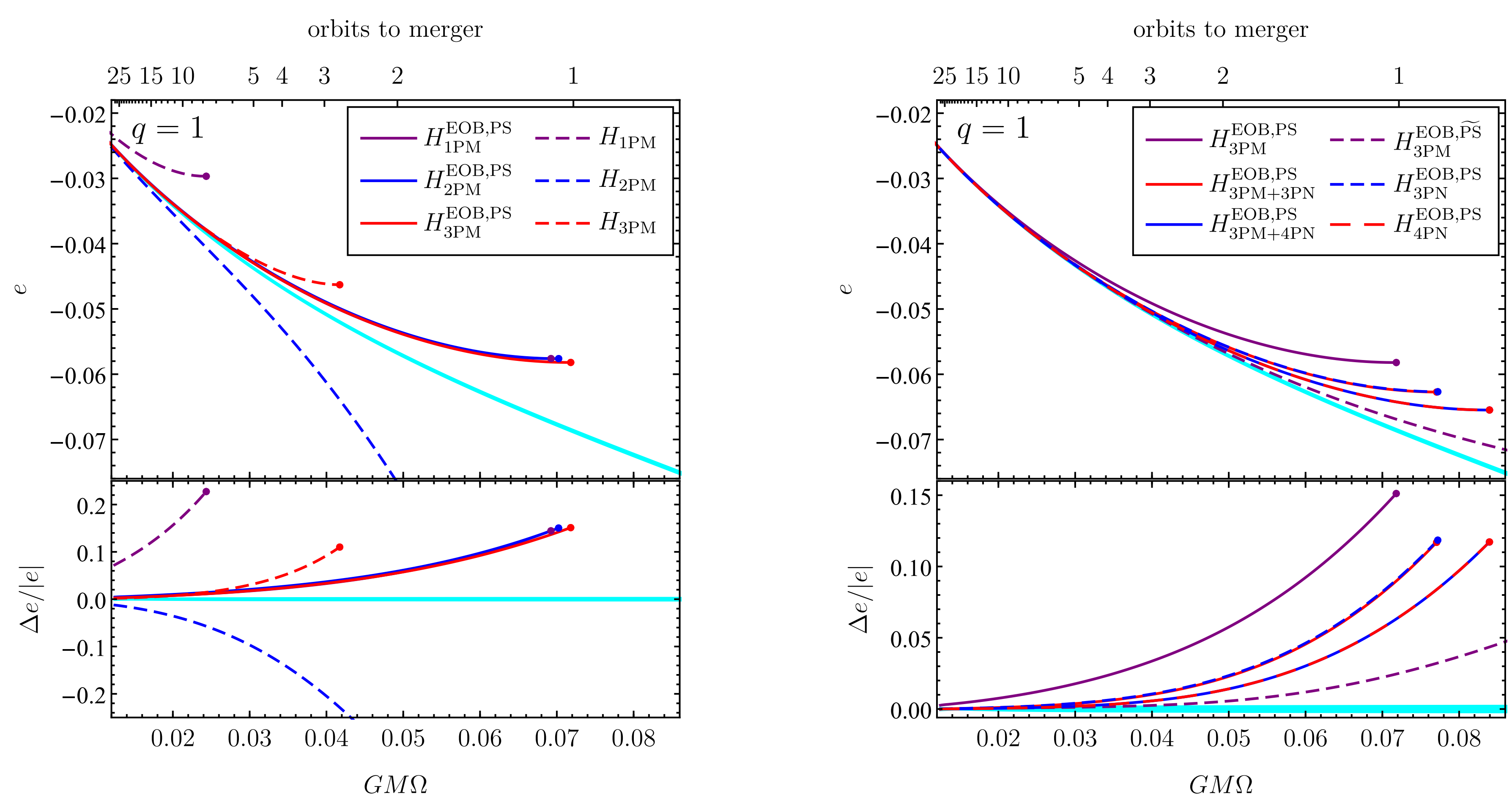
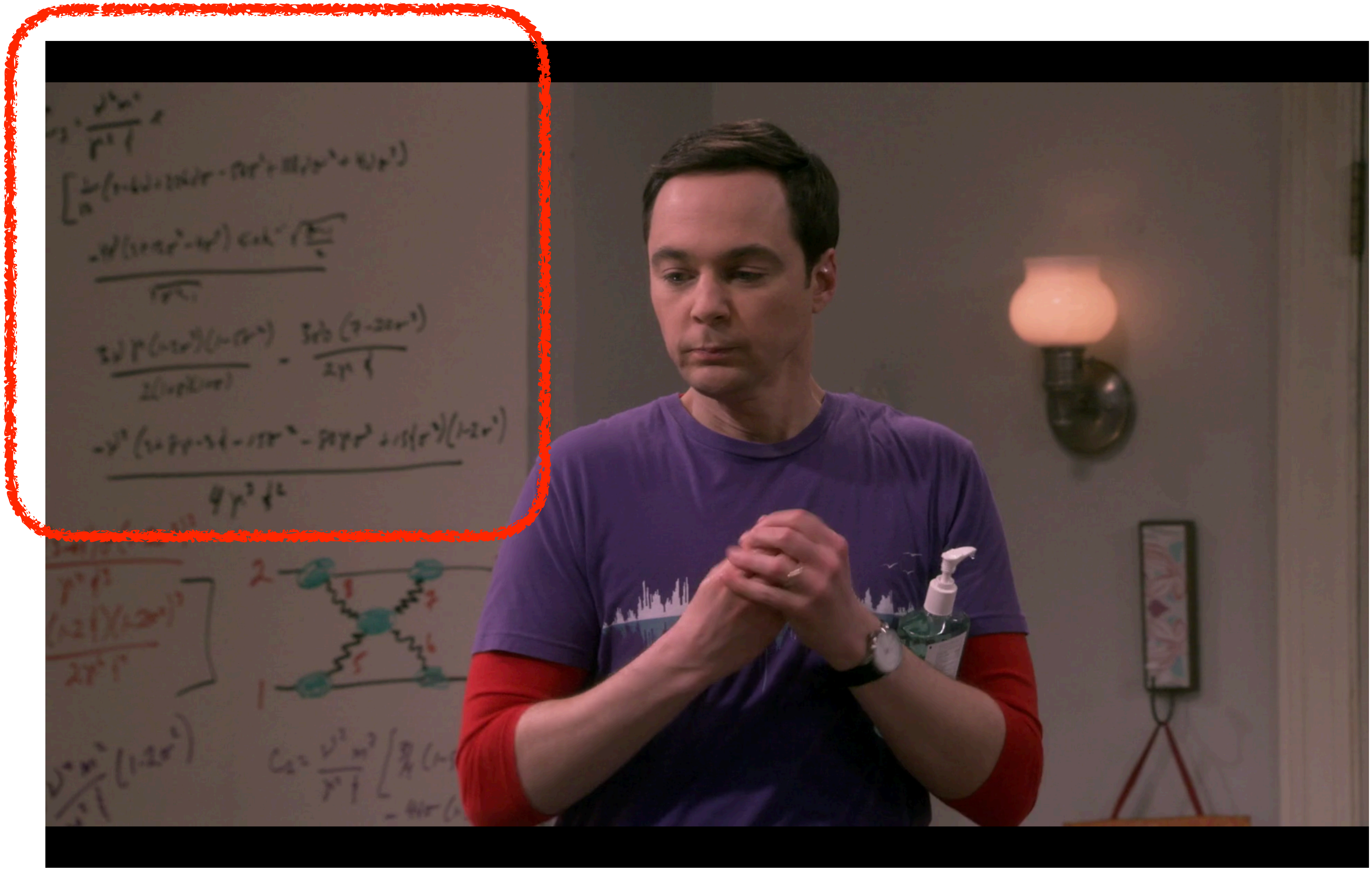
[2] *Department of Physics, University of Maryland, College Park, MD 20742, USA*
(Dated: January 23, 2019)





conclusions

summary of results

- There is a concrete and scaleable path to connect progress in amplitudes to LIGO.

- We computed the scattering amplitude at 3PM utilizing old and new methods.

- We obtain the 3PM potential via EFT matching, which passes nontrivial checks.

future directions

- Extend these results to incorporate spin, finite size effects, radiation, higher loops.

- Implement a systematic bootstrap for PM amplitudes from RPI, locality, etc.

- Apply EFT and amplitudes methods to "self-force" regime where NR and PN fail.

map of perturbation theory

0PN 1PN 2PN 3PN 4PN 5PN 6PN 7PN

1PM $$\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + v^{14} + \cdots \right) G$$

2PM $$\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots \right) G^2$$

3PM $$\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots \right) G^3$$

4PM $$\left(1 + v^2 + v^4 + v^6 + v^8 + \cdots \right) G^4$$

5PM $$\left(1 + v^2 + v^4 + v^6 + \cdots \right) G^5$$

$$\vdots$$

1808.02489

1901.04424

looks doable

thank you!