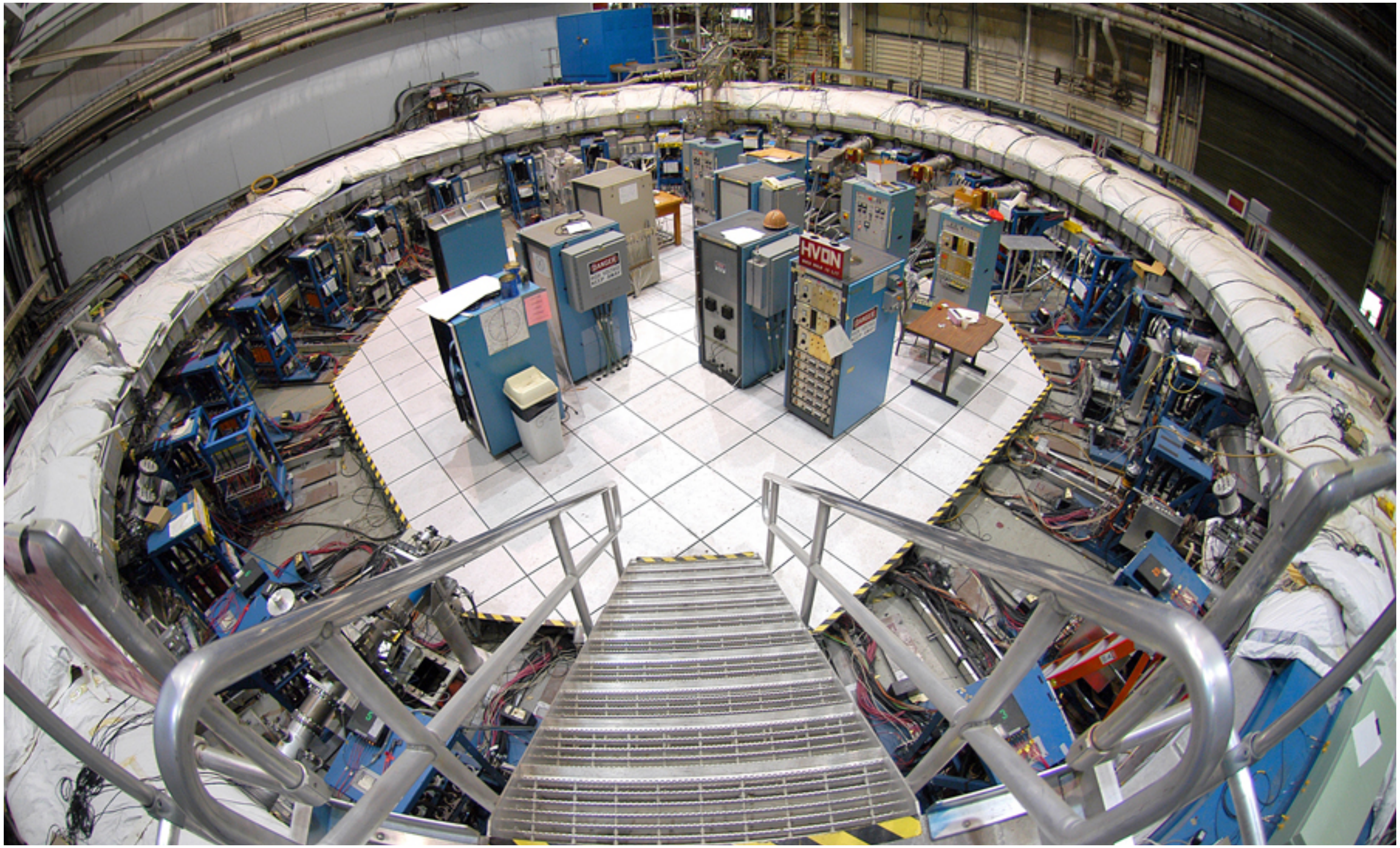
Theory status and prospects for the muon g-2 (/2 = a_μ)

Tom Blum (UConn/RBRC)

Bay Area Particle Theory Seminar

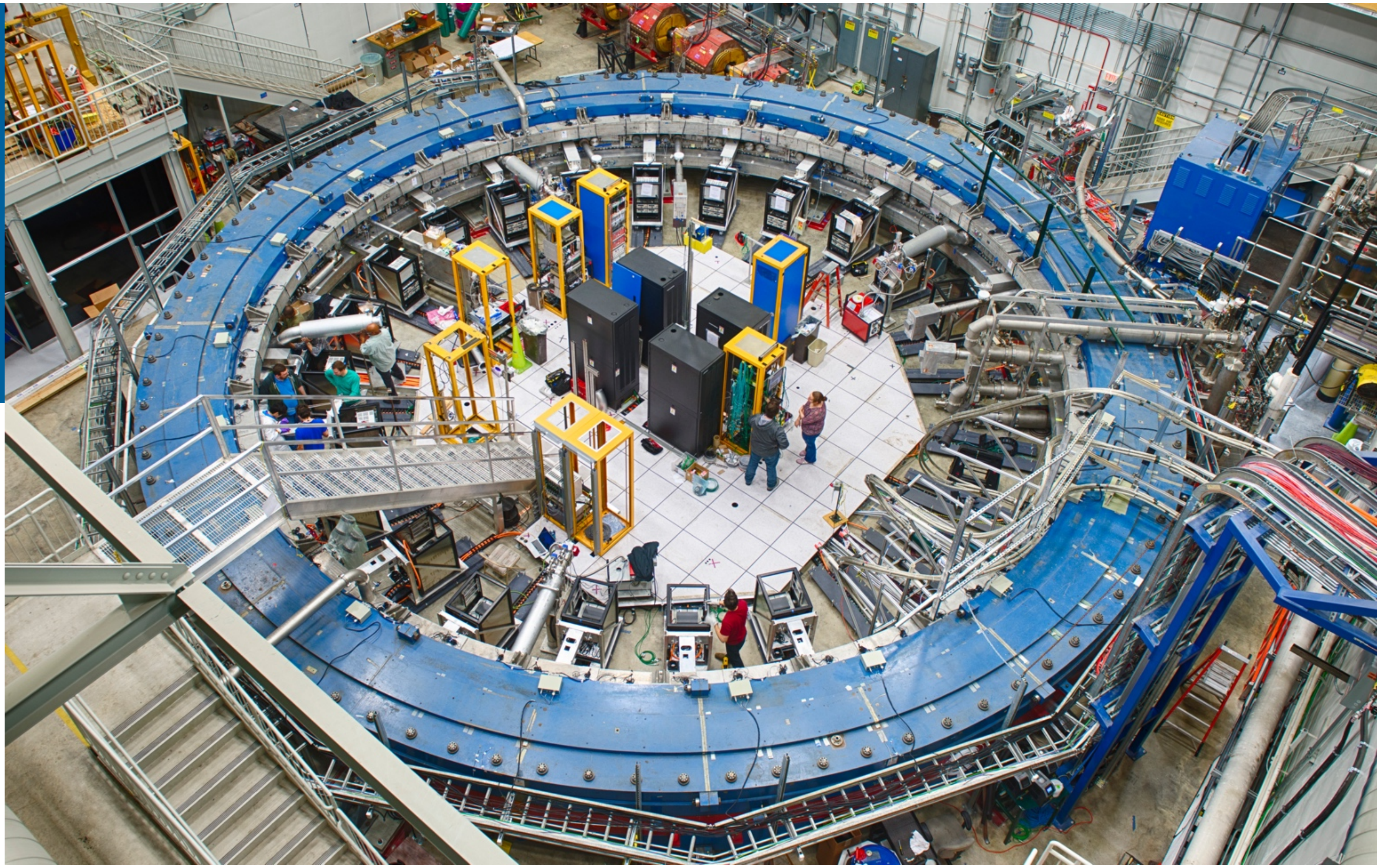
March 5, 2021



BNL E821 Measurement

a_μ = 116592089(63) 10^{-11} (0.54 ppm)

THE MUON g-2 EXPERIMENT AT FERMILAB



PETER WINTER
High Energy Physics Division
Argonne National Laboratory

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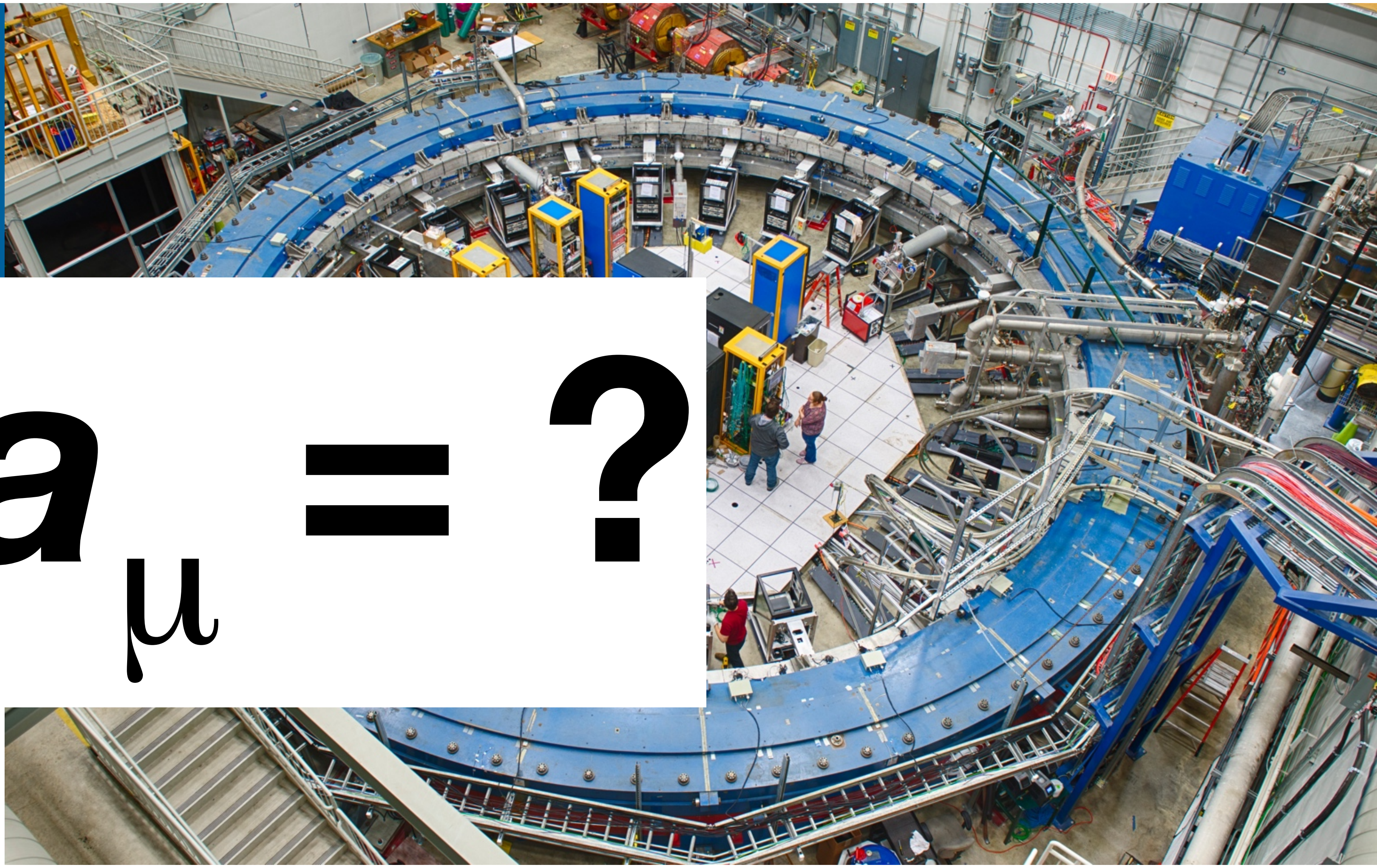

$$a_\mu = ?$$

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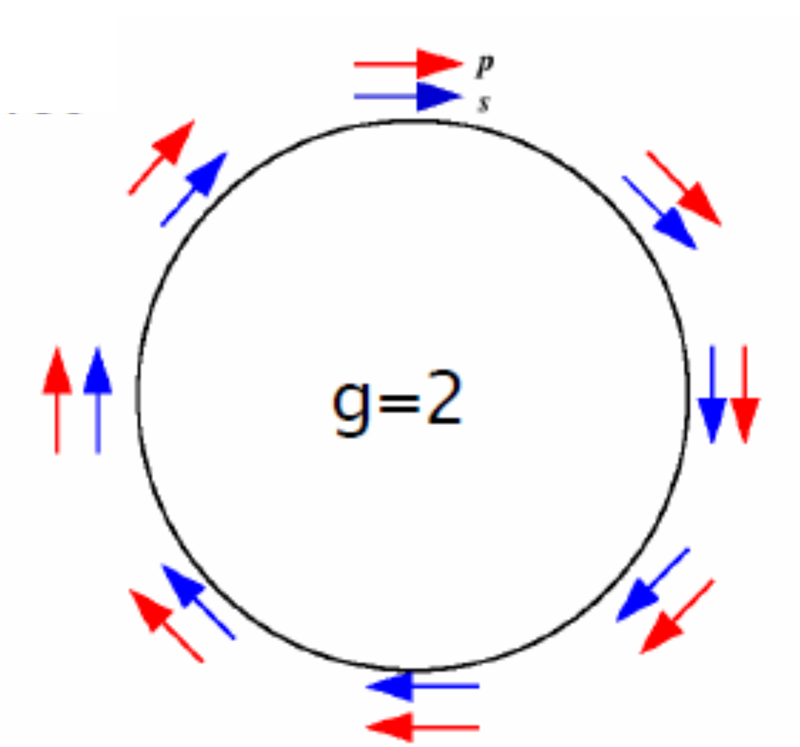
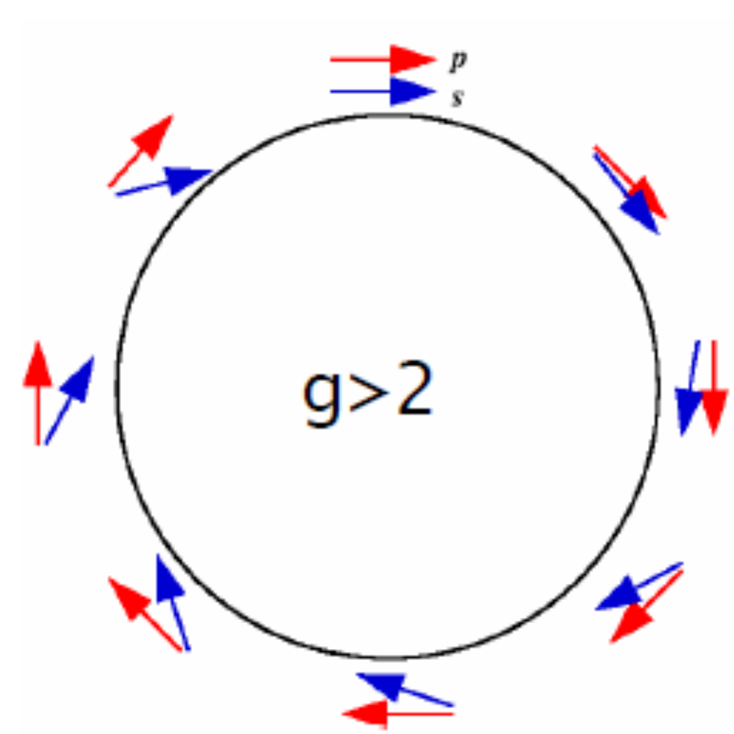
MUONS IN A STORAGE RING

- Cyclotron frequency: $$\omega_c = \frac{e}{m\gamma} B$$
- Spin precession frequency: $$\omega_S = \frac{e}{m\gamma} B(1 + \gamma\ a_\mu)$$

Larmor + Thomas precession

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_c = \frac{e}{m}\left(a_\mu \vec{B}\right)$$





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MUONS IN B AND E FIELD

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- In presence of additional E-field:

$$\vec{\omega}_a = \frac{e}{m}\left(a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right)$$

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Magic momentum (γ = 29.3, p=3.094 GeV/c)

E field for vertical focusing

CERN-III, BNL E821, Fermilab E989

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CERN-III, BNL E821, Fermilab E989

No E field: E = 0
Weak magnetic focusing
J-PARC E34

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$$\omega_a = e/m \, a_\mu \, B$$

- Measuring the anomalous moment a_μ requires both
  1. the spin precession frequency ω_a
  2. the magnetic field **B** (through NMR spectroscopy of proton)

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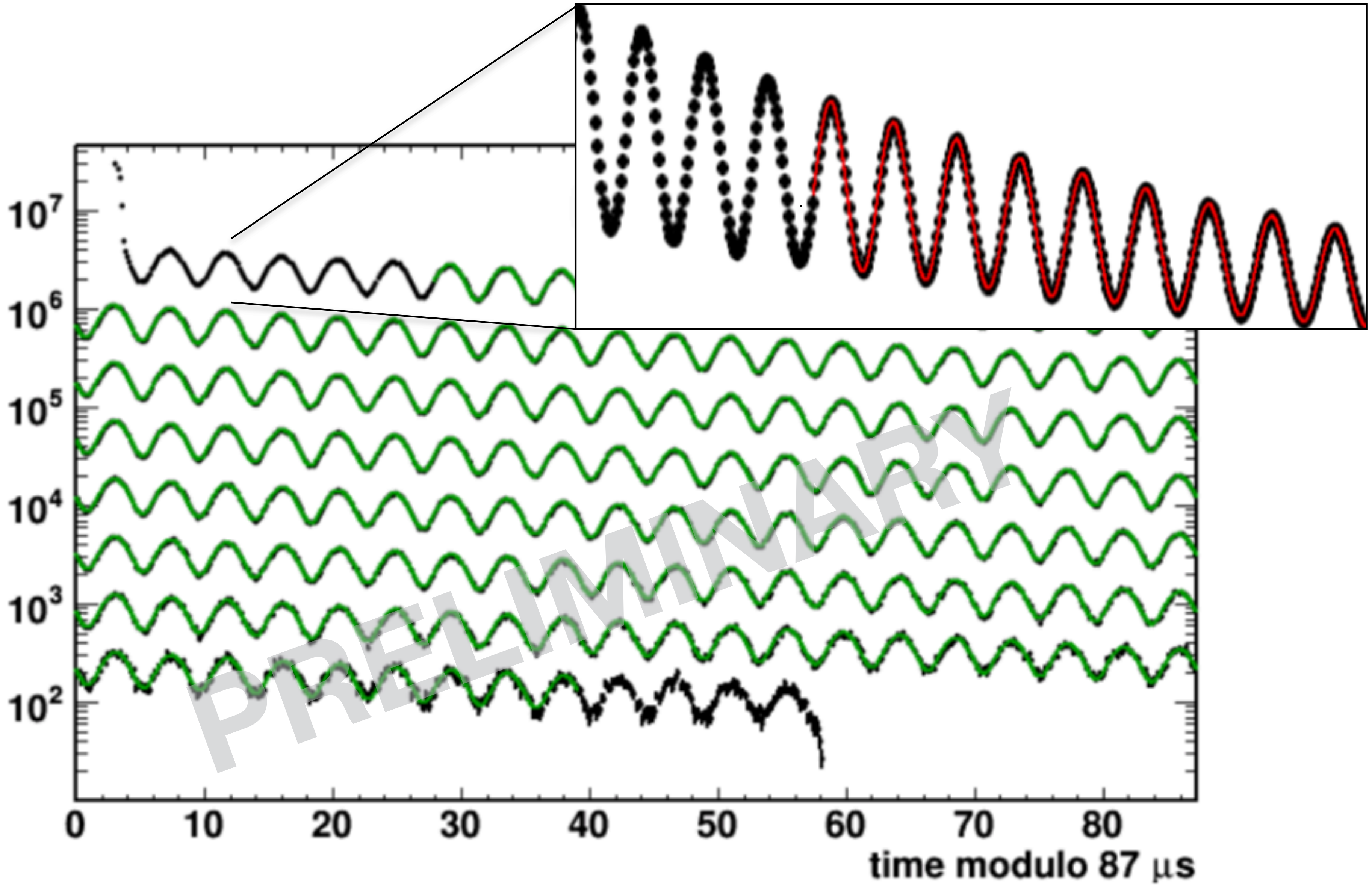
RUN-1 ANALYSIS STATUS: ω_a

- Simple 5-parameter fit captures the main features of the "wiggle plot":

$$N(t) = N_0\, e^{-t/\tau}\left[1 - A\cos\left(\omega_a t + \phi\right)\right]$$



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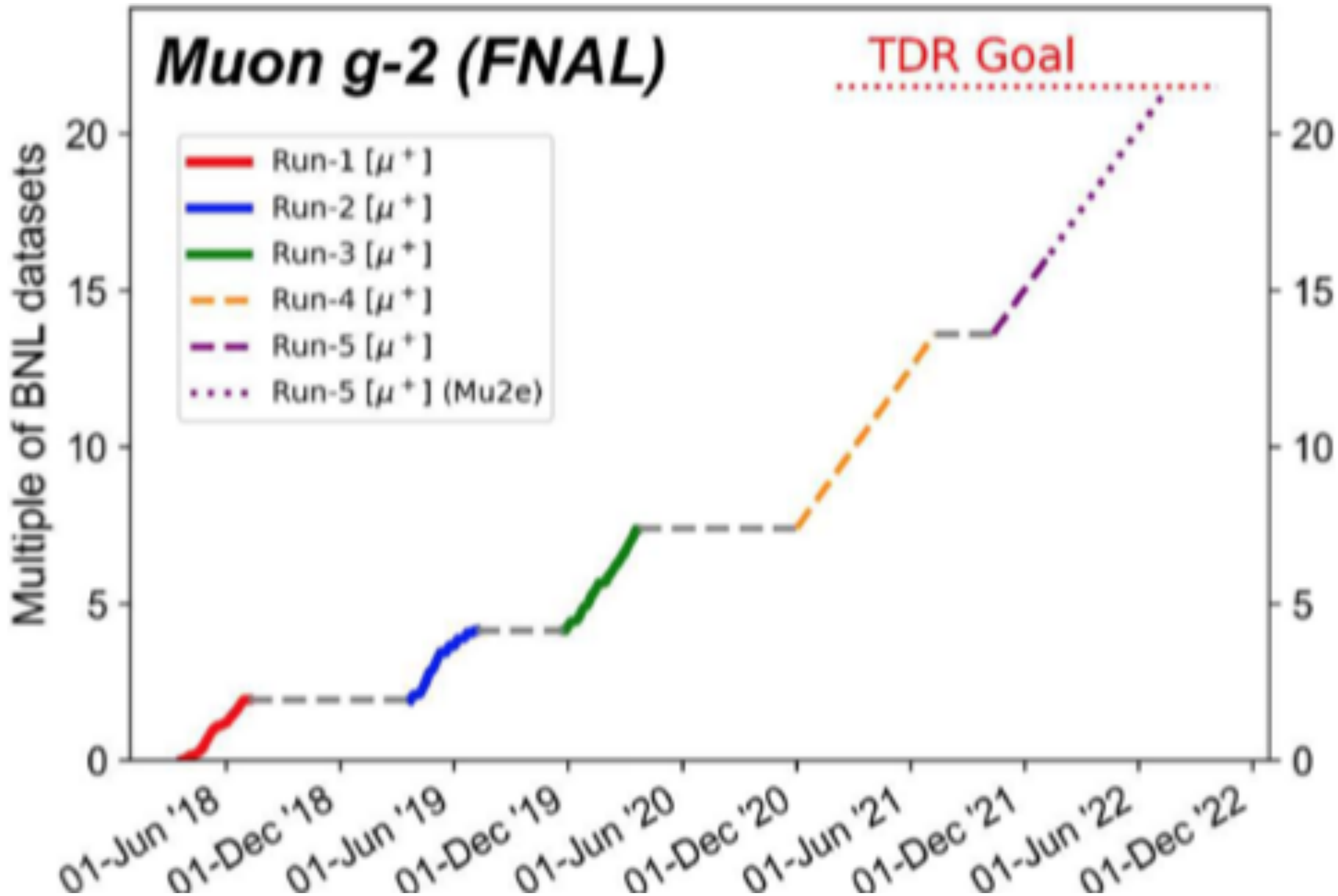
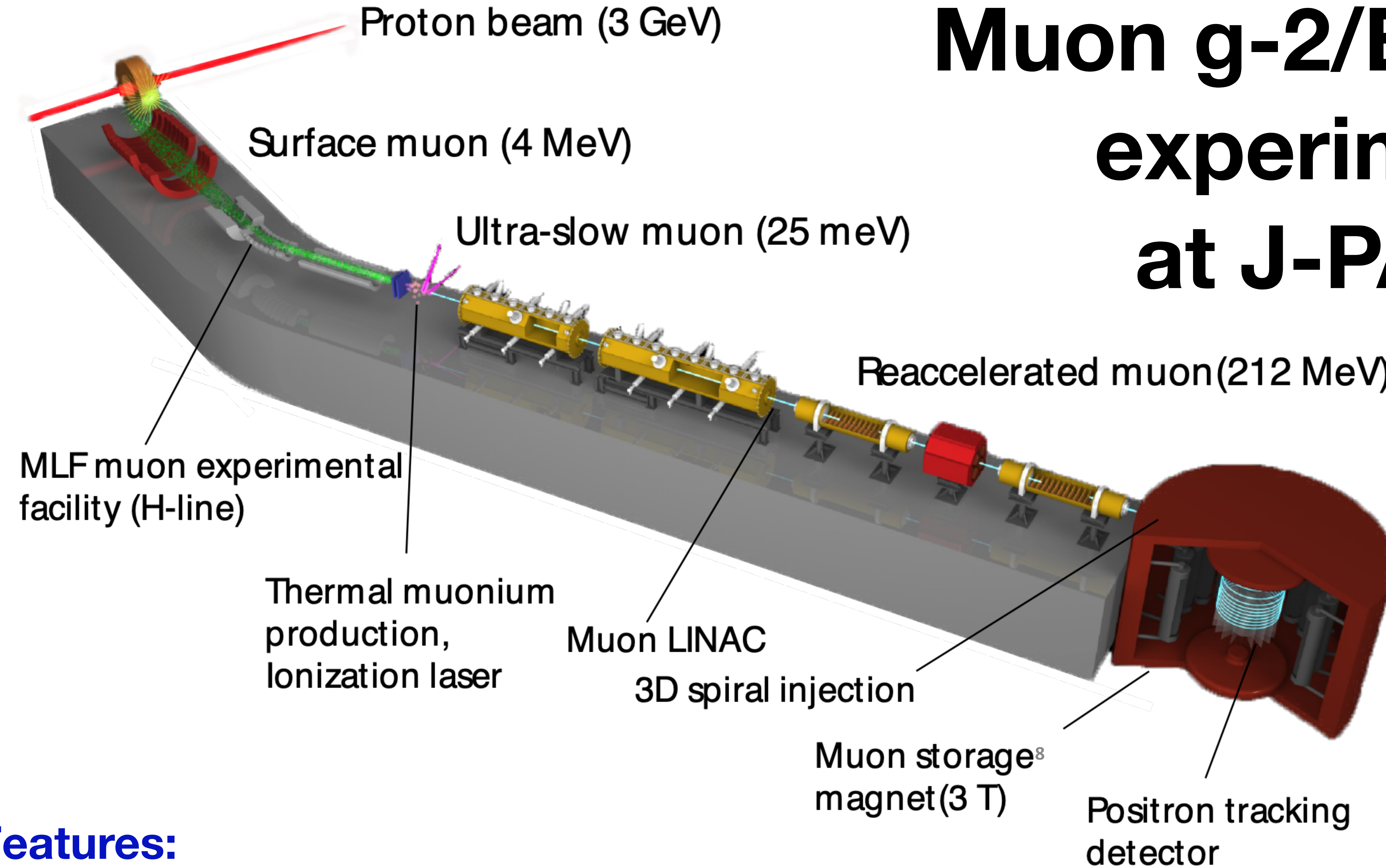
Run plan and expected statistics



Muon g-2/EDM experiment at J-PARC



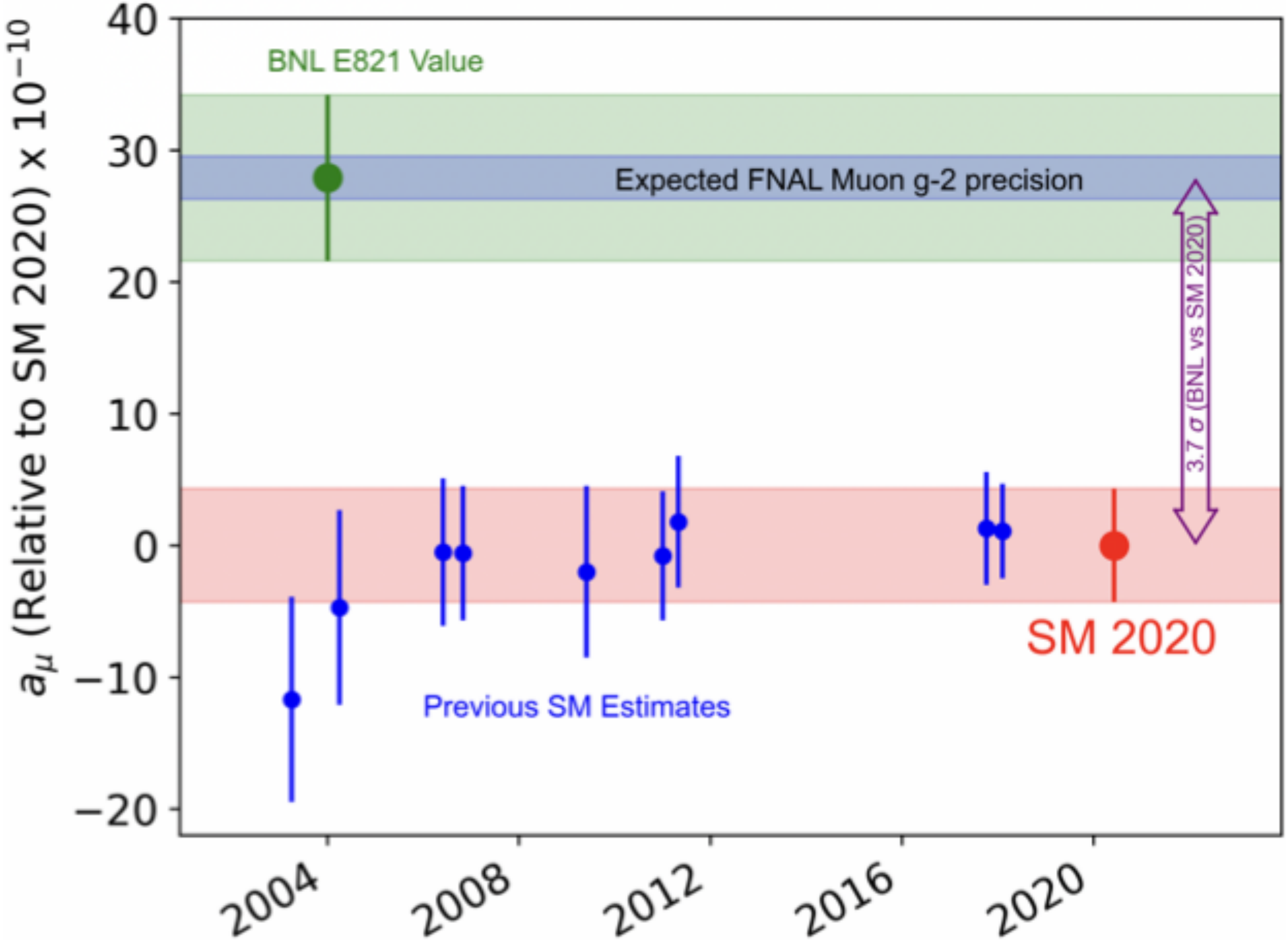
Features:

- **Low emittance muon beam (1/1000)**
- **No strong focusing (1/1000) & good injection eff. (x10)**
- **Compact storage ring (1/20)**
- **Tracking detector with large acceptance**

Credit: T. Mibe

Theory v. experiment (muon g-2 theory initiative baseline)

From T. Teubner: Plot from Mark Lancaster

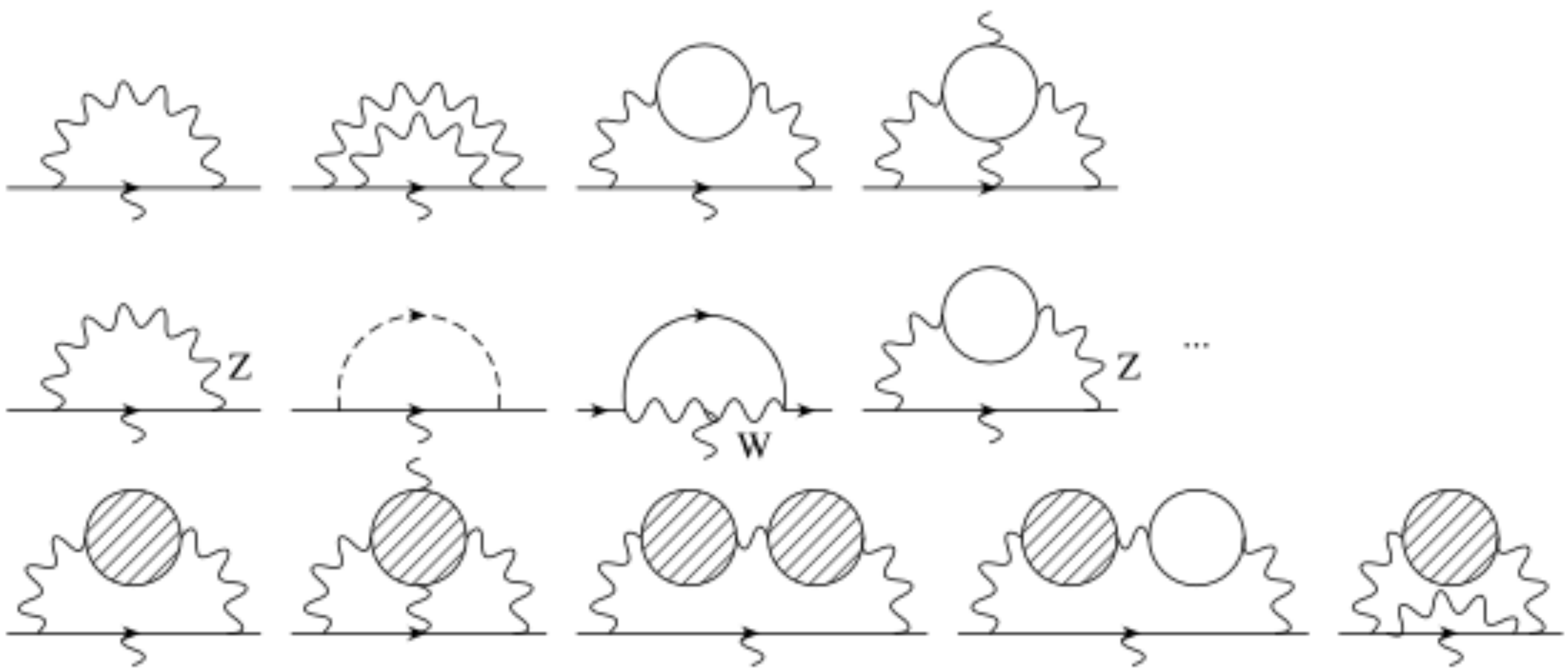


magnetic moment of free muon is $spin \times \frac{e}{2m}\ g,\ g=2$

Standard Model Theory: QED+EW+QCD

$$\langle \mu(\vec{p}')|J_\nu(0)|\mu(\vec{p})\rangle = -e\bar{u}(\vec{p}')\left(F_1(q^2)\gamma_\nu + i\frac{F_2(q^2)}{4m}[\gamma_\nu,\gamma_\rho]q_\rho\right)u(\vec{p})$$

$$a_\mu \equiv (g-2)/2 = F_2(0) \qquad (q = p' - p)$$



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Muon g-2 Theory Initiative

(2006.04822 [hep-ph])

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: nmscmsr jhrsr SuSIjSajc Ss DbIrcmbcChpcbs

Ogxrlbr Qdonqsr

int pmSj gnl coSec/ v v v cjrculcpbnl .jnbSsc.ogwrpco

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L -G_x_j _v _ +F-Gdqcnëy_ +C-V -Gdqsynf +@Gndbj dq +
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Y-Fdkydq +I-Q-Fqddm +R-Ft dkk_sh, J gdkhe_ +C-G_ssnm +
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I-V hgdkl +Q-V hkh_l r +@R-Ygduk j nu

Muon g-2 Theory Initiative

(2006.04822 [hep-ph])

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Table 1: Summary of the contributions to a_μ^{SM}. After the experimental number from E821, the first block gives the main results for the hadronic contributions from Secs. 2 to 5 as well as the combined result for HLbL scattering from phenomenology and lattice QCD constructed in Sec. 8. The second block summarizes the quantities entering our recommended SM value, in particular, the total HVP contribution, evaluated from e^+e^- data, and the total HLbL number. The construction of the total HVP and HLbL contributions takes into account correlations among the terms at different orders, and the final rounding includes subleading digits at intermediate stages. The HVP evaluation is mainly based on the experimental Refs. [37–89]. In addition, the HLbL evaluation uses experimental input from Refs. [90–109]. The lattice QCD calculation of the HLbL contribution builds on crucial methodological advances from Refs. [110–116]. Finally, the QED value uses the fine-structure constant obtained from atom-interferometry measurements of the Cs atom [117].

Muon g-2 Theory Initiative

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0.54 PPM

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Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

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Muon g-2 Theory Initiative

(2006.04822 [hep-ph])

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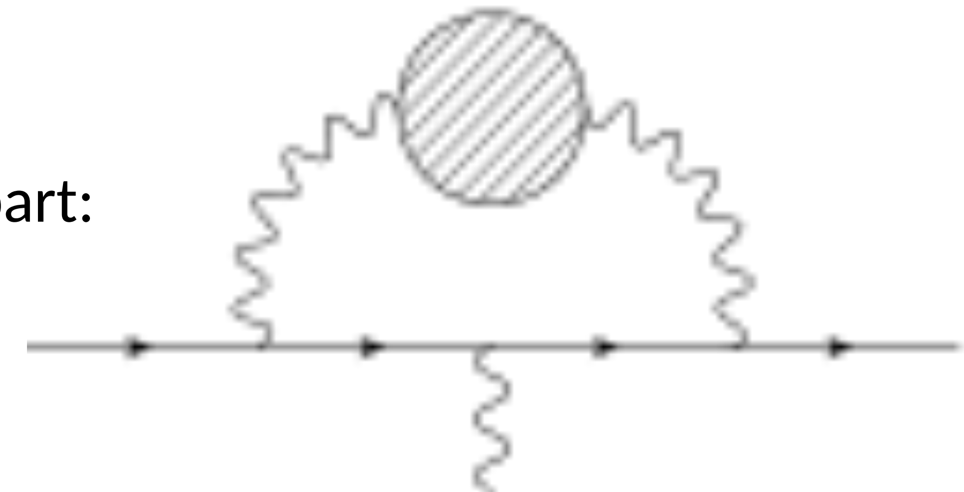
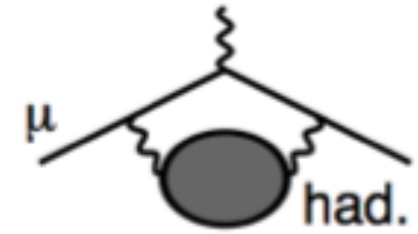
Hadronic vacuum polarization (HVP) I: data driven (e^+e^-)

Credit: T. Teubner

Magnetic part:

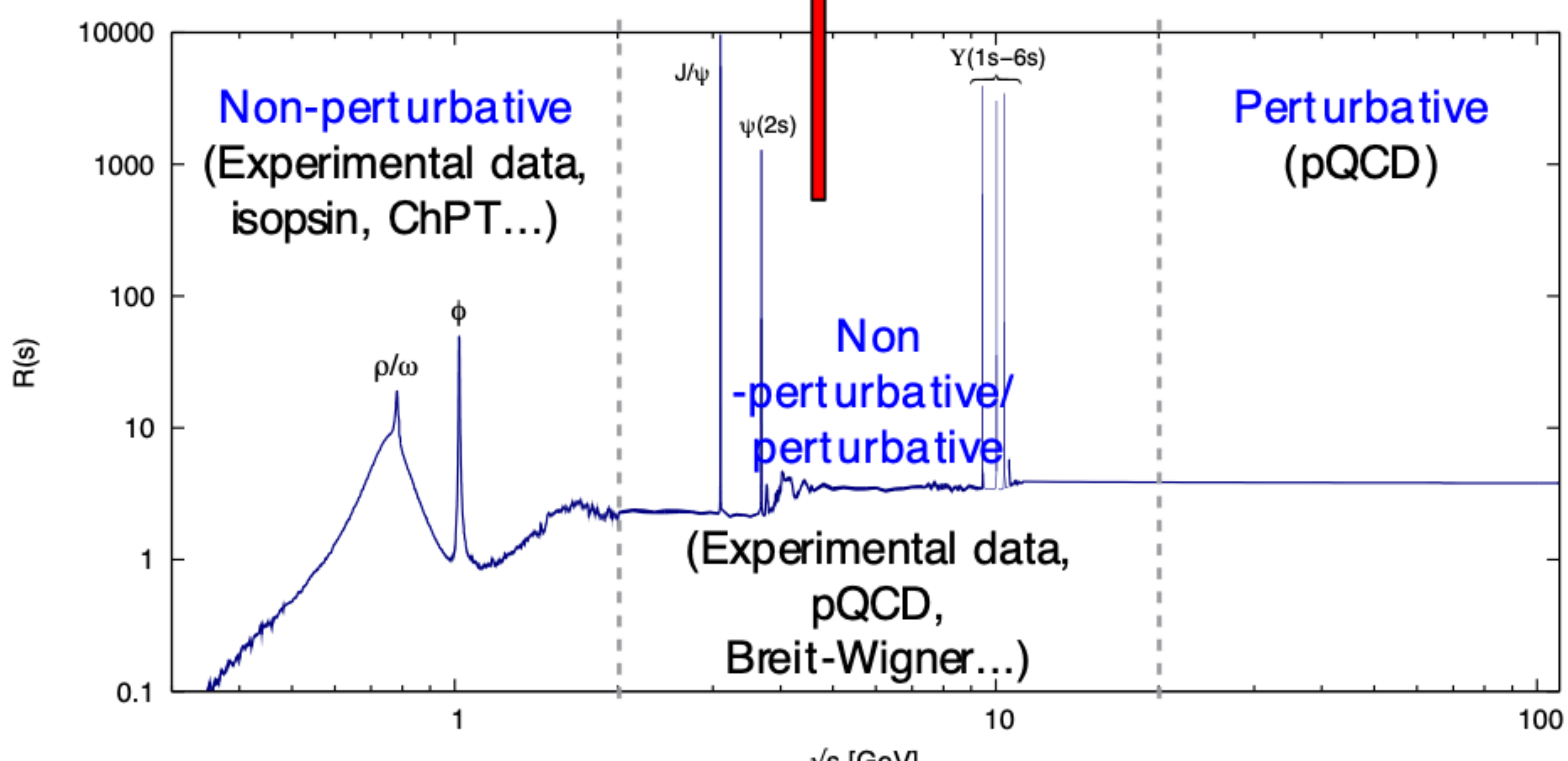


…e diagram to be evaluated:



…QCD not useful. Use the dispers…

$$a_\mu^{had,LOVP} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{ds}{s}R(s)K(s), \quad \text{where } R(s) = \frac{\sigma^0_{had,\gamma}(s)}{4\pi\alpha^2/3s}$$



a_μ-HVP from data

$$a_\mu^{\text{HVP, LO}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{10}$$

More precise than lattice determination. Total error larger than DHMZ and KNT separately.

Data from BABAR, BESIII, CMD-2, KLOE, SND

“Merged” value from DHMZ, KNT, and CHHKS (simple average in each channel for central value, conservative combination of errors). Errors statistical and systematic

Data sets disagree outside of quoted errors, leads to differences in analysis too.

Some differences cancel in integrated quantities

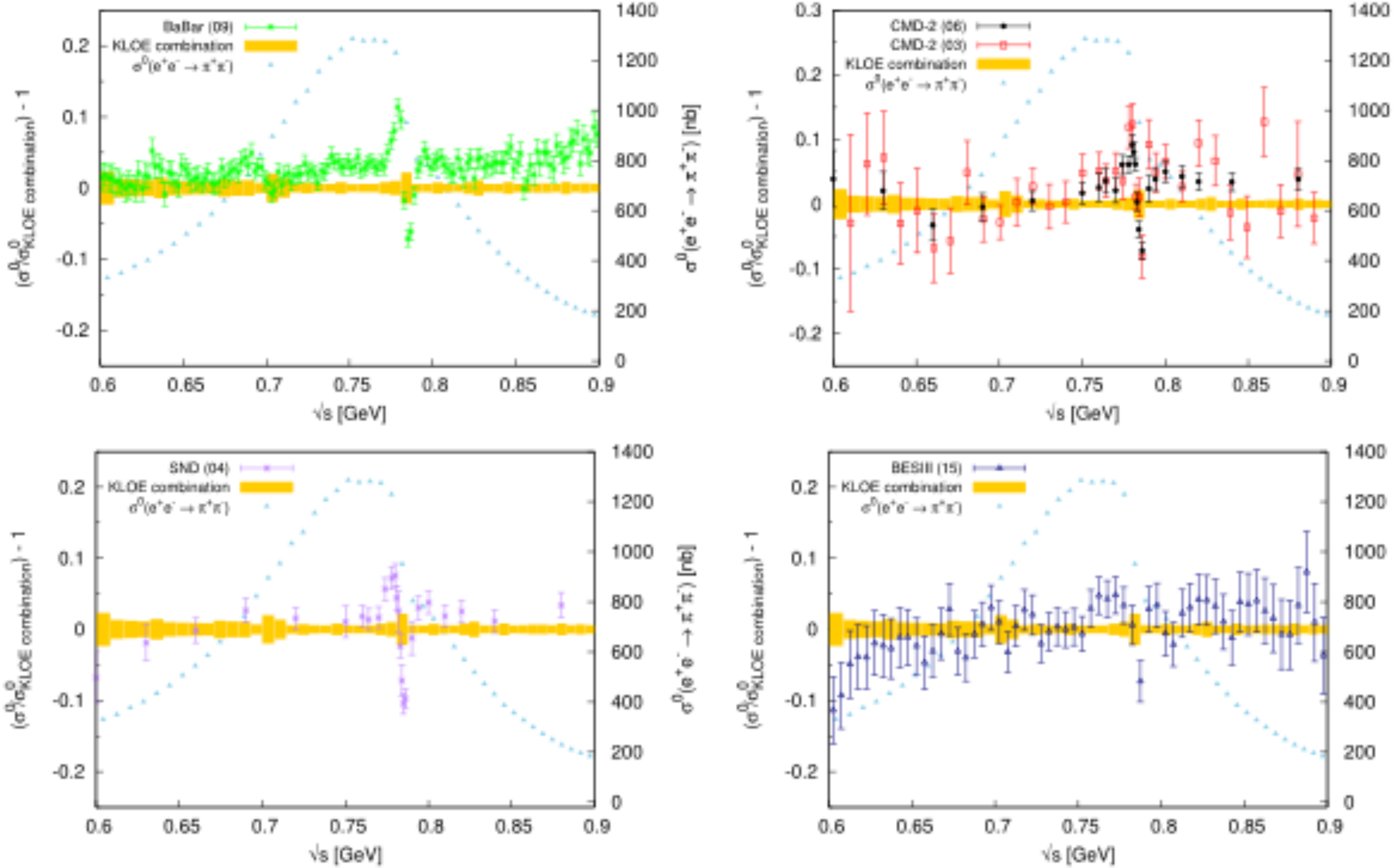
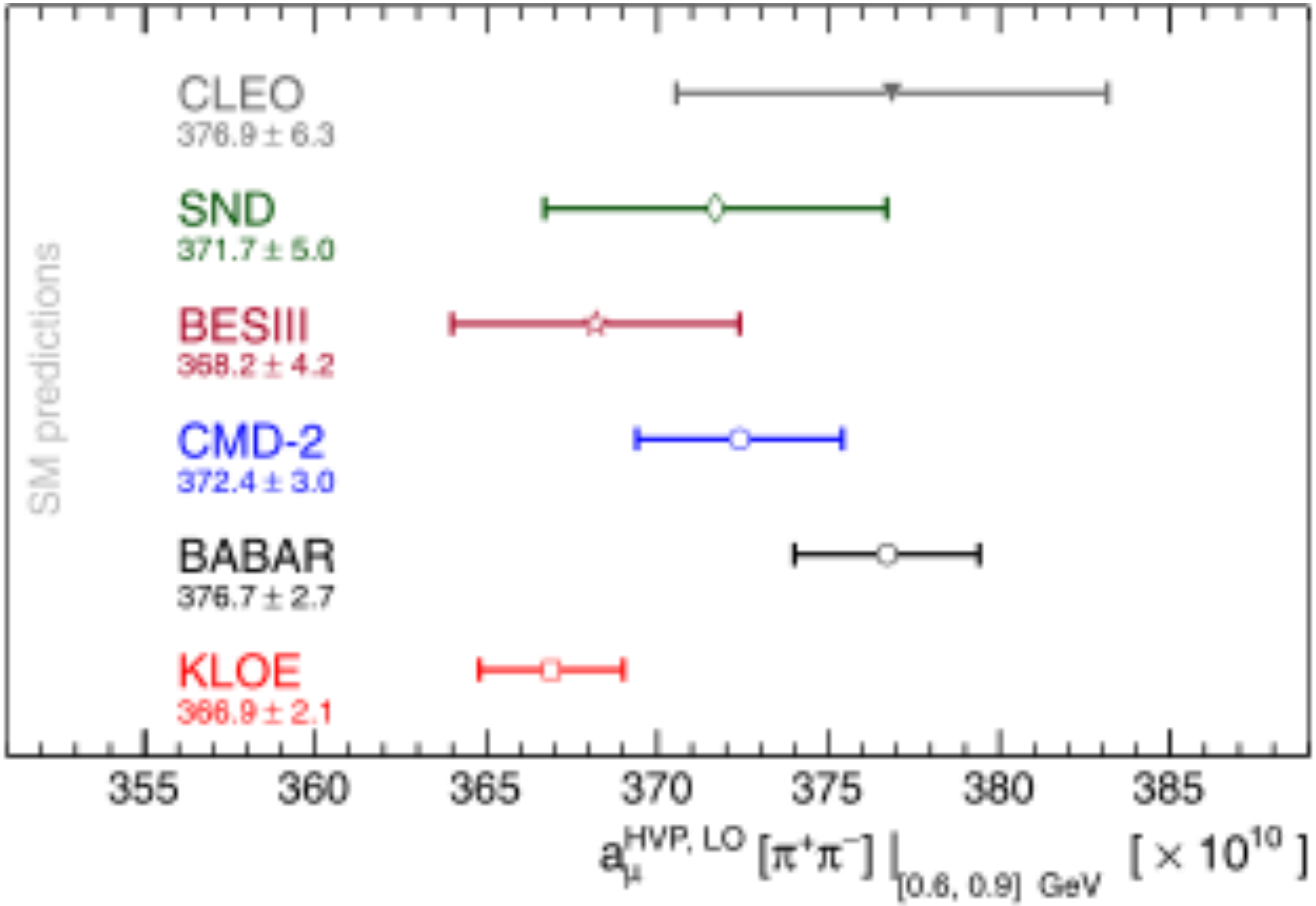


Fig. 13. The $\pi^+\pi^-$ cross section from the KLOE combination compared to the BABAR, CMD-2, SND, and BESIII data points in the 0.6–0.9 GeV range [82]. The KLOE combination is represented by the yellow band. The uncertainties shown are the diagonal statistical and systematic uncertainties summed in quadrature.
Source: Reprinted from Ref. [82].



Comparison of results for $a_\mu^{\mathrm{HVP,\ LO}}[\pi\pi]$, evaluated between 0.6 GeV and 0.9 GeV for the various experiments..

Table 5
Selected exclusive-mode contributions to $a_\mu^{\mathrm{HVP,\ LO}}$ from DHMZ19 and KNT19, for the energy range $\leq$ 1.8 GeV, in units of 10^{-10}. Where three (or more) uncertainties are given for DHMZ19, the first is statistical, the second channel-specific systematic, and the third common systematic, which is correlated with at least one other channel. For the $\pi^+\pi^-$ channel, the uncertainty accounting for the tension between BABAR and KLOE (amounting to 2.76×10^{-10}) is included in the channel-specific systematic.

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\mathrm{HVP,\ LO}}$	694.0(1.0)(3.5)(1.6)(0.1)$_\psi$(0.7)$_{\mathrm{DV+QCD}}$	692.8(2.4)	1.2

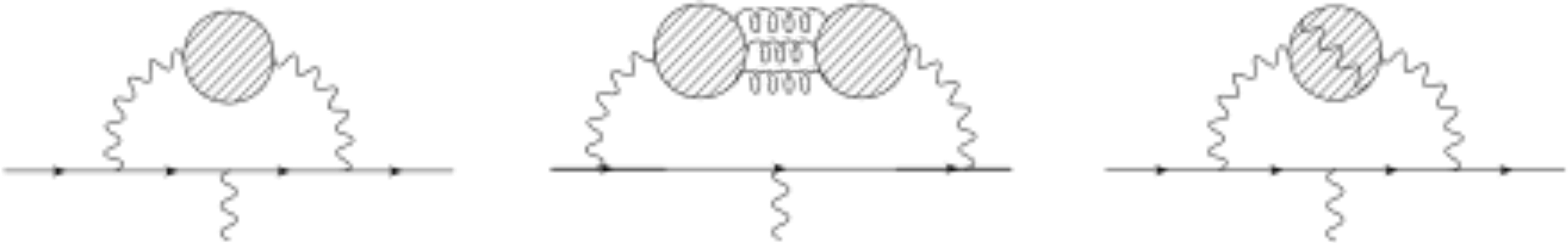
a_μ-HVP from data

$$a_\mu^{\text{HVP, LO}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10}$$

Prospects for improvement:

- Better/more analysis of existing data (BABAR, KLOE)
- More data from CMD-3, Belle II, BESIII, BESCII, SND
- Include τ-decay data when isospin breaking properly unde
  (lattice may help)

Hadronic vacuum polarization II: Lattice QCD



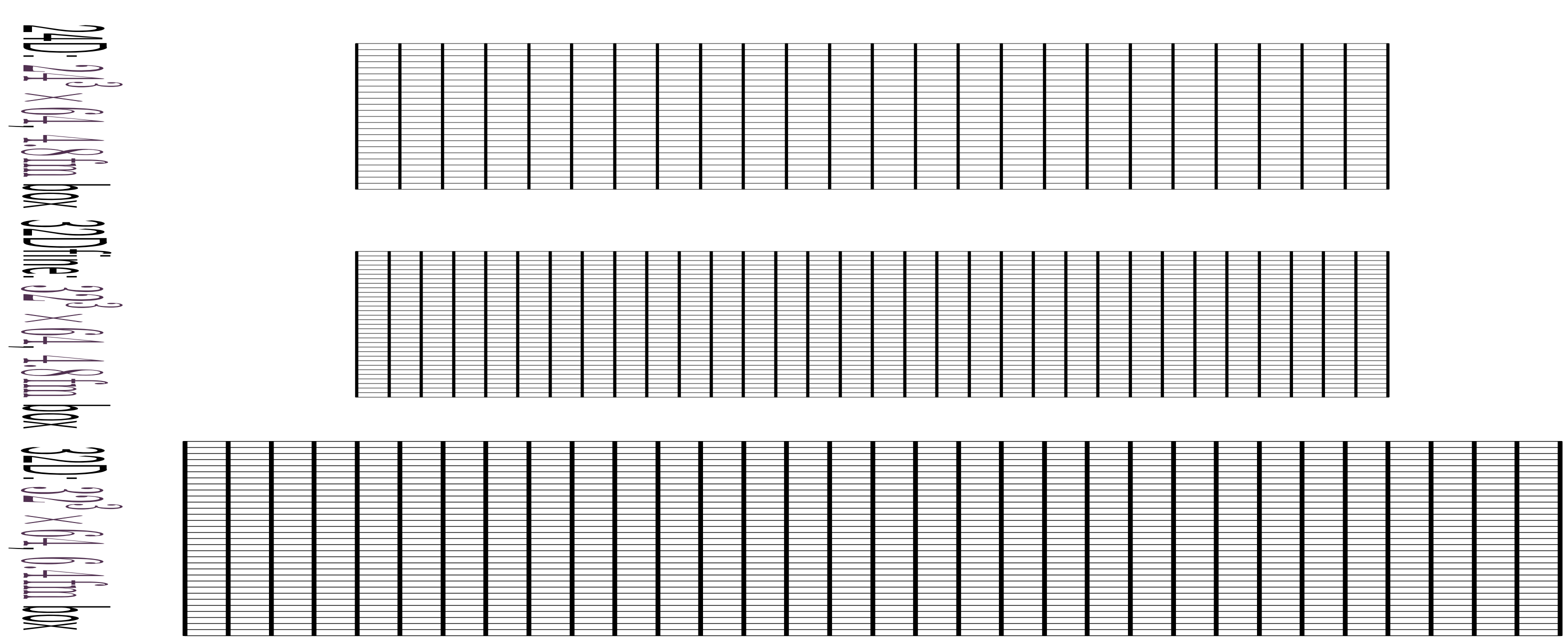
$$\Pi^{\mu\nu}(q) = \int d^4x\, e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = \hat{\Pi}(q^2)\left(q_\mu q_\nu - q^2 \delta_{\mu\nu}\right)$$

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\, f(q^2)\, \hat{\Pi}(q^2)$$

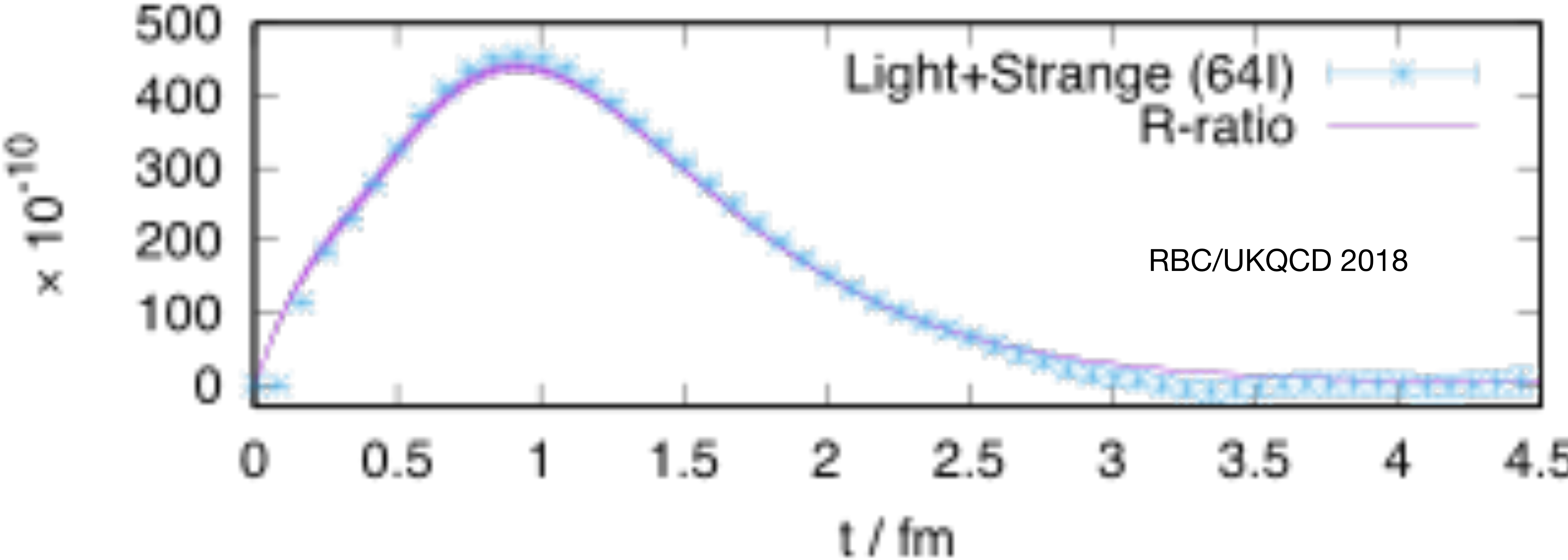
Time Momentum Representation (TMR) [Bernecker and Meyer, 2011]

$$a_\mu^{\mathrm{HVP}} = \sum_t w(t) C(t), \qquad C(t) = \frac{1}{3} \sum_{i,\vec{x}} \langle j_i(\vec{x}, t) j_i(0) \rangle$$

Calculate $C(t)$ on 4d Euclidean space-time lattice



Hadronic vacuum polarization II: Lattice QCD



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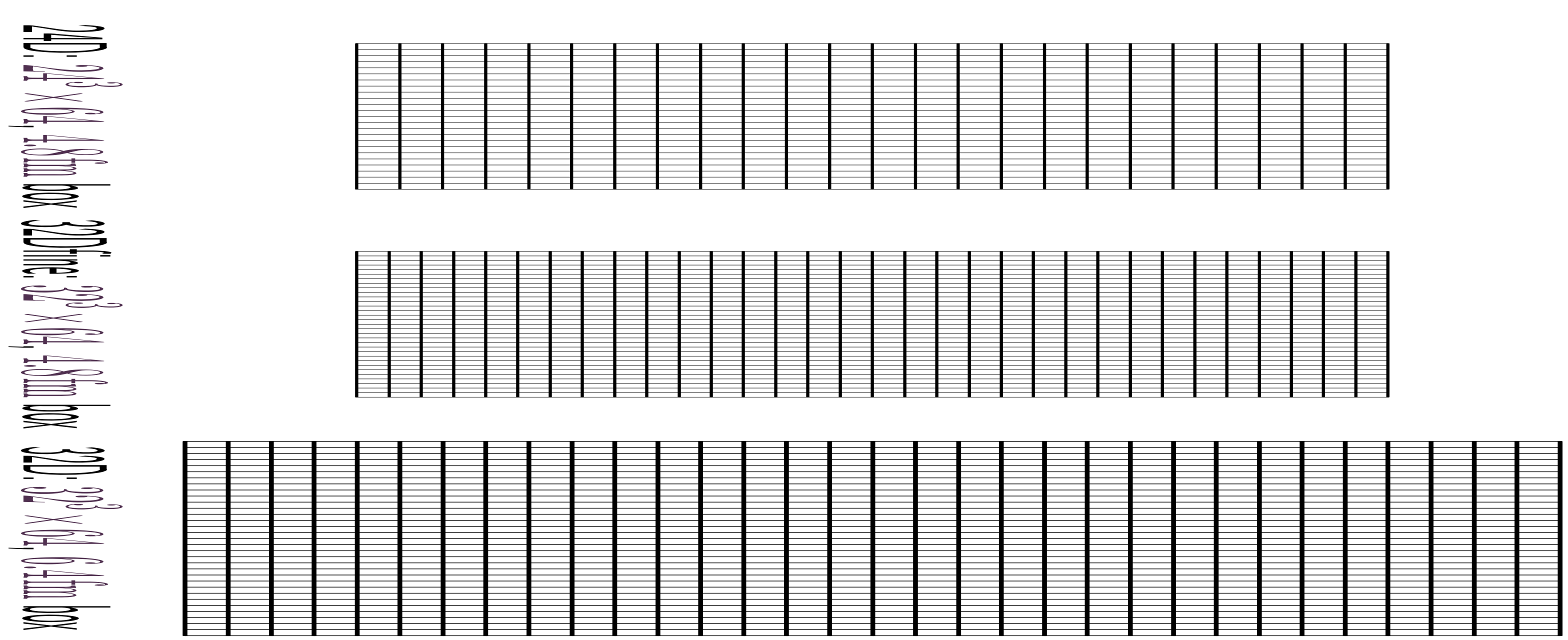
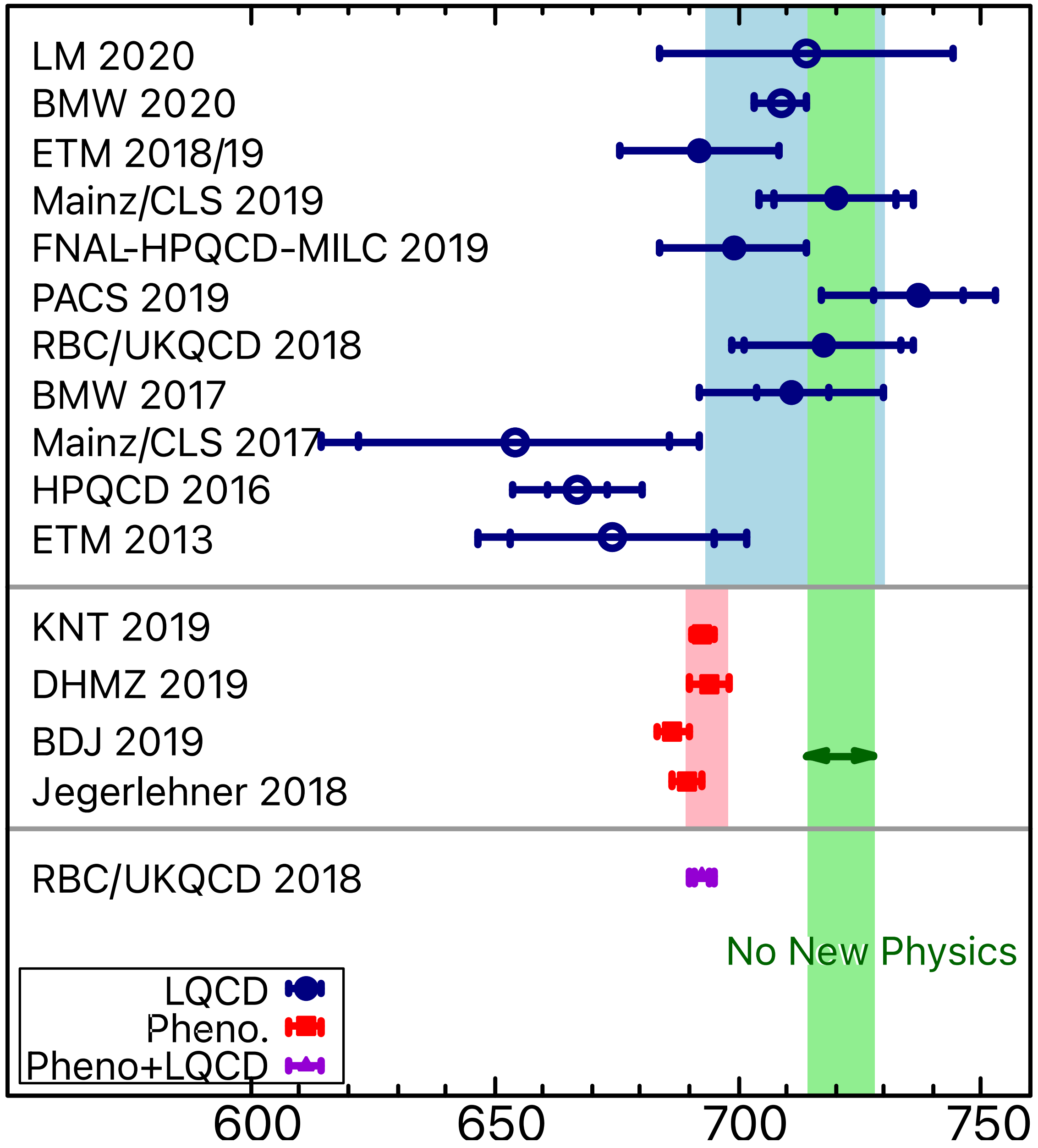


a_μ-HVP from Lattice QCD+QED



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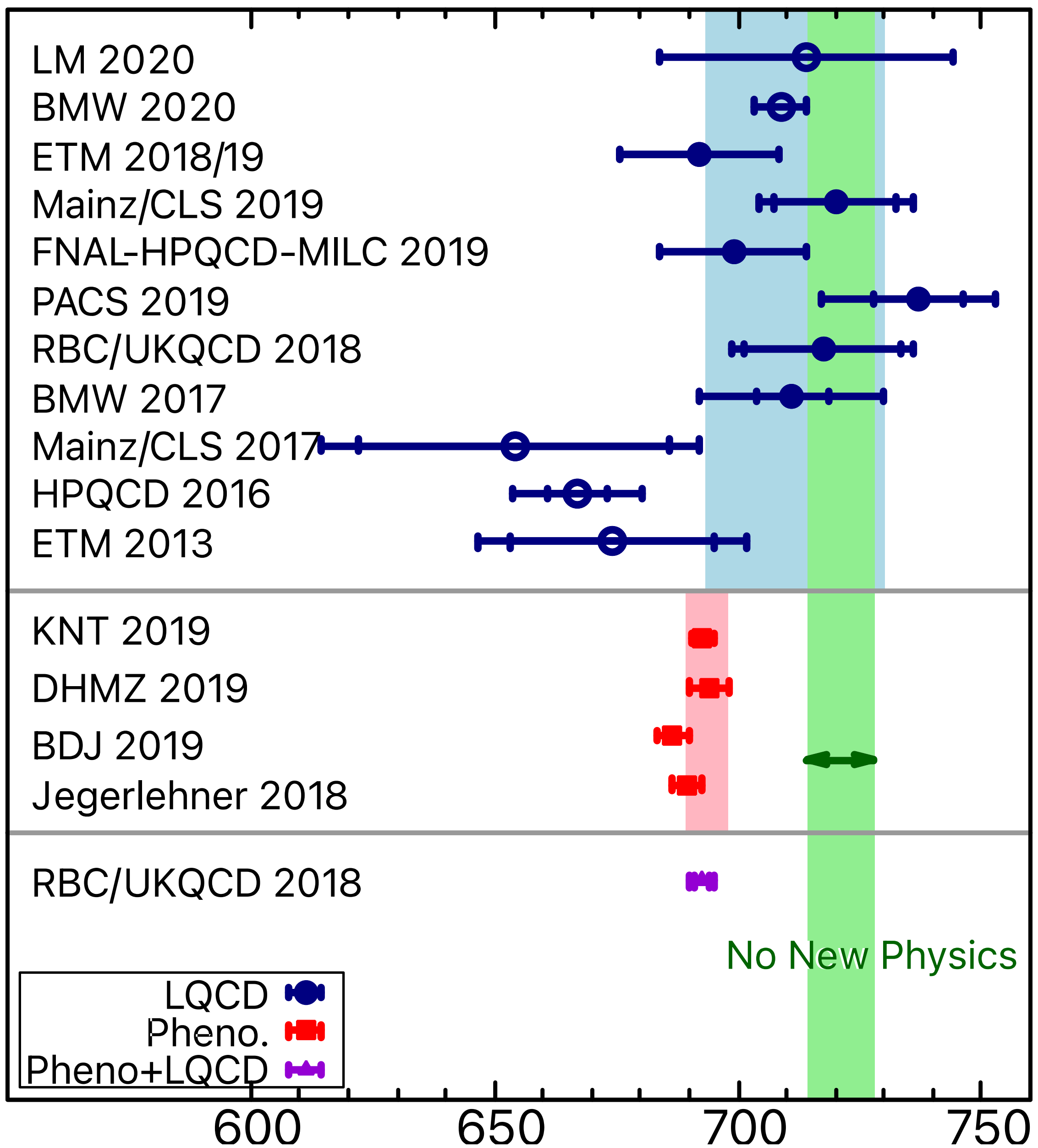
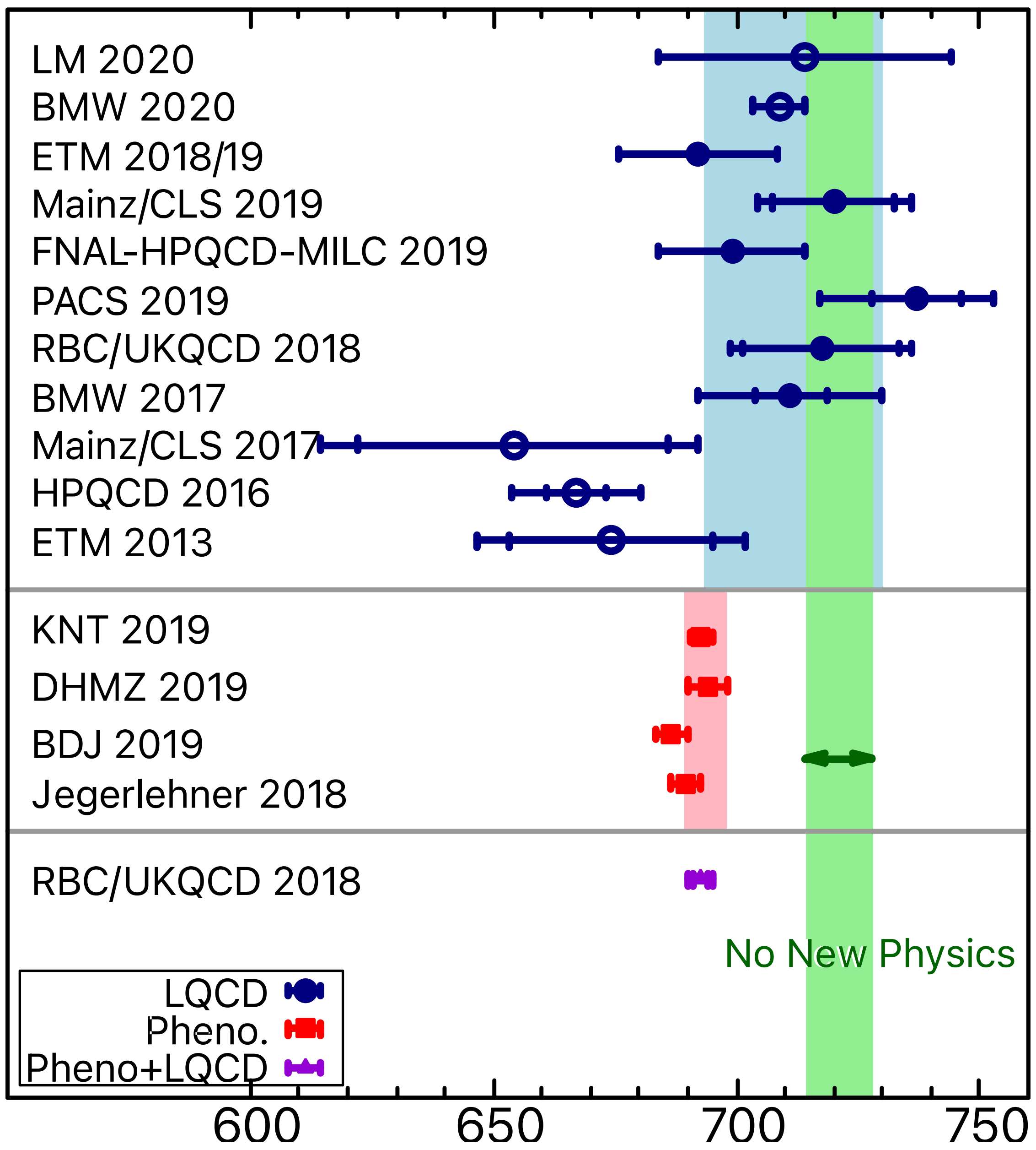


(2.6%)

HVP (Lattice): a_μ = 7116 (184) x 10^{-11}

a_μ-HVP from Lattice QCD+QED



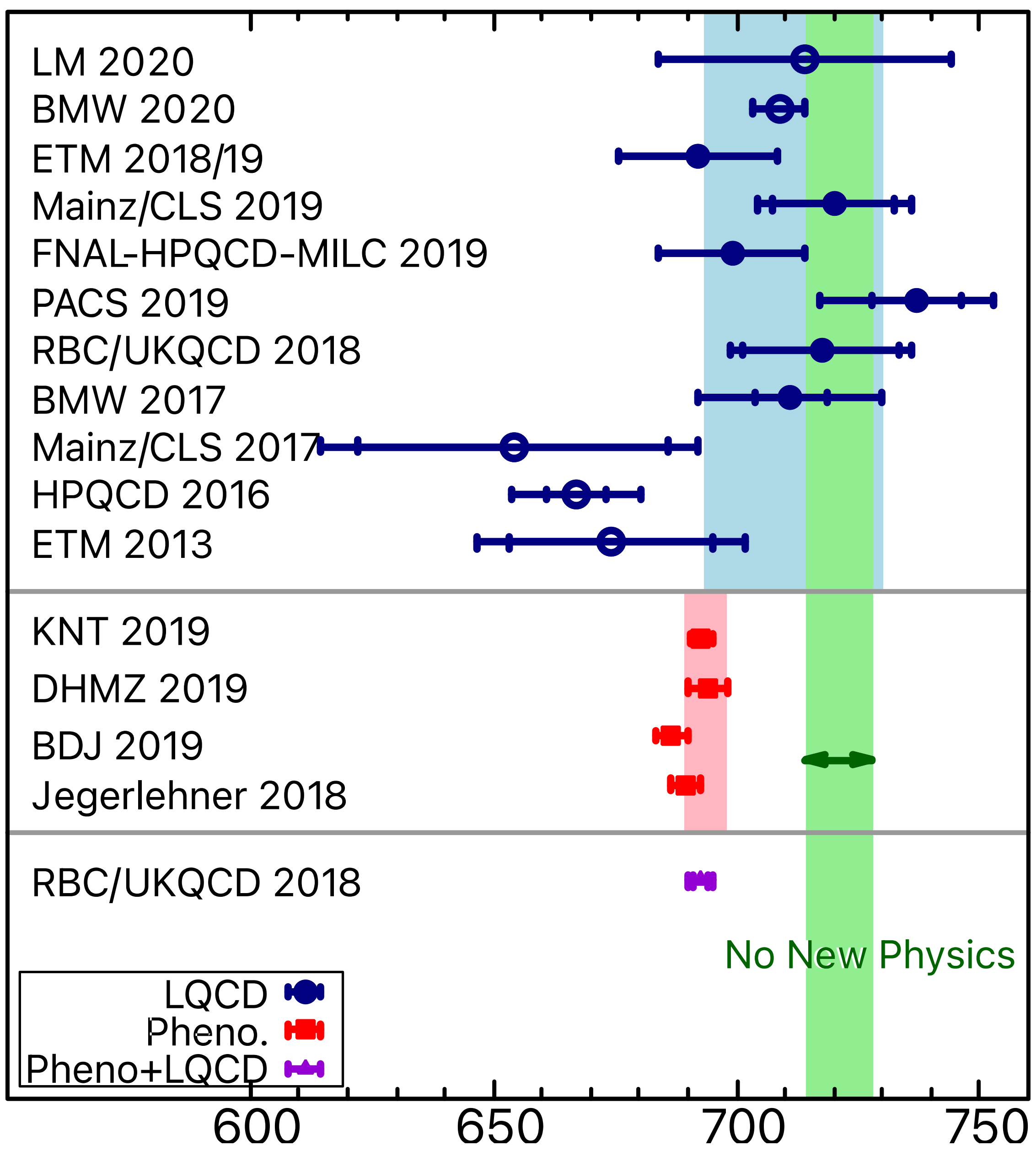
(2.6%)

HVP (Lattice): a_μ = 7116 (184) x 10^{-11}

(0.58%)

HVP (pheno): a_μ = 6931 (40) x 10^{-11}

a_μ-HVP from Lattice QCD+QED



(0.75%)

HVP (BMW-20): a_μ = 7087 (53) x 10^{-10}

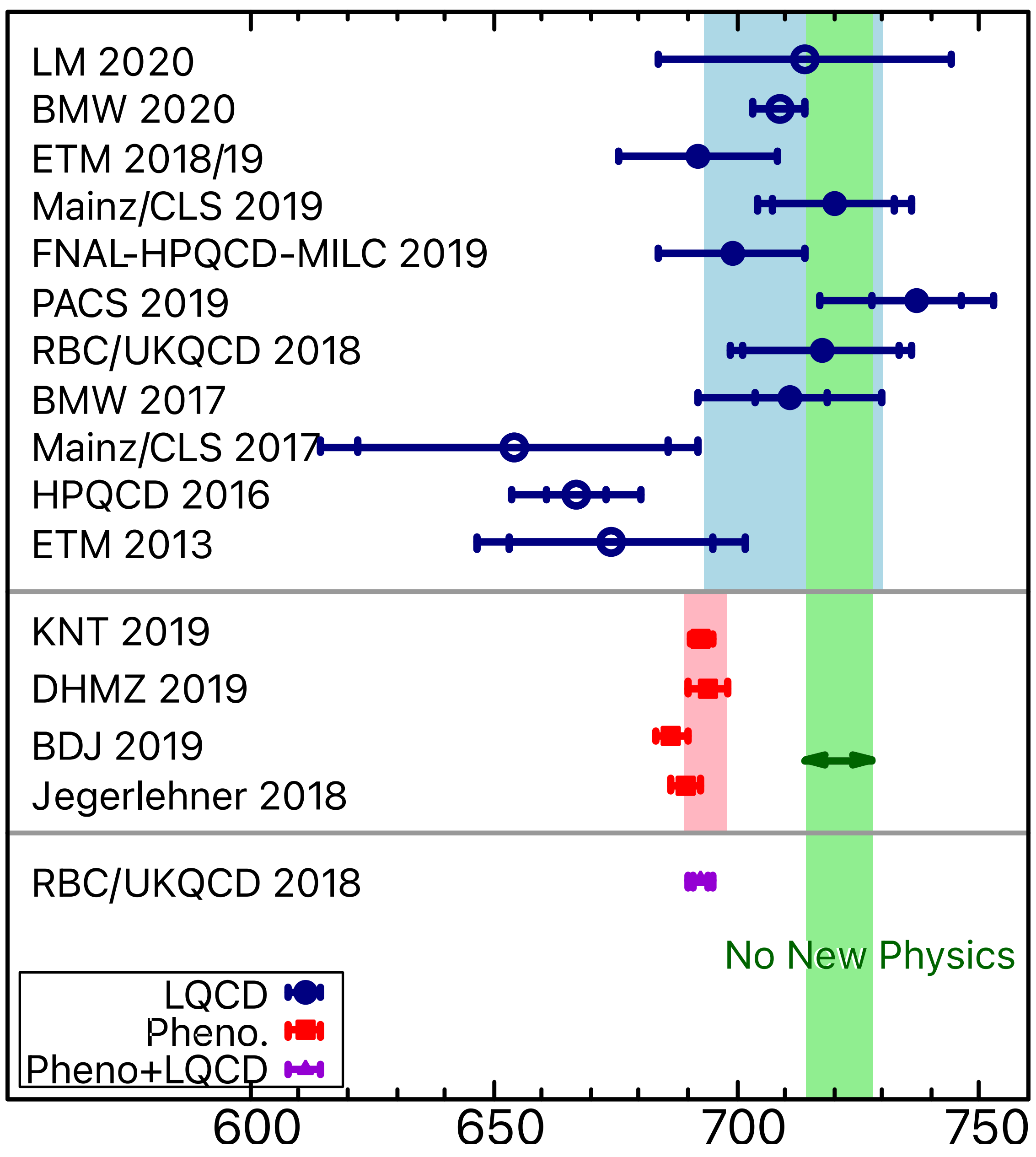
(2.6%)

HVP (Lattice): a_μ = 7116 (184) x 10^{-11}

(0.58%)

HVP (pheno): a_μ = 6931 (40) x 10^{-11}

a_μ-HVP from Lattice QCD+QED



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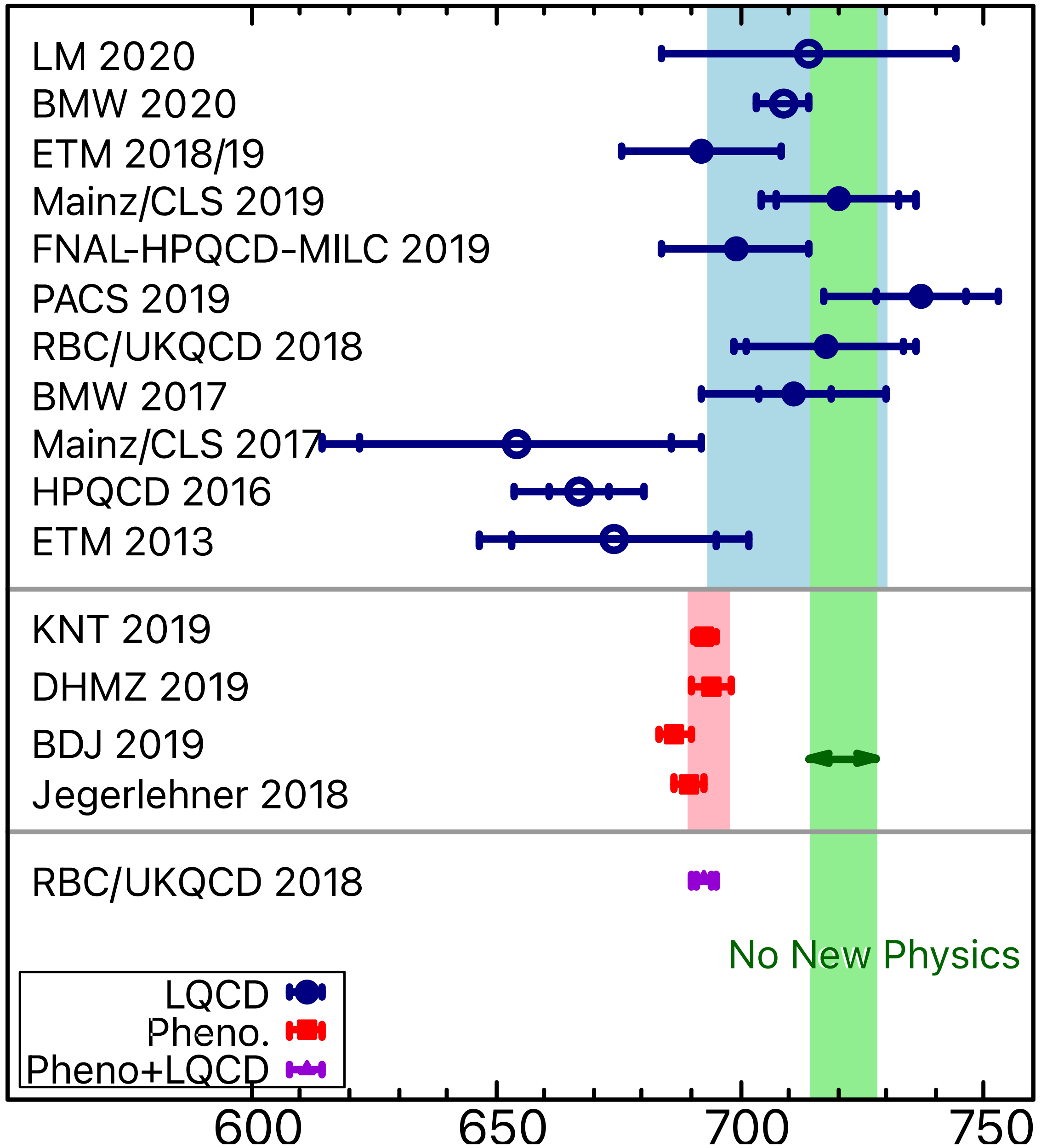
Lattice – pheno $\approx 18.5\ (18.8)$

a_μ-HVP from Lattice QCD+QED



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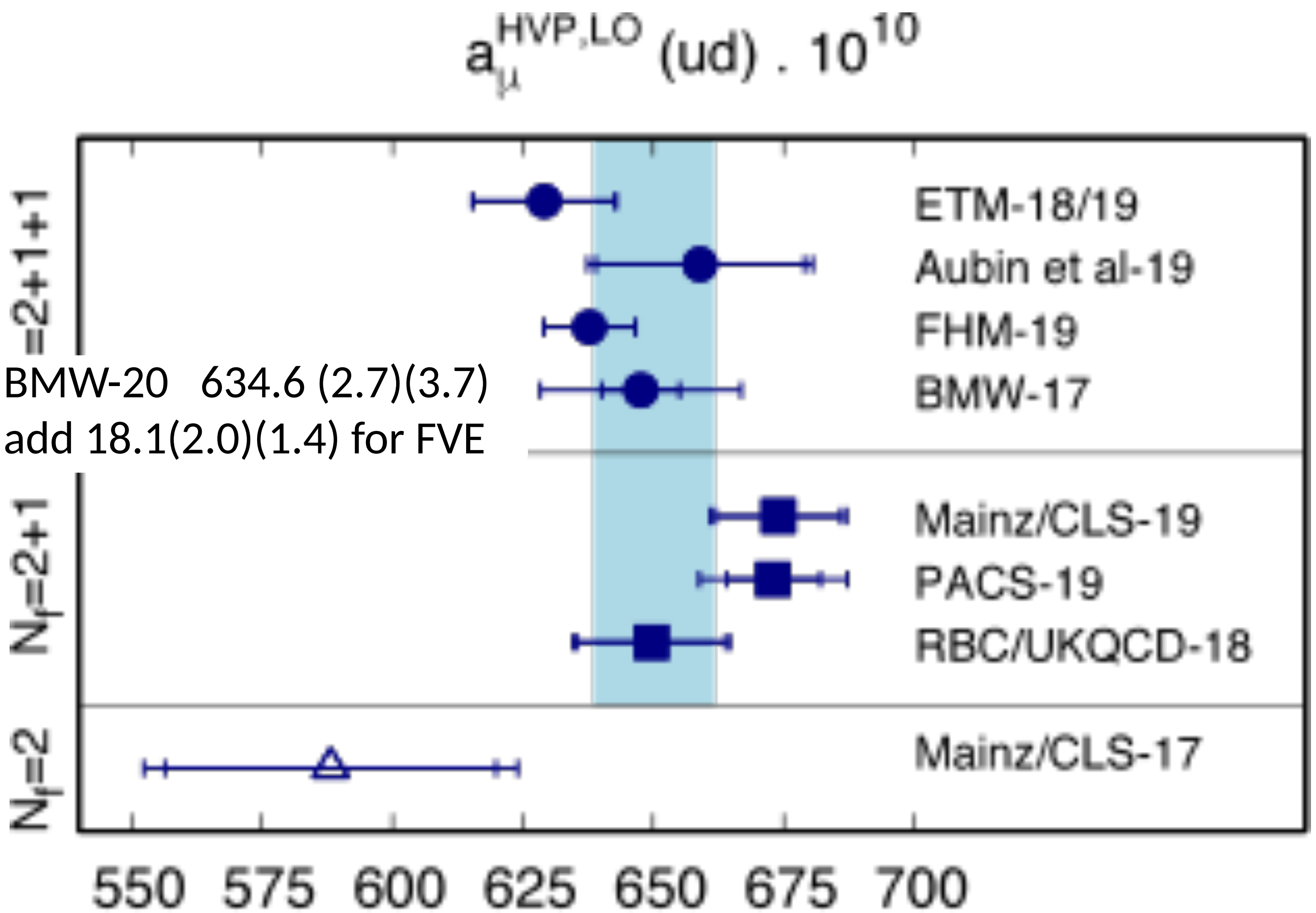
Lattice – pheno $\approx 18.5\ (18.8)$

BMW-20 – pheno $\approx 15.6\ (6.6)$

The connected light quark contribution



$a_\mu^{\text{HVP, ll}} = 650.2\ (11.6) \times 10^{-10}$

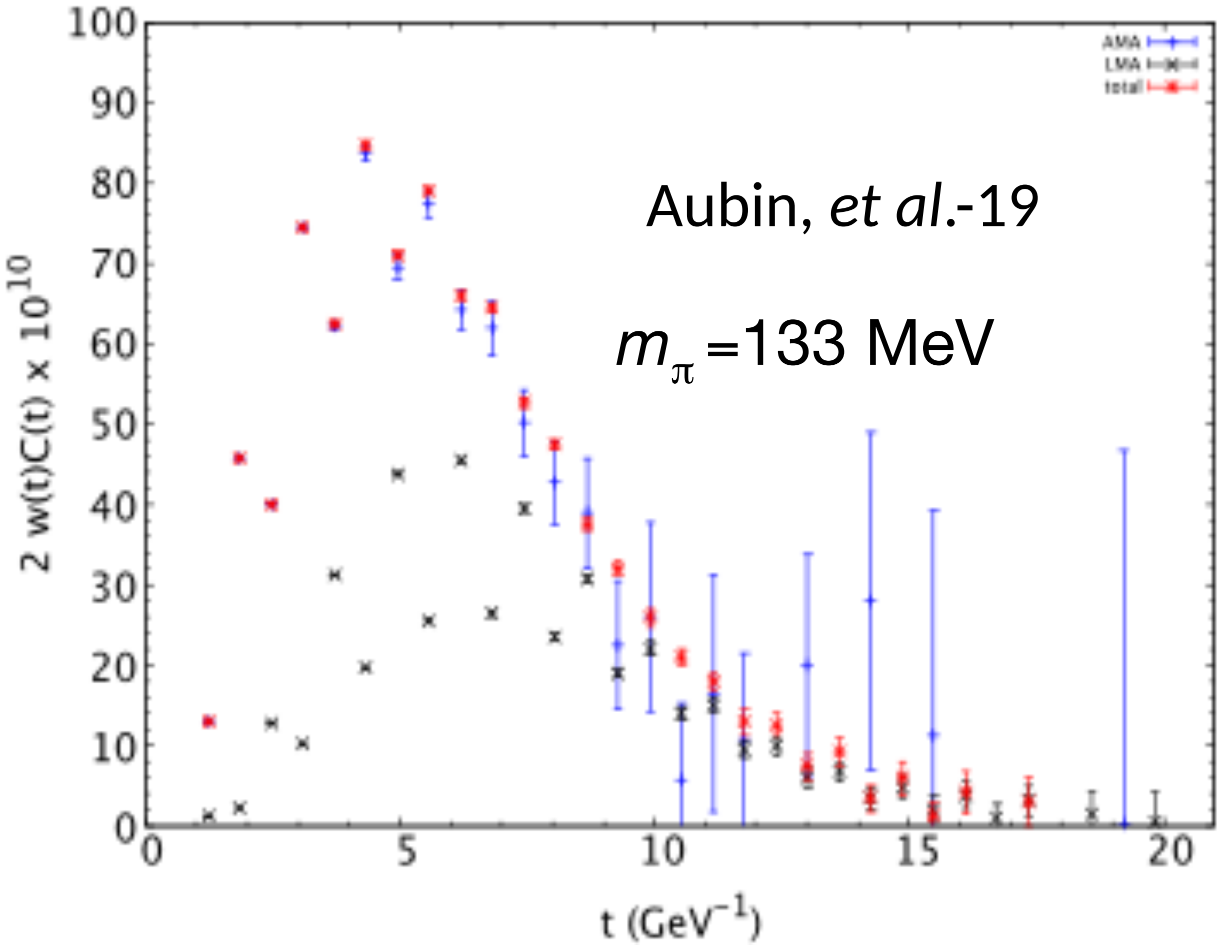
Large spread ~630-675 x 10^{-10}

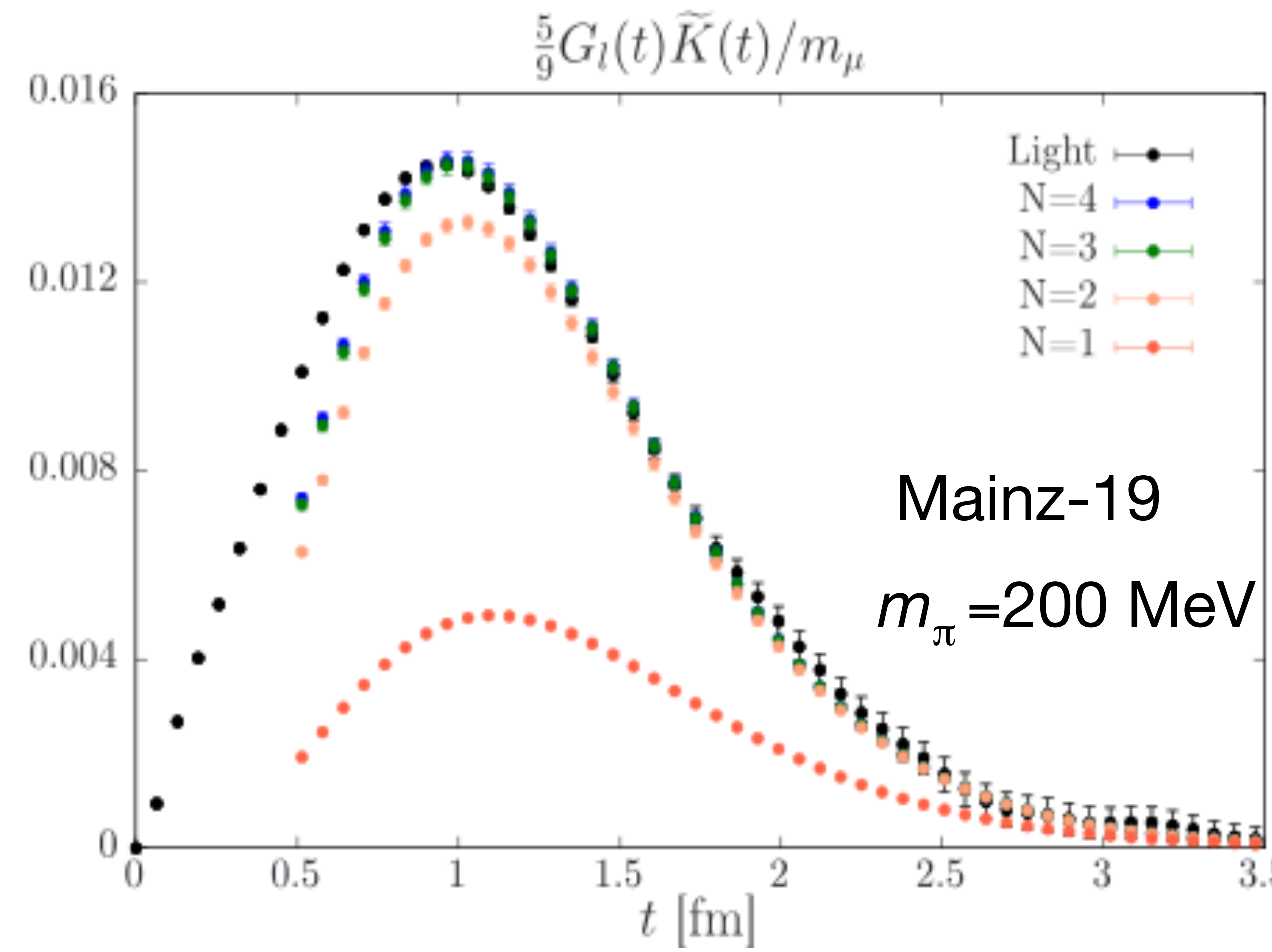
(Does not include BMW-20)

State-of-the-art is to use physical quark masses
(Significant chiral extrap in ETM and Mainz/CLS)

Long distance contributions
and the statistical error

Correlator reconstruction: Mainz, RBC/UKQCD





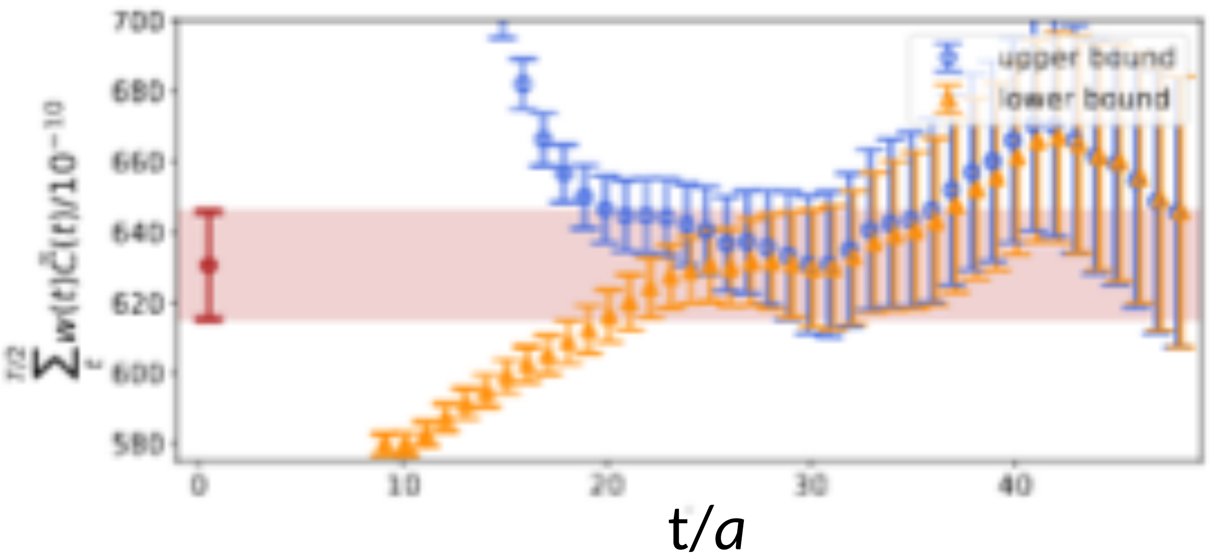
Low Mode Average: RBC/UKQCD-18, Aubin, *et al*.-19, BMW-20 (*C*(t) averaged over all EM current source-sink pairs)

$$C(x_0) = \sum_{n=1}^{\infty} A_n e^{-E_n x_0}$$

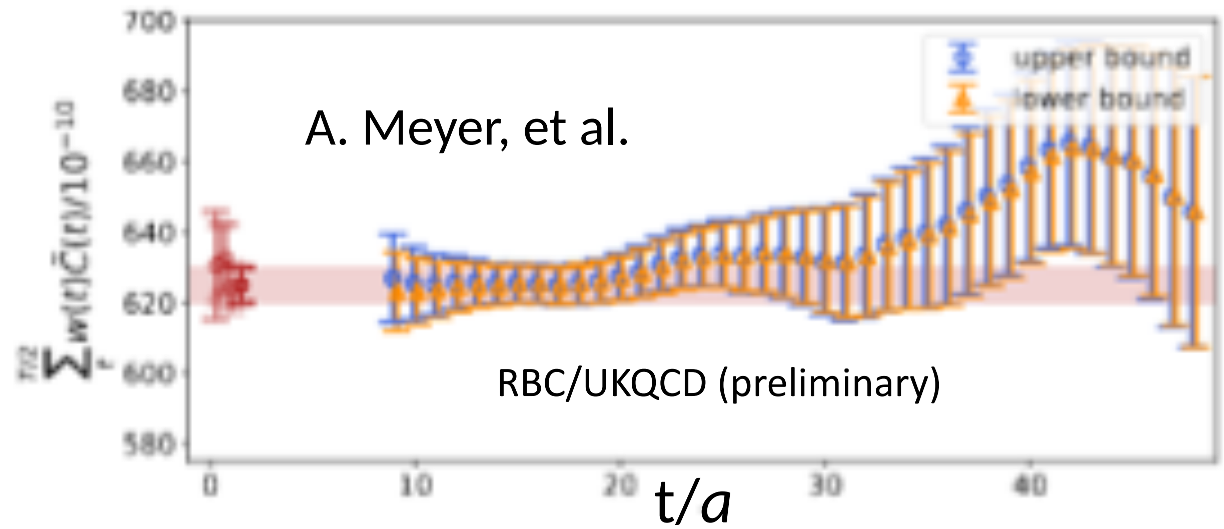
Dominated by two pion states at large time

Bounding method

Original method:
BMW-17,20, RBC/UKQCD-18



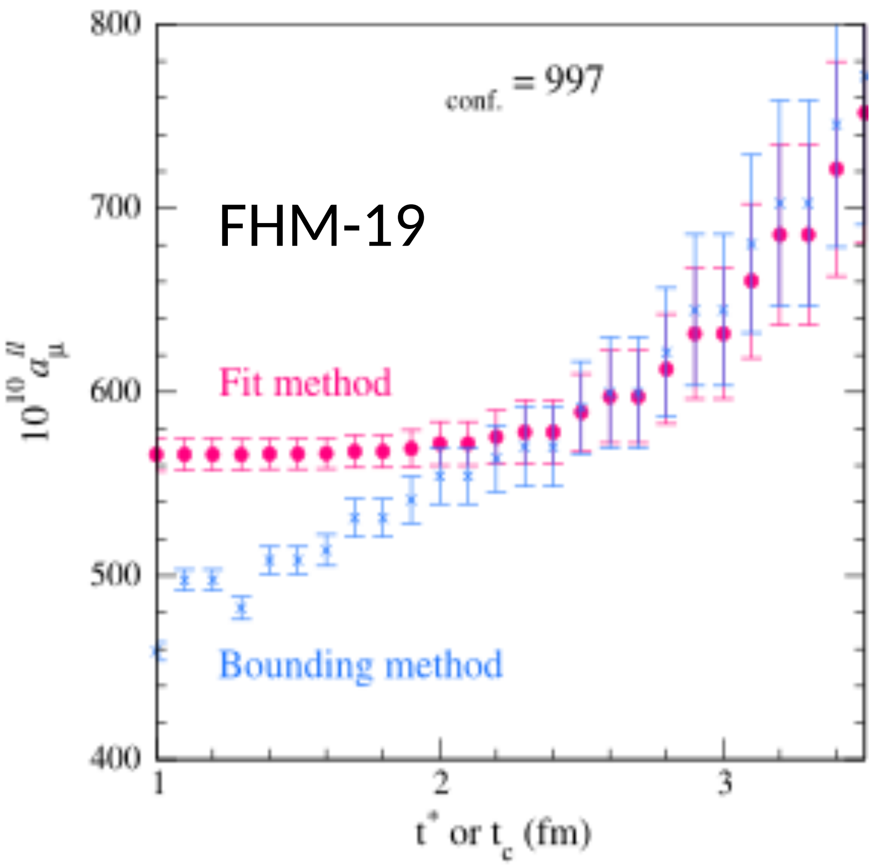
Mainz-19, RBC/UKQCD: Improved method using long distance correlator reconstruction



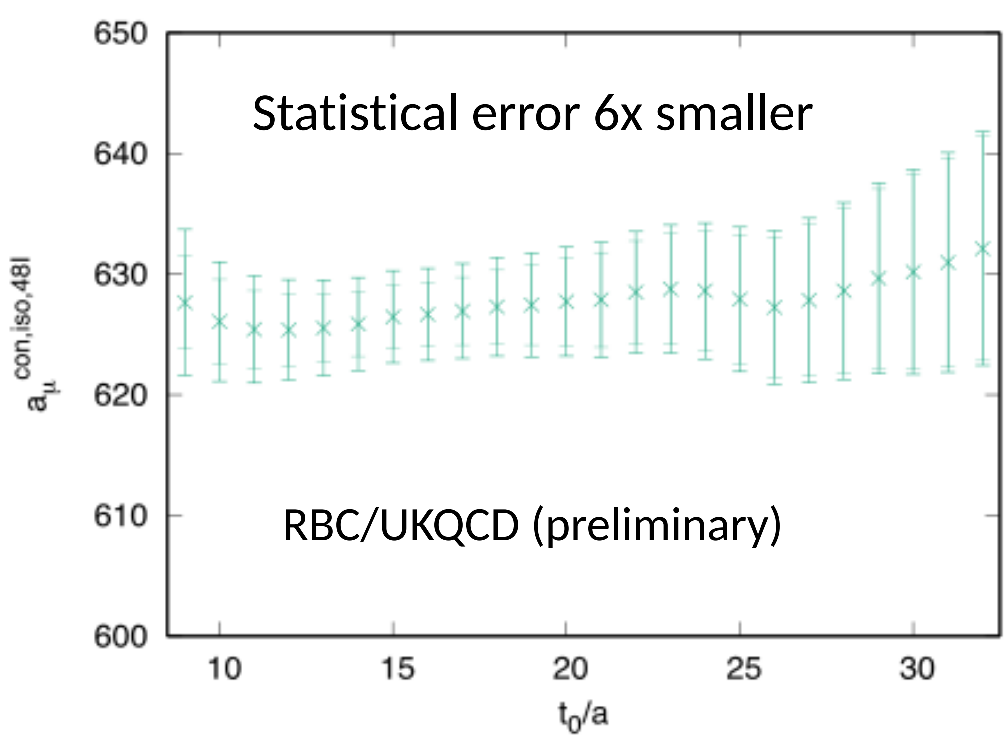
Fit method

FHM-19, ETM-19

replace data beyond t* with multi-exponential, multi-operator fit

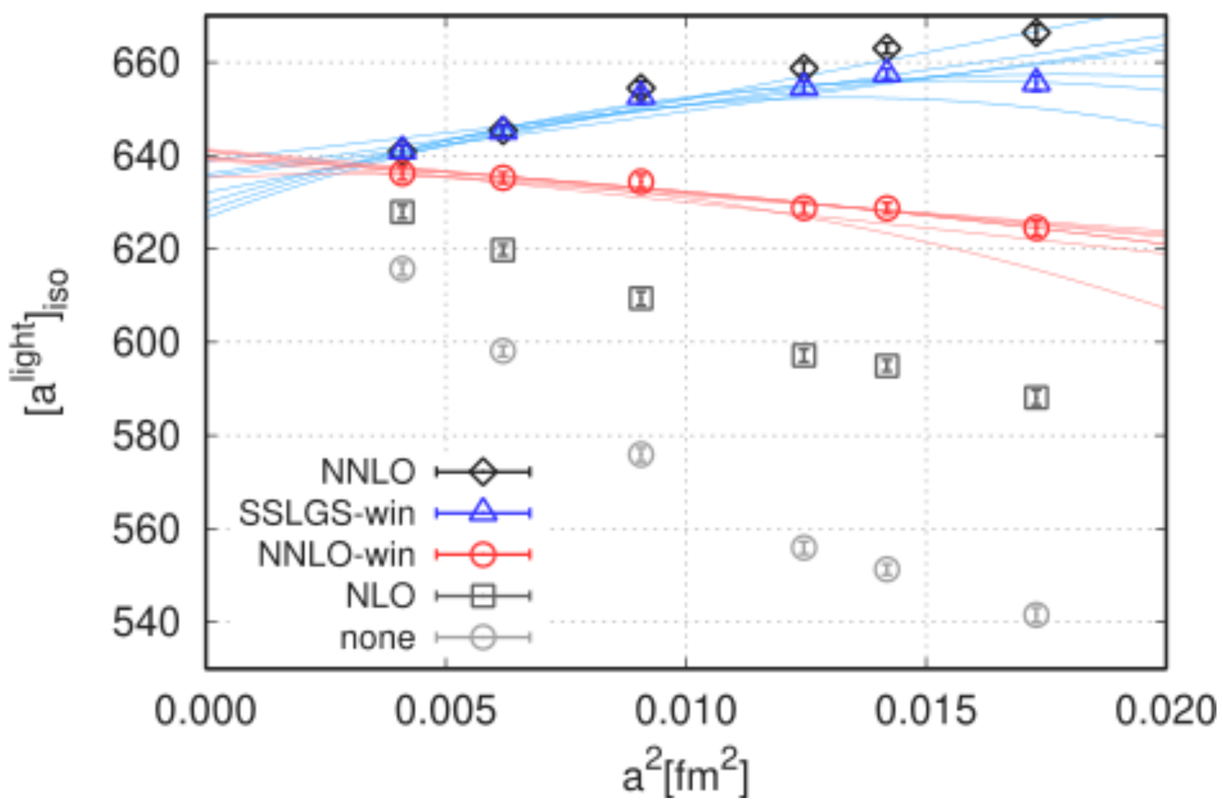


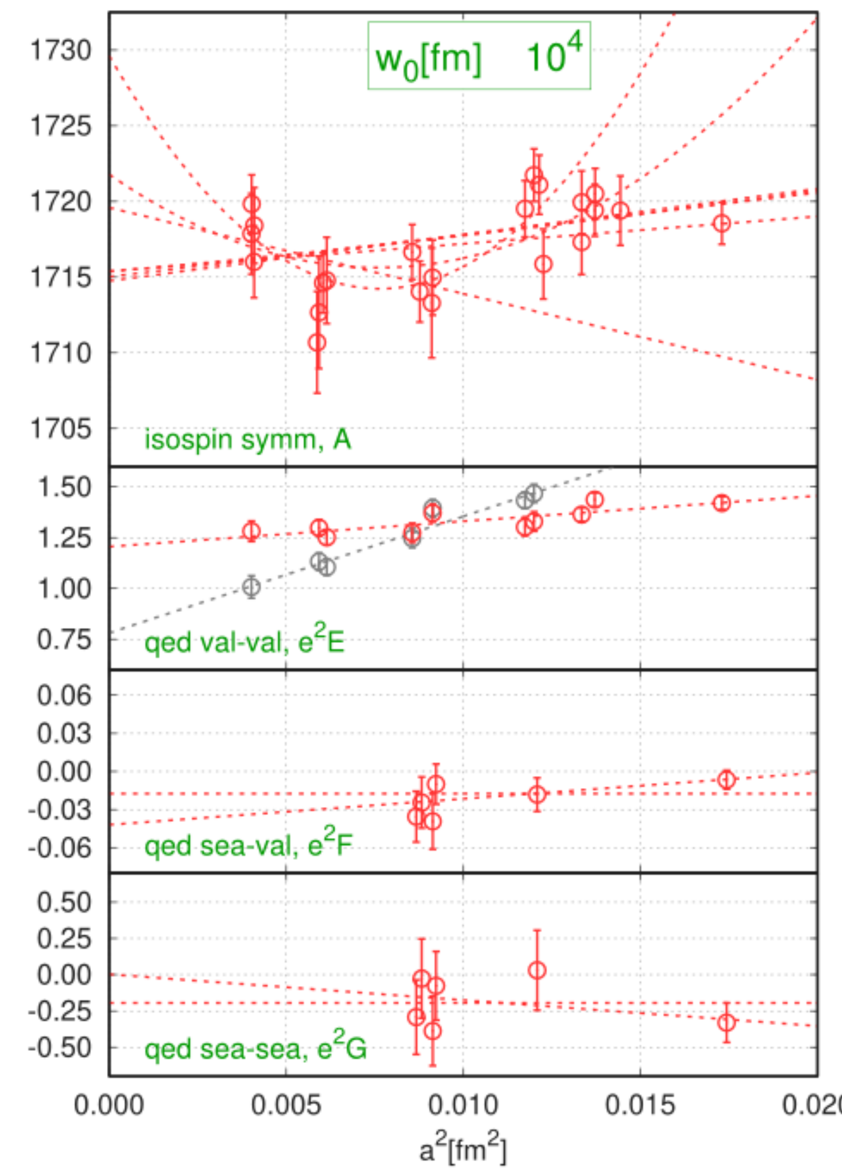
(much) Shorter distances possible



BMW-20 continuum extrapolation ($M_\Omega + w_0$)

$a_\mu + a^2 \; (+a^4)$





634.6 (2.7)(3.7)(0.5)(...) x 10^{-10} (stat, taste, CL, …)

$L_{\mathrm{ref}} = 6.272 \text{ fm} \; = \; \frac{2}{3} T_{\mathrm{ref}}$

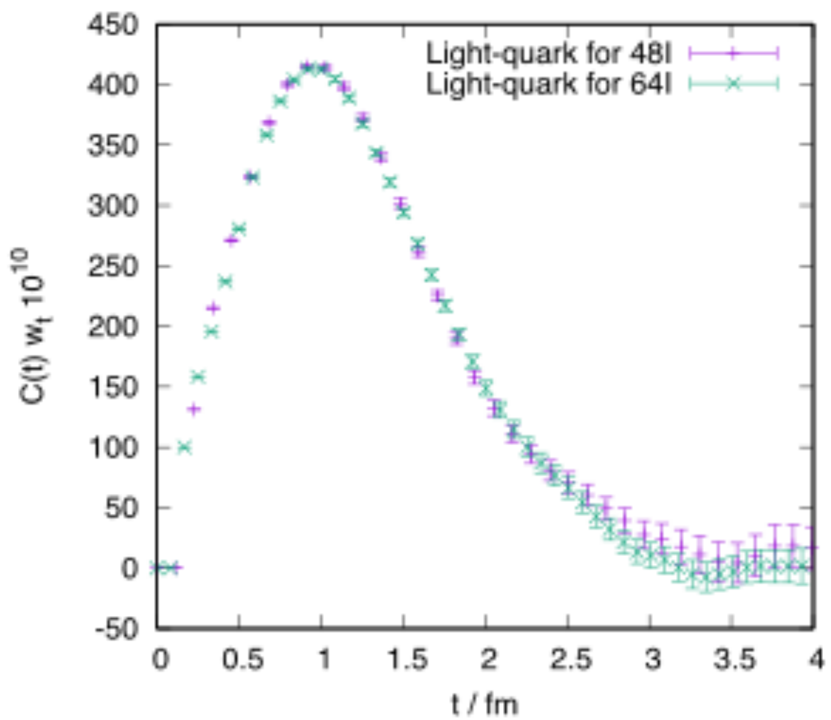
(FV correction 18.7(2.5), IB 5.7(…))

$w_0 = 0.17236(29)(63)$ fm

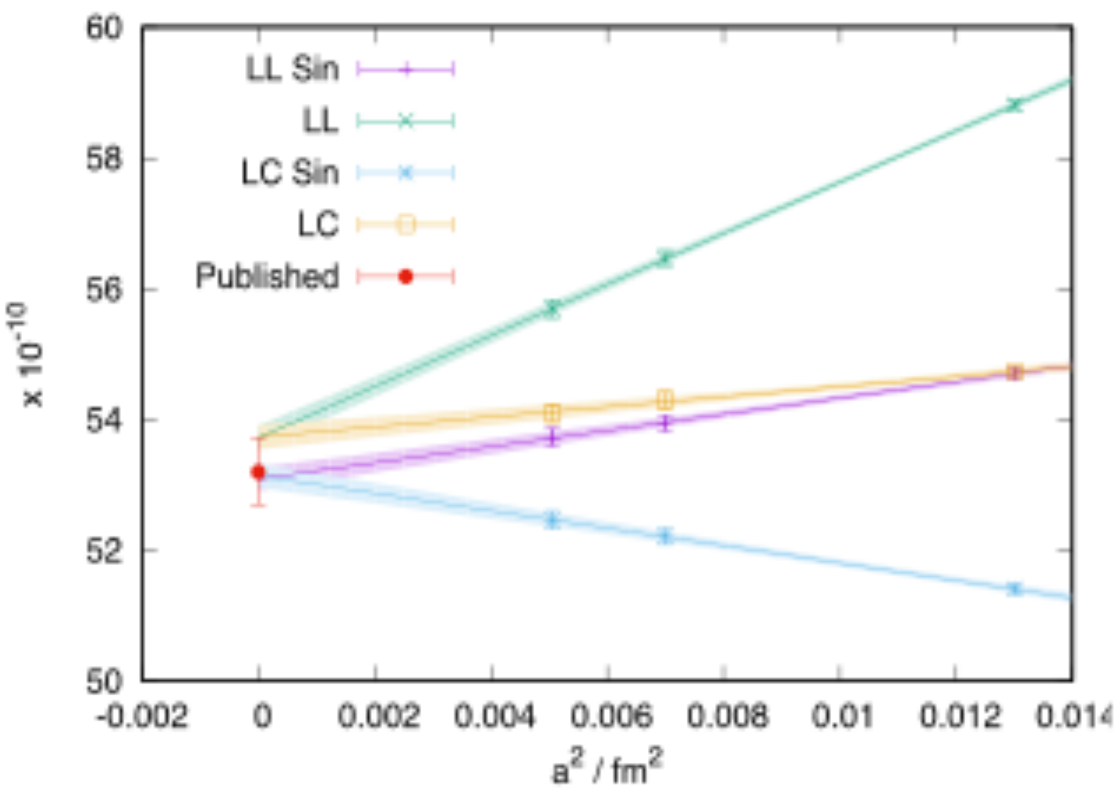
(0.4%)

RBC/UKQCD-18 continuum limit (M_Ω)

$a_\mu + a2$



- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):



(strange quark contribution)

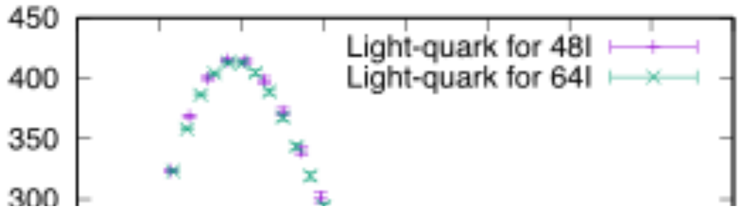
- a^{-1} = 1.730 GeV results differ by a few percent from continuum limit
- 2.7 GeV lattice underway to compliment 1.730, and 2.359 GeV

Light quark: 647.9 (14.2)(2.8)(1.5)(3.7)(…) x 10^{-10} (statistical, a^4, scale setting, FV, …)

total: 705.9 (14.6)(2.9)(1.8)(3.7)(…) x 10^{-10} (statistical, a^4, scale setting, FV, …)

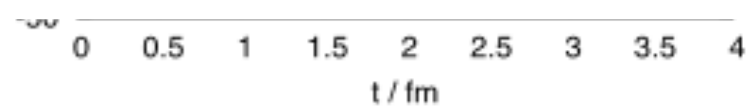
RBC/UKQCD-18 continuum limit (M_Ω)

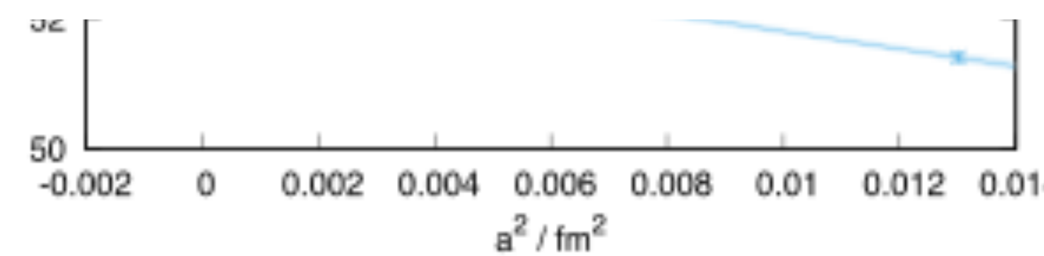
$a_\mu + a2$



▶ Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):

- More than doubled statistics on current ensembles
- Improved bounding method
- Estimate sub-percent total error with these improvements
- Need to add 3rd lattice spacing to reach 0.7 % error





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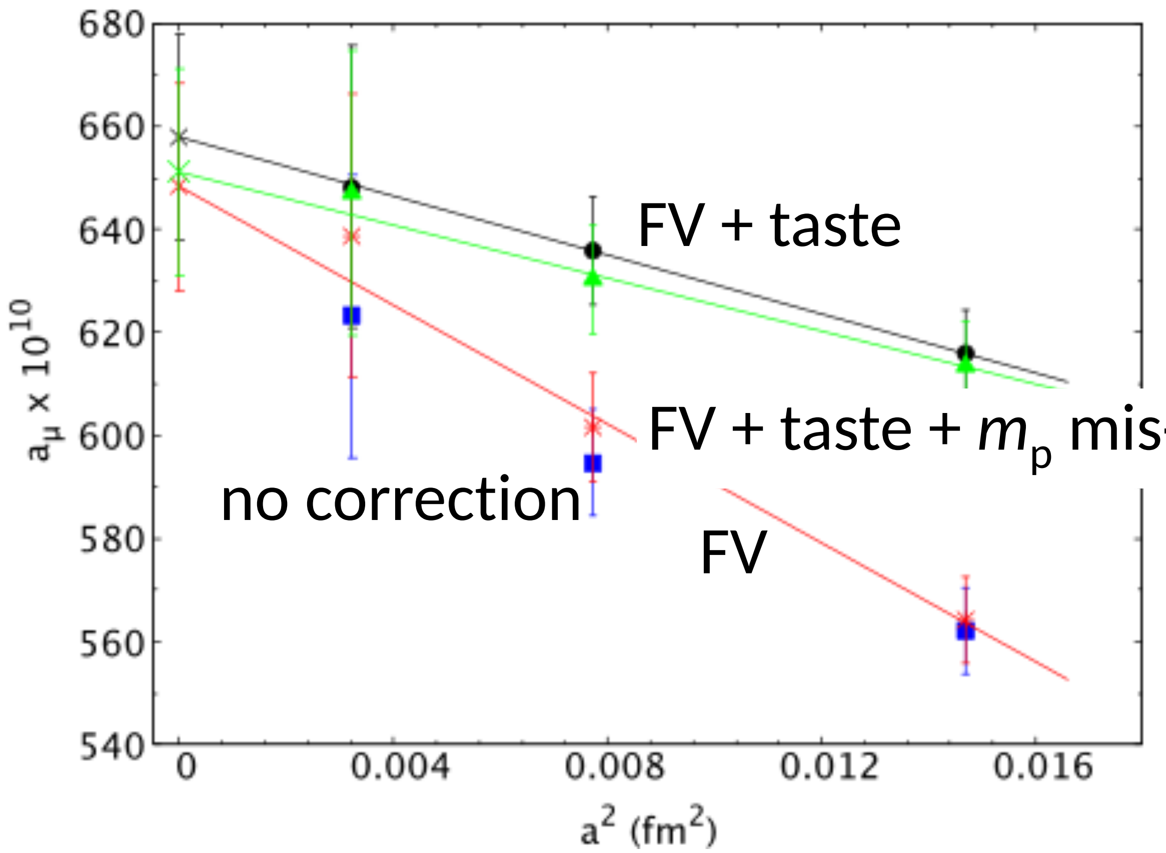
total: 705.9 (14.6)(2.9)(1.8)(3.7)(...) x 10^{-10} (statistical, a^4, scale setting, FV, ...)

Aubin, *et al.*-20 continuum limit (f_π + w_0, from FHM 19)

(Light quark contribution)



a_μ Fit $+ a2$

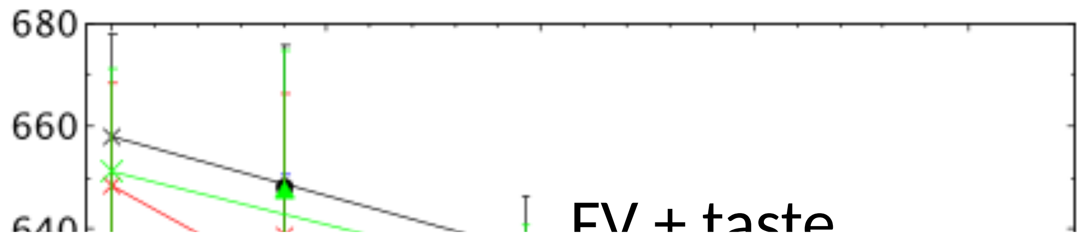
- taste symmetry breaking large
- corrected at NLO in SChPT

651 (20)(5)(5) x 10^{-10}

Statistical, CL fit, scale setting (includes NLO taste breaking (and FV) corrections)

Aubin, *et al.*-20 continuum limit ($f_\pi + w_0$, from FHM 19)

(Light quark contribution)



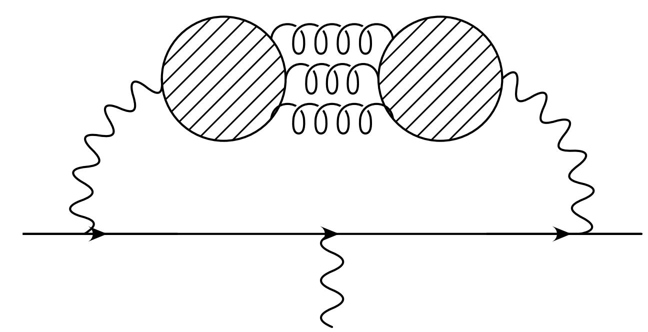
a_μ Fit

- More than doubled statistics on finest ensemble
- Improved low-mode average on finest ensemble
- Added new coarsest ensemble
- Adding statistics for two finest ensembles
- Aiming for 1% total

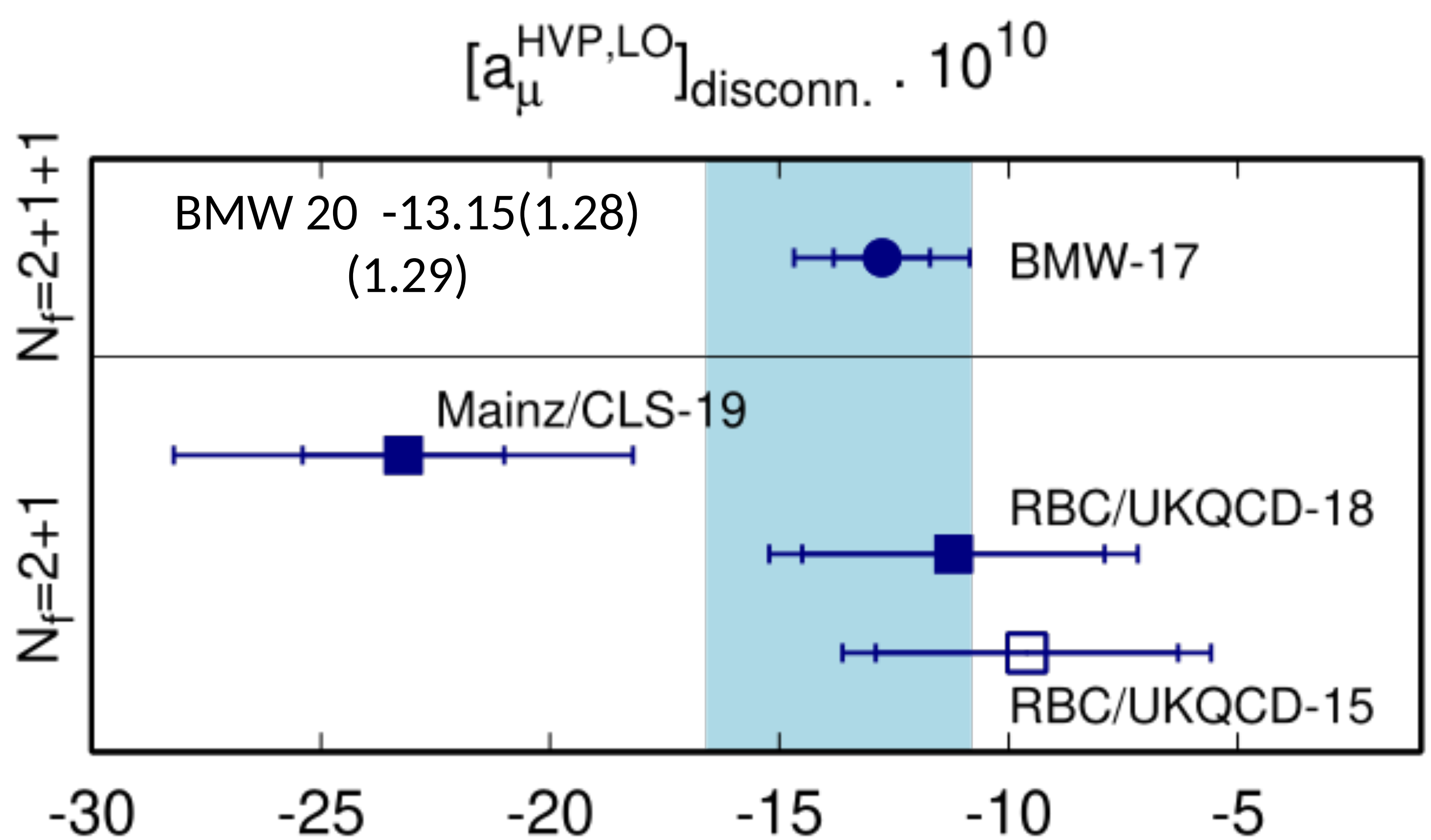
651 (20)(5)(5) x 10^{-10}

Statistical, CL fit, scale setting (includes NLO taste breaking (and FV) corrections)

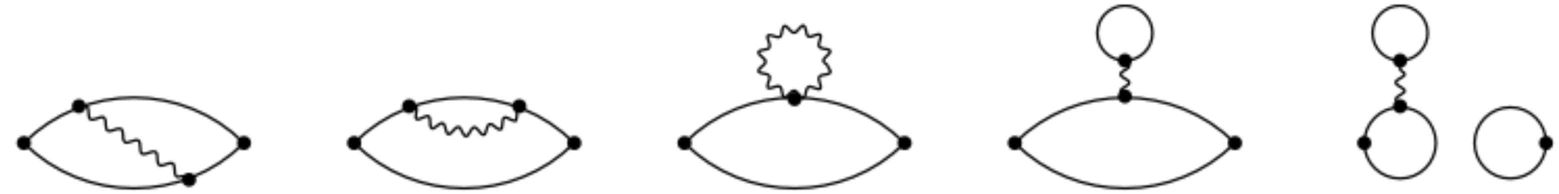
Disconnected contributions



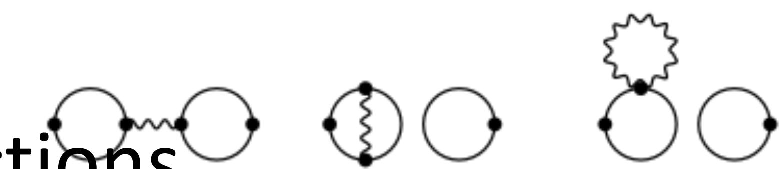
- More groups needed
- Includes strange contribution [Mainz]
- Statistical and systematic errors important



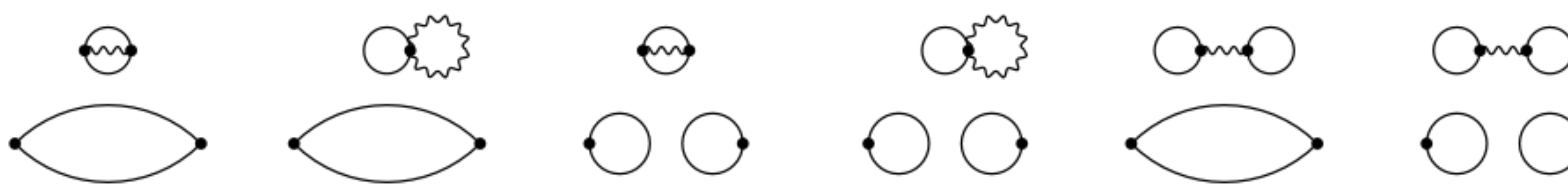
Isospin symmetry breaking corrections



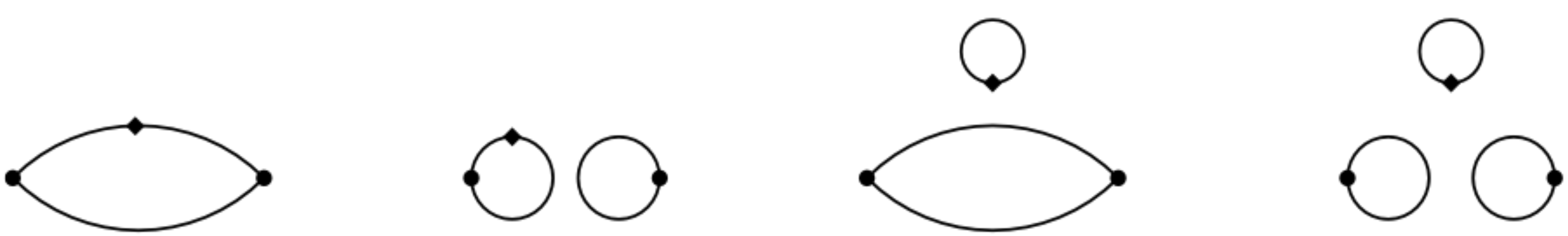
(a) V (b) S (c) S_T (d) T (e) T_d



(f) F (g) D3 (h) $D3_T$



(i) D1 (j) $D1_T$ (k) $D1_d$ (l) $D1_{d,T}$ (m) D2 (n) $D2_d$



(a) M (b) O (c) R (d) R_d

Isospin symmetry breaking corrections

Collaboration	QED			Strong IB	
	V+S (+S_T)		F (+D3)	M	O
BMW-20	-1.27(40)(33)		-0.55(15)(11)	6.59(63)(53)	-4.63(54)(69)
ETM-19	1.1(1.0)			6.0(2.3)	
RBC/UKQCD-18	5.9(5.7)(…)		-6.9(2.1)(1.4)(…)	10.6(4.3)(…)	
FHM-19				1.5(7) % a_μ^{ll}	
LM-20				9.0(0.8)(1.2)	

- statistical errors large (except BMW-20, LM-20)
- Spread is relatively large
- FV effects can be very large (*e.g.,* see LM-20)
- large cancelations

Towards precise comparisons: the window method

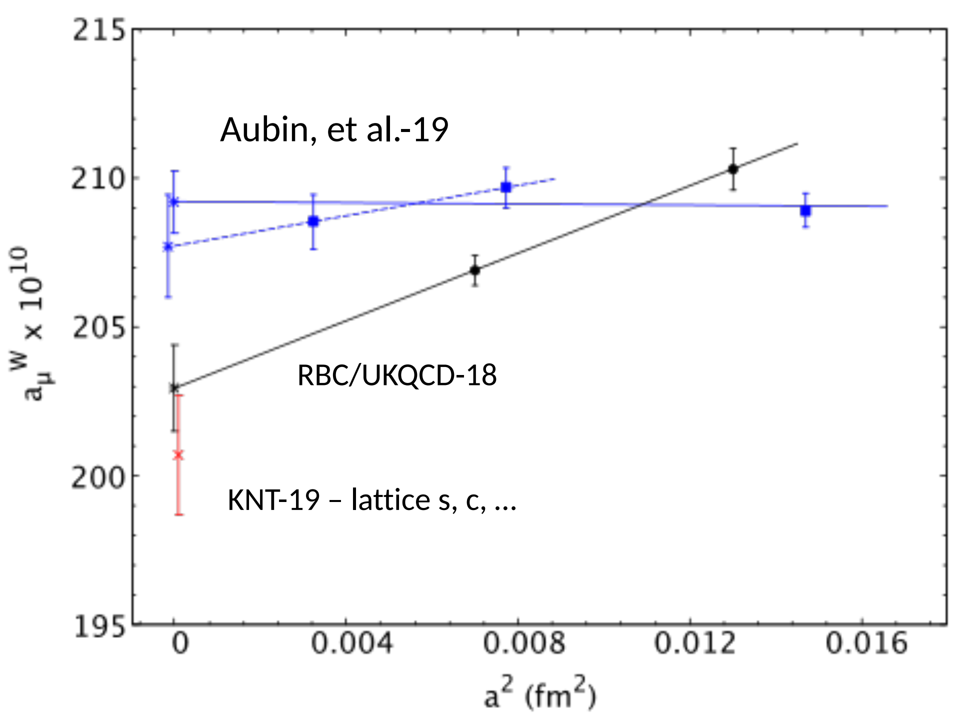
[RBC/UKQCD-18]

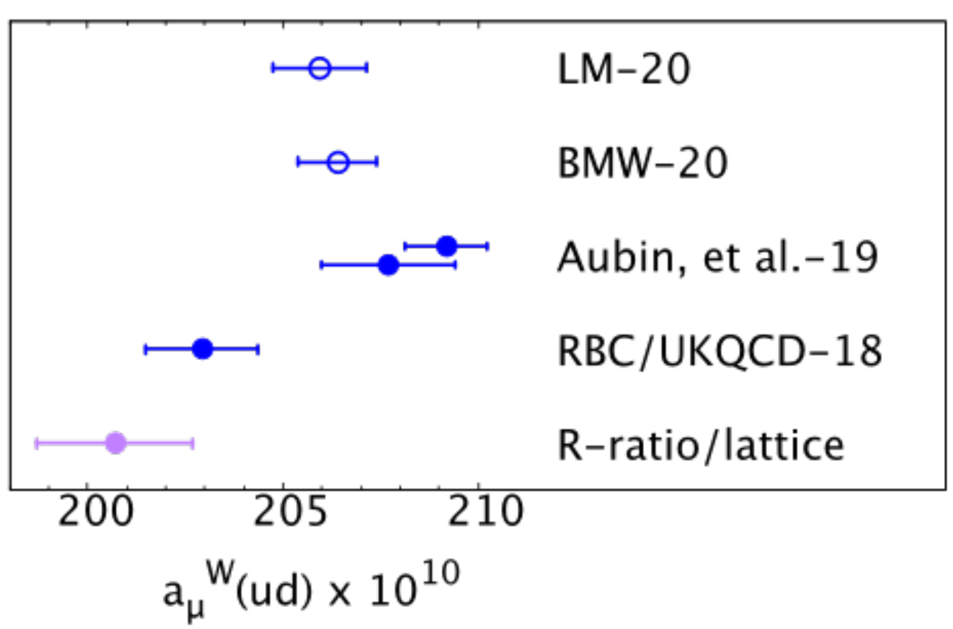
$$a_\mu^{\text{HVP, LO}} = a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}} ,$$

$$a_\mu^{\text{SD}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\widetilde{f}(x_0)[1 - \Theta(x_0, t_0, \Delta)] ,$$

$$a_\mu^{\text{W}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\widetilde{f}(x_0)[\Theta(x_0, t_0, \Delta) - \Theta(x_0, t_1, \Delta)] , \quad \Theta(t, t', \Delta) = [1 + \tanh\left(\frac{t - t'}{\Delta}\right)]/2$$

$$a_\mu^{\text{LD}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\widetilde{f}(x_0)\Theta(x_0, t_1, \Delta) ,$$





(ud connected, 0.4 – 1.0 fm window)

Towards precise comparisons: the window method

[RBC/UKQCD-18]

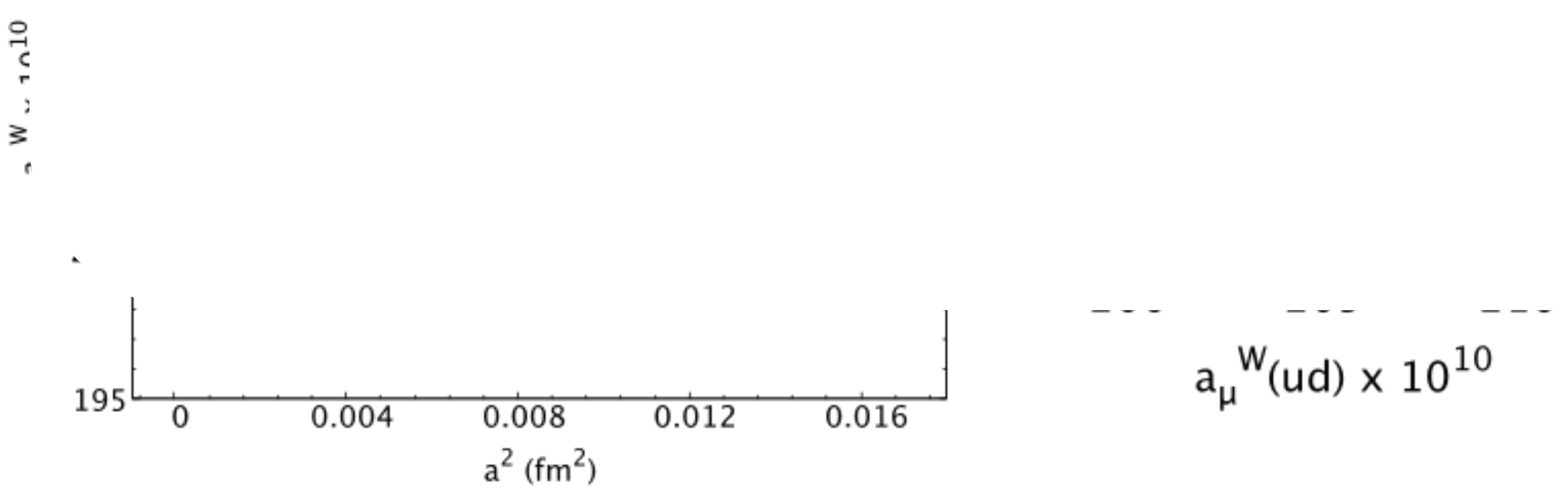
$$a_\mu^{\mathrm{HVP,\ LO}} = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}\,,$$

$$a_\mu^{\mathrm{SD}} = \left(\frac{\alpha}{\pi}\right)^2 \int^\infty$$

$$a_\mu^{\mathrm{W}} = \left(\frac{\alpha}{\pi}\right.$$

$$a_\mu^{\mathrm{LD}} = \left(\frac{\alpha}{\pi}\right.$$

- Staggered results lie above DWF, R-ratio
- Conserved v. local currents
- More groups need to investigate
- Gradually increase size of window
- Compare with R-ratio (*e.g.*, investigate BaBar KLOE discrepancy)



(ud connected, 0.4 – 1.0 fm window)

Towards precise comparisons: the window method

[RBC/UKQCD-18]

November 2020 workshop on HVP, Harmut Wittig

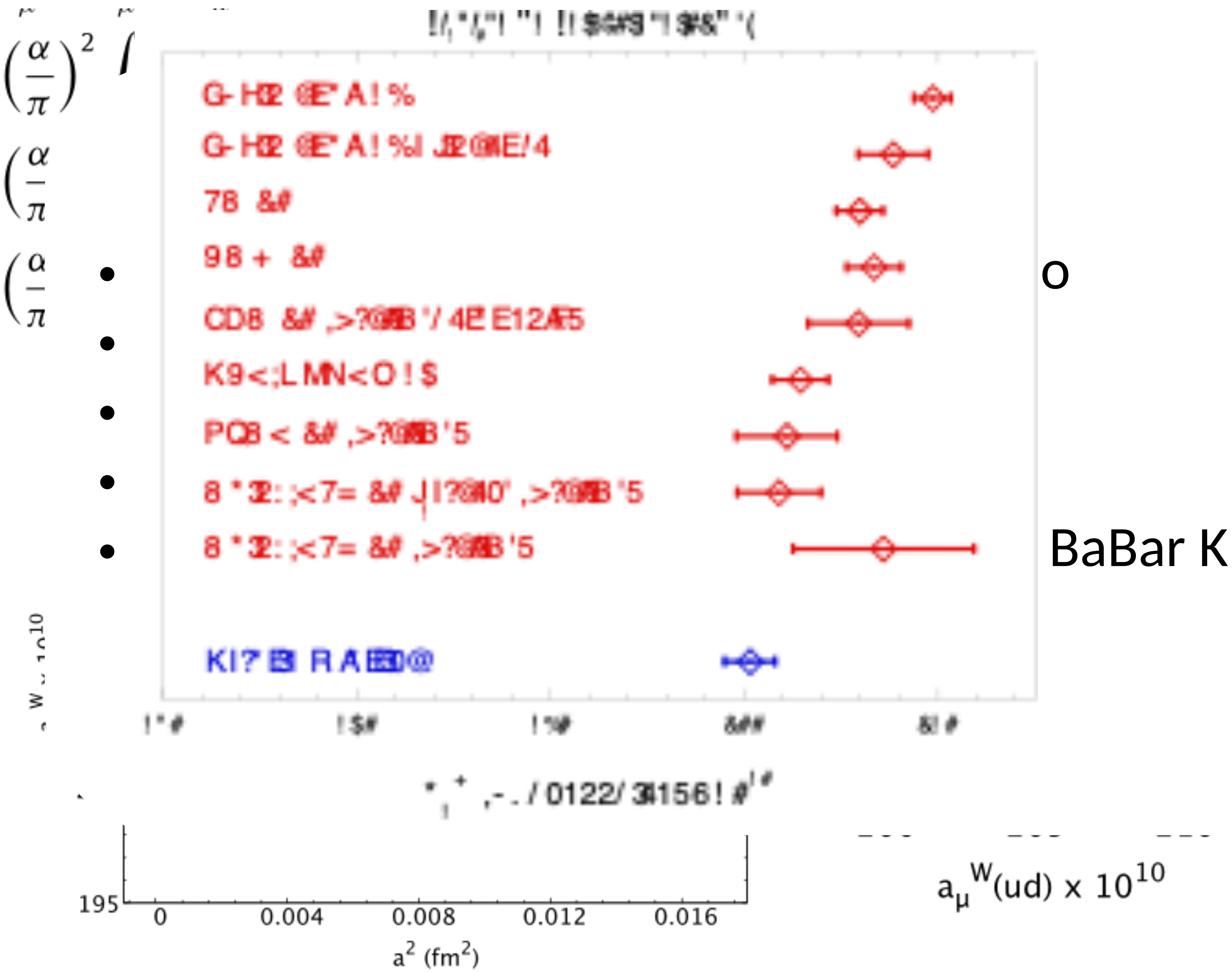
$a_\mu^{\mathrm{HVP,\,LO}} =$

$a_\mu^{\mathrm{SD}} = \left(\frac{\alpha}{\pi}\right)^2 \int$

$a_\mu^{\mathrm{W}} = \left(\frac{\alpha}{\pi}\right.$

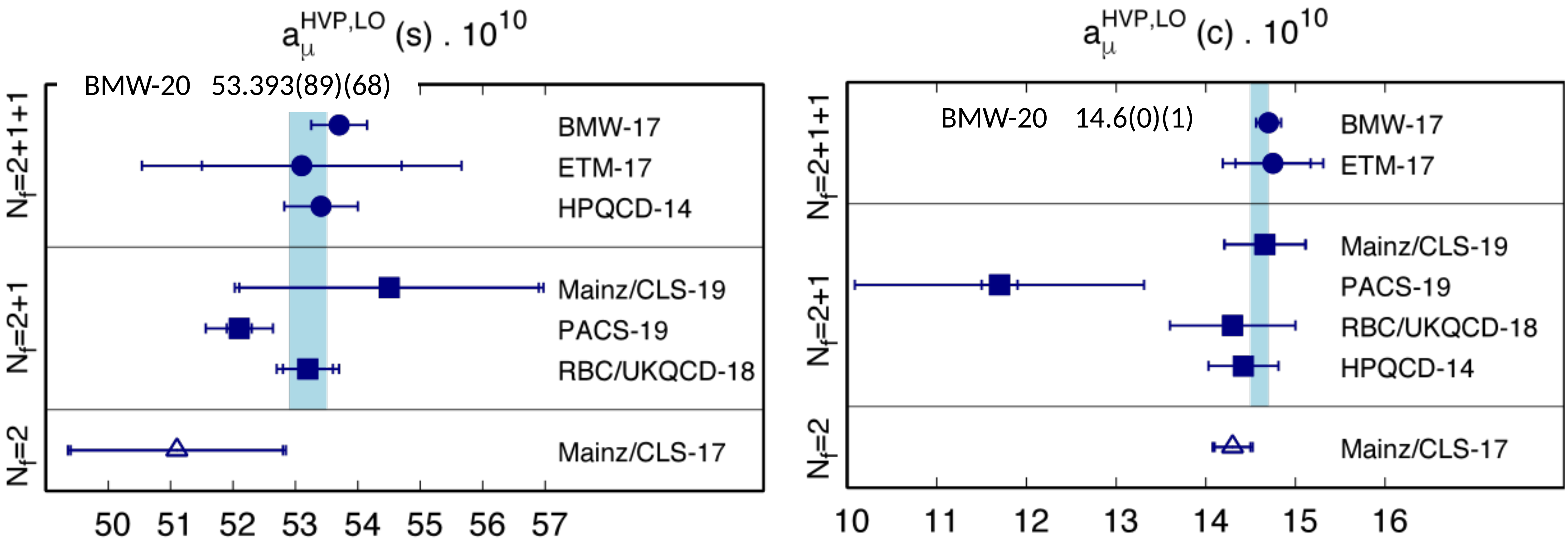
$a_\mu^{\mathrm{LD}} = \left(\frac{\alpha}{\pi}\right.$

- o
-
-
-
- BaBar KLOE discrepancy)



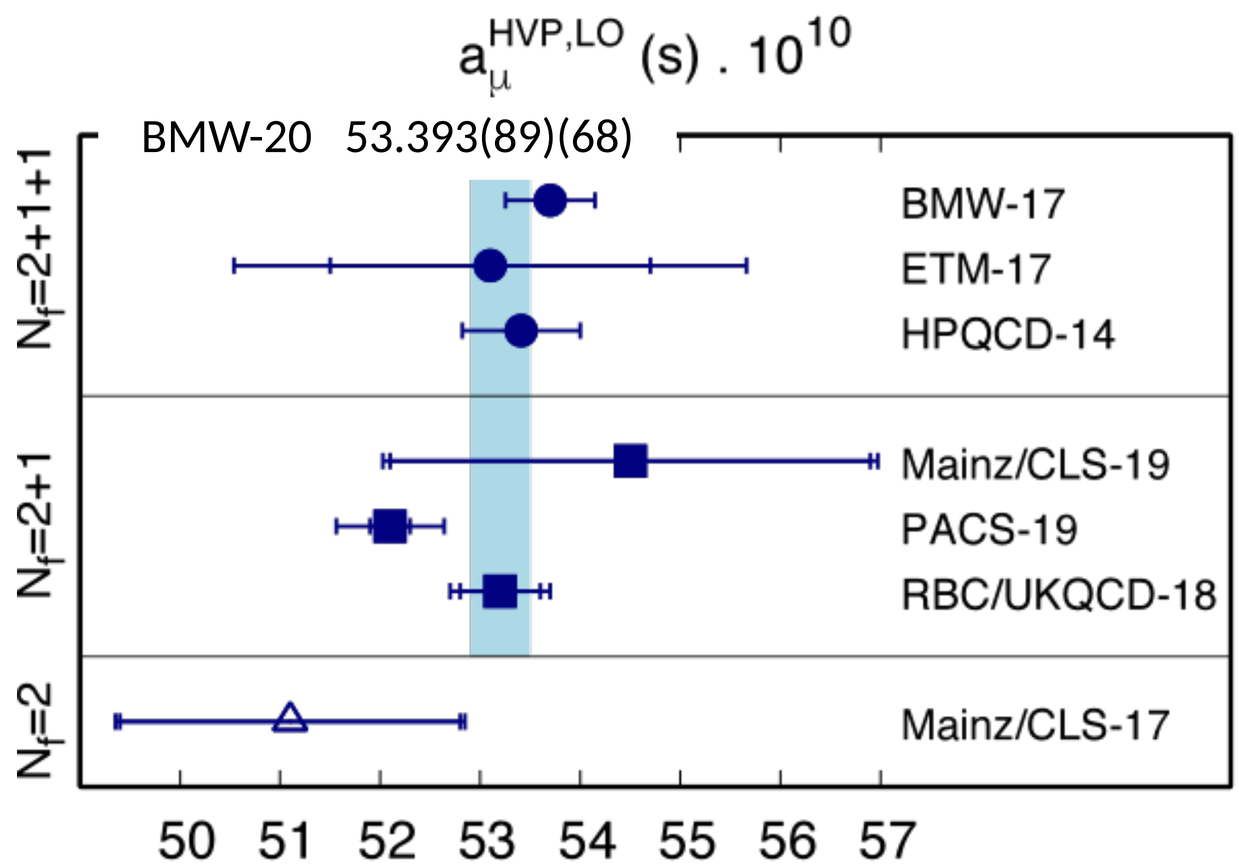
(ud connected, 0.4 – 1.0 fm window)

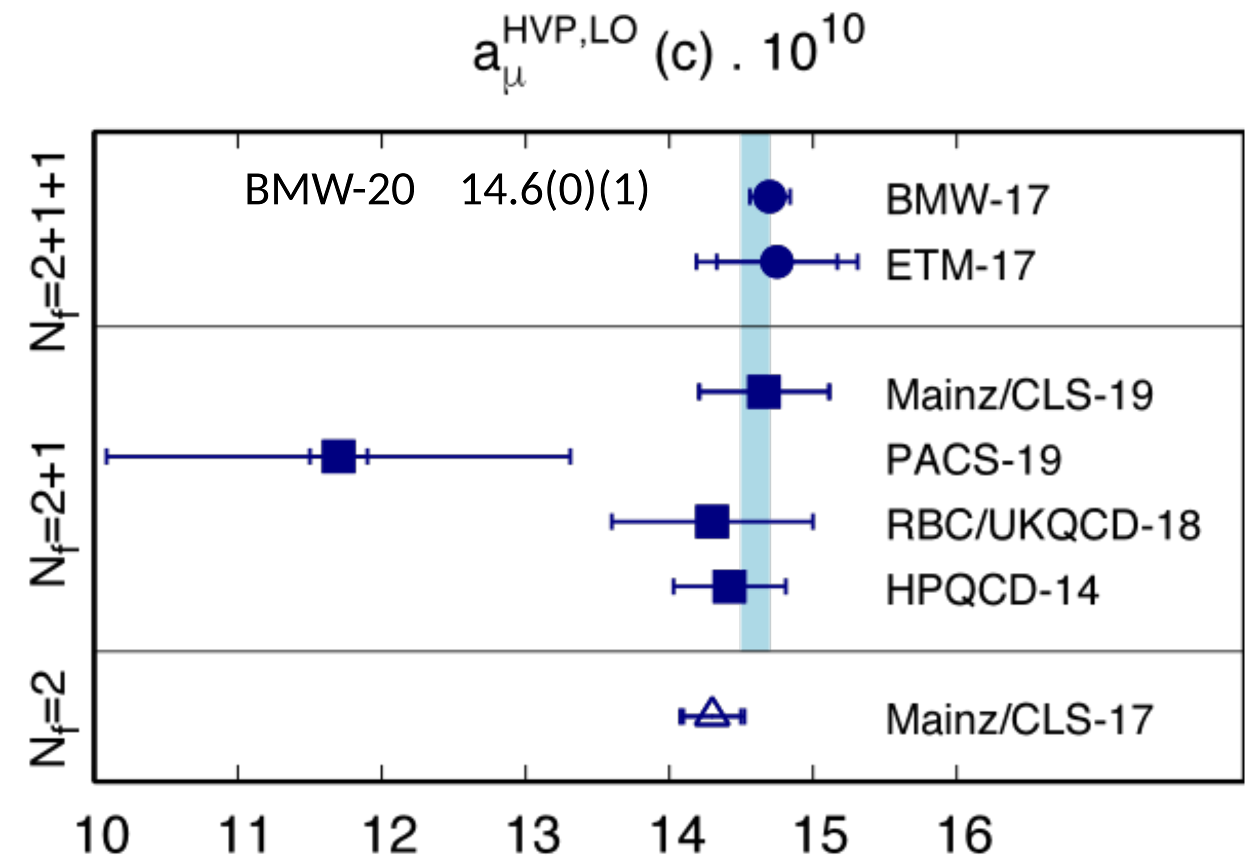
Strange and charm contributions



Strange and charm contributions

Seem to be in good shape





To reach desired precision (2-5 per-mil?)

- Strange, charm contributions in good shape (will not resolve issues)

- FV corrections $(L > 6\ \mathrm{fm})$ reliable (NNLO χPT, LLGS, HP)
  Important to have a big box (BMW, PACS use L = 10 fm)

- Statistical precision top priority for DW, TM, Wilson (in the works)
  Improved bounding method using low-lying states for long distance tail

- Must work directly with physical masses (most groups already)

- More, more precise disconnected and IB calculations needed
  Some spread in results, not all diagrams computed

- Continuum limit and scale setting (per-mil) crucial.
  At least 3 lattice spacings in a^2-scaling regime
  Are (N)NLO and LLGS taste corrections enough?
  All groups need to investigate windows in Euclidean time
  Is f_π good enough (EM corrections)?

a_μ-HLbL from data and models

T. Aoyama, N. Asmussen, M. Benayoun et al. Physics Reports 887 2020) 1–166

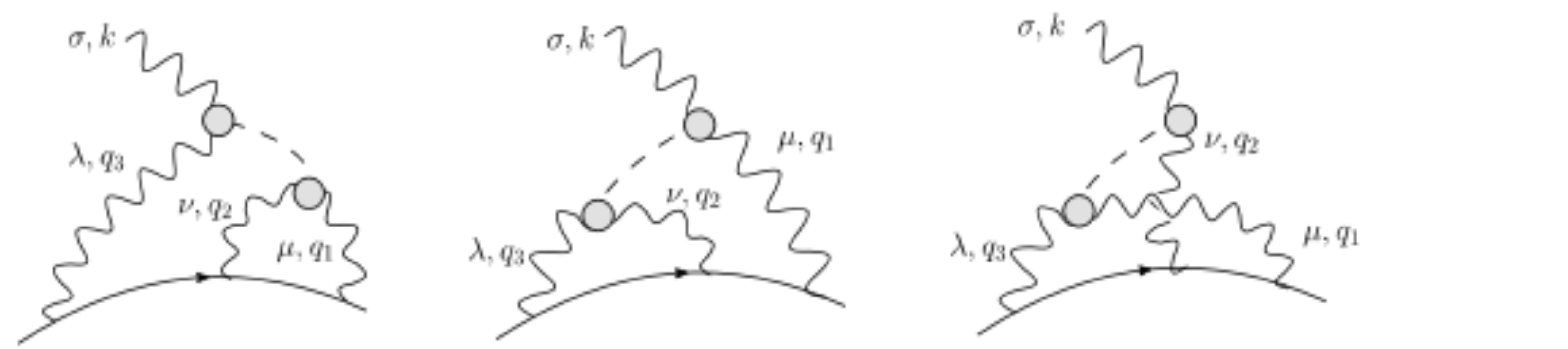
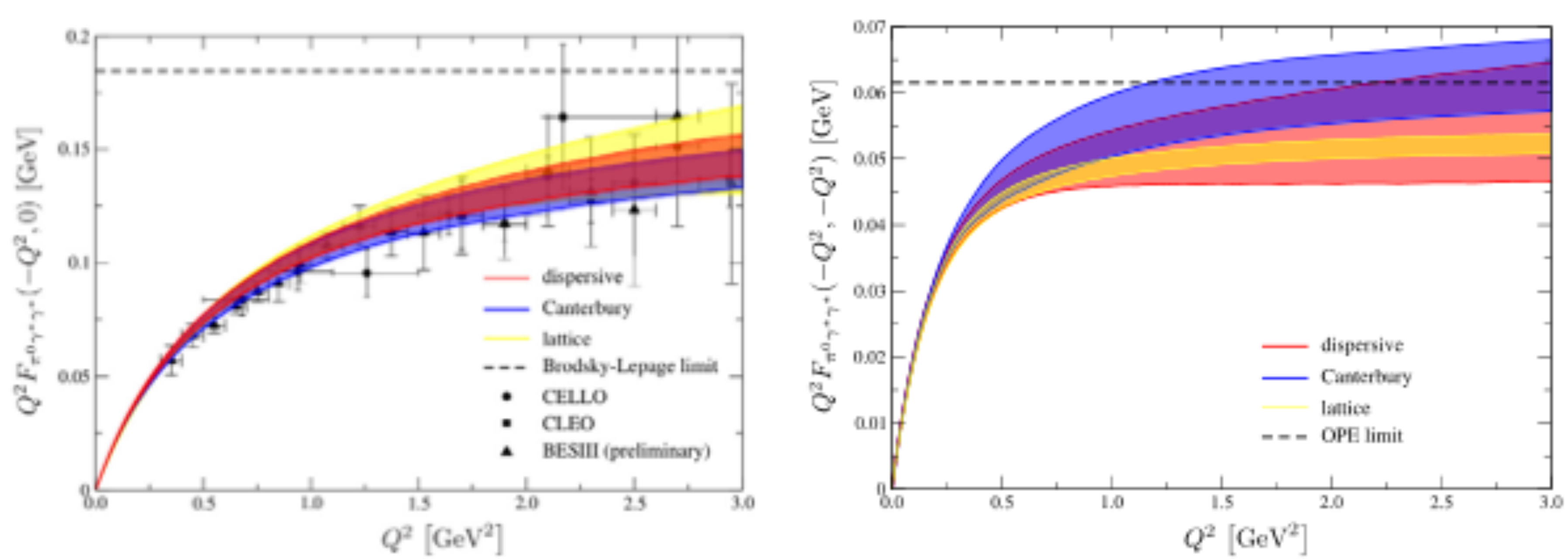


Fig. 57. The pseudoscalar-pole contribution: the dashed lines stand for the pseudoscalar meson, while the blobs can be unambiguously related to the TFFs.
Source: Reprinted from Ref. [19].

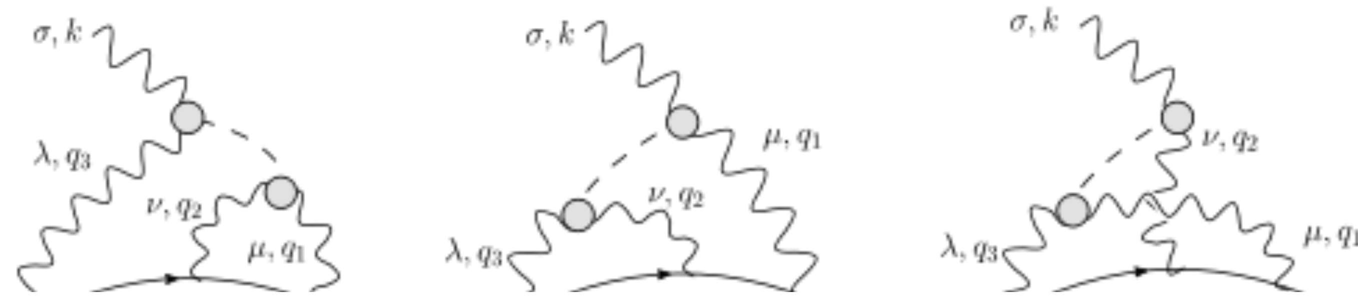
S- : nxZl Z+M- : rl t rrdm+L - AdmZxnt m ds Zk Qgxrlbr Qdonqgr 776 1/ 1/ (0ż055



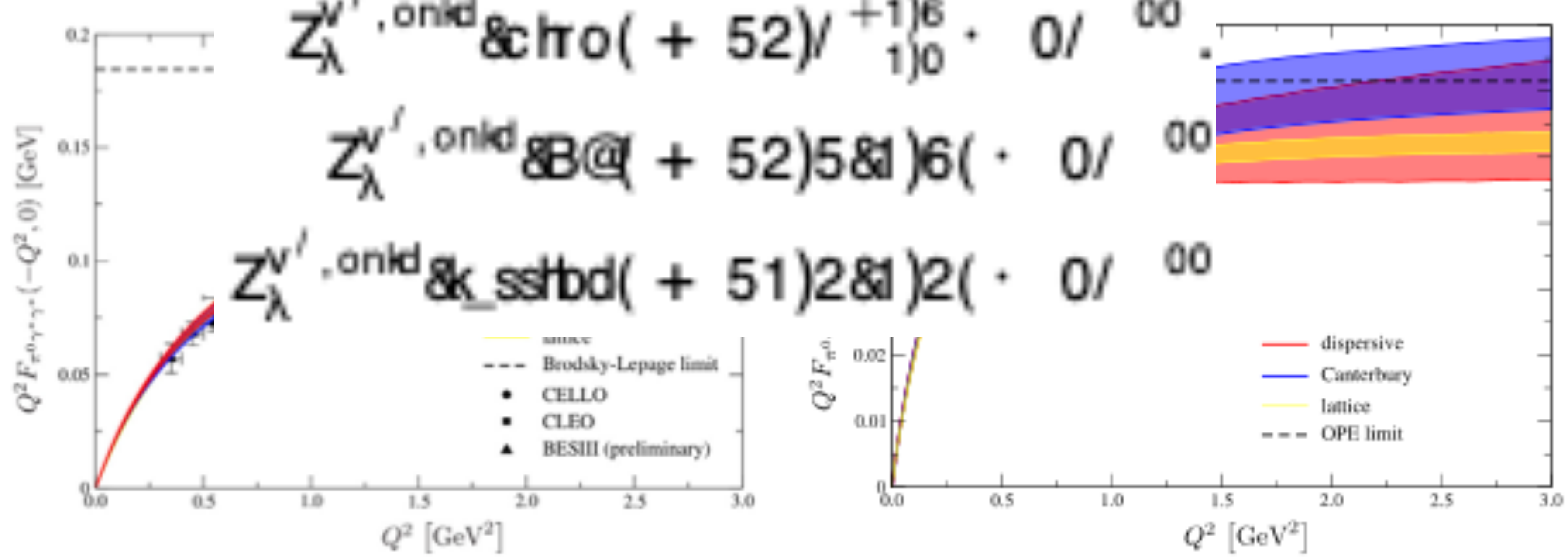
Ehf - 5. - Bnl o_qhrnm ne sgd v' SEE eqnl chrodqrhnm sgdnqx Z10-886[&qdc(+B@Z08[&akt d(+_mc k_sshbd PBC Z11[&xdkknv (- V d rgnv ansg sgd rhmfkx, &kdes(_mc sgd cnt akx,uhqst _k &qhfgs(end e_bsnq-

a_μ-HLbL from data and models

T. Aoyama, N. Asmussen, M. Benayoun et al. Physics Reports 887 2020) 1–166



Good agreement between
data/dispersive and lattice approaches



a_μ-HLbL from data and models

T. Aoyama, N. Asmussen, M. Benayoun et al. *Physics Reports 887 (2020) 1–166*

Table 15
Comparison of two frequently used compilations for HLbL in units of 10^{-11} from 2009 and a recent update with our estimate. Legend: PdRV = Prades, de Rafael, Vainshtein ("Glasgow consensus"); N/JN = Nyffeler / Jegerlehner, Nyffeler; J = Jegerlehner.

Contribution	PdRV(09) [475]	N/JN(09) [476,596]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	19(19)	19(13)	20(5)	16.4(2)
S-wave $\pi\pi$ rescattering	7(7)	7(2)	5.98(1.20)	8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars				1(3) (scalars + tensors)
tensors			1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance		21(3)	20(4)	15(10)
c-loop	2.3		2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

a_μ-HLbL from data and models

T. Aoyama, N. Asmussen, M. Benayoun et al. Physics Reports 887 2020) 1–166

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S-wave $\pi\pi$ rescatter				8(1)
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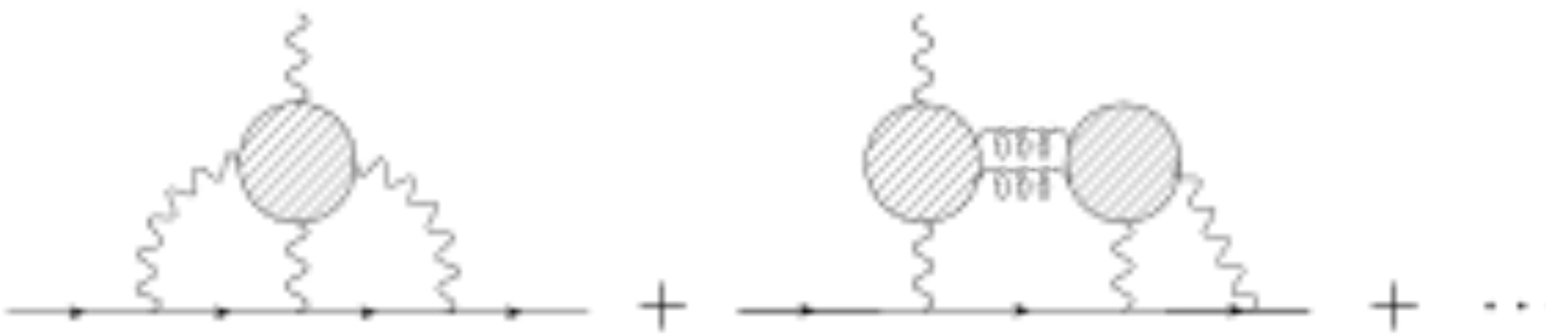
tensors				
axial vectors				6(6)
u, d, s-loops / short-				15(10)
c-loop	2.3		2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

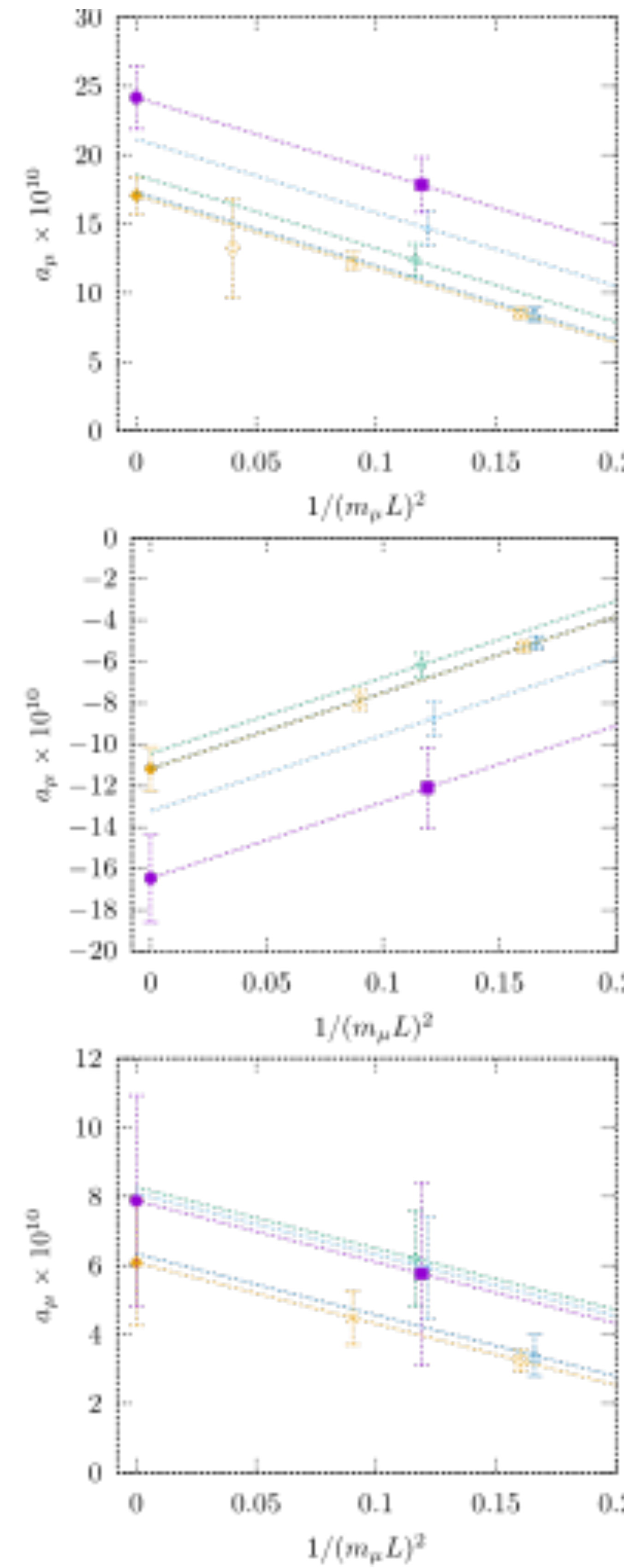
- Huge improvement in pole contributions
- All contributions computed or estimated
- Errors added in quad for dispersive results
- Errors added linearly for model-dependent results

Lattice HLbL

Blum, *et al.* (RBC) PRL 124 (2020)
Editor's Suggestion



$$a_\mu^{\rm HLBL} = 7.87 \pm 3.06 \pm 1.77 \times 10^{-10}$$



- RBC: first lattice calculation with all errors controlled.
- 1 G core-hours on ALCF's Mira (BG/Q).
- 1st HLbL calculation was done on USQCD resources (Blum, *et al*., PRL 114 (2015))
- Crucial for Standard Model Comparison
- Included in Muon g-2 Theory Initiative average
  - 92(19)x 10^{-11} (phenomenology)
  - 90(17)x 10^{-11} (phenomenology+lattice)
- Unlikely to explain discrepancy with experiment

a_μ-HLbL outlook

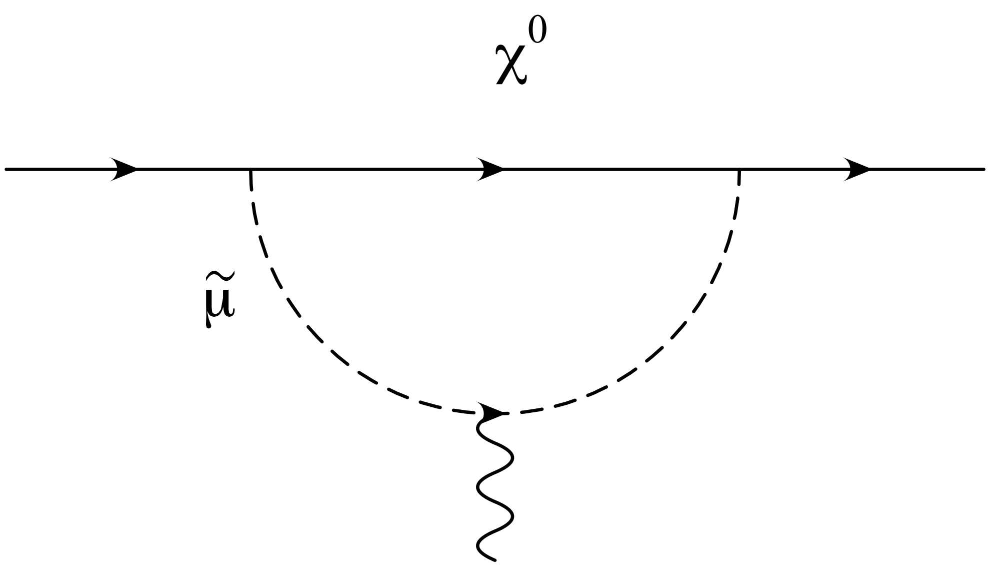
- More data for pheno (…)
- RBC: QED∞ calculation, 10-20% accuracy (5 yrs)
- Other lattice groups starting (Mainz, BMW, FHM, …)
- Combined 10% result (or better) within 5 years possible

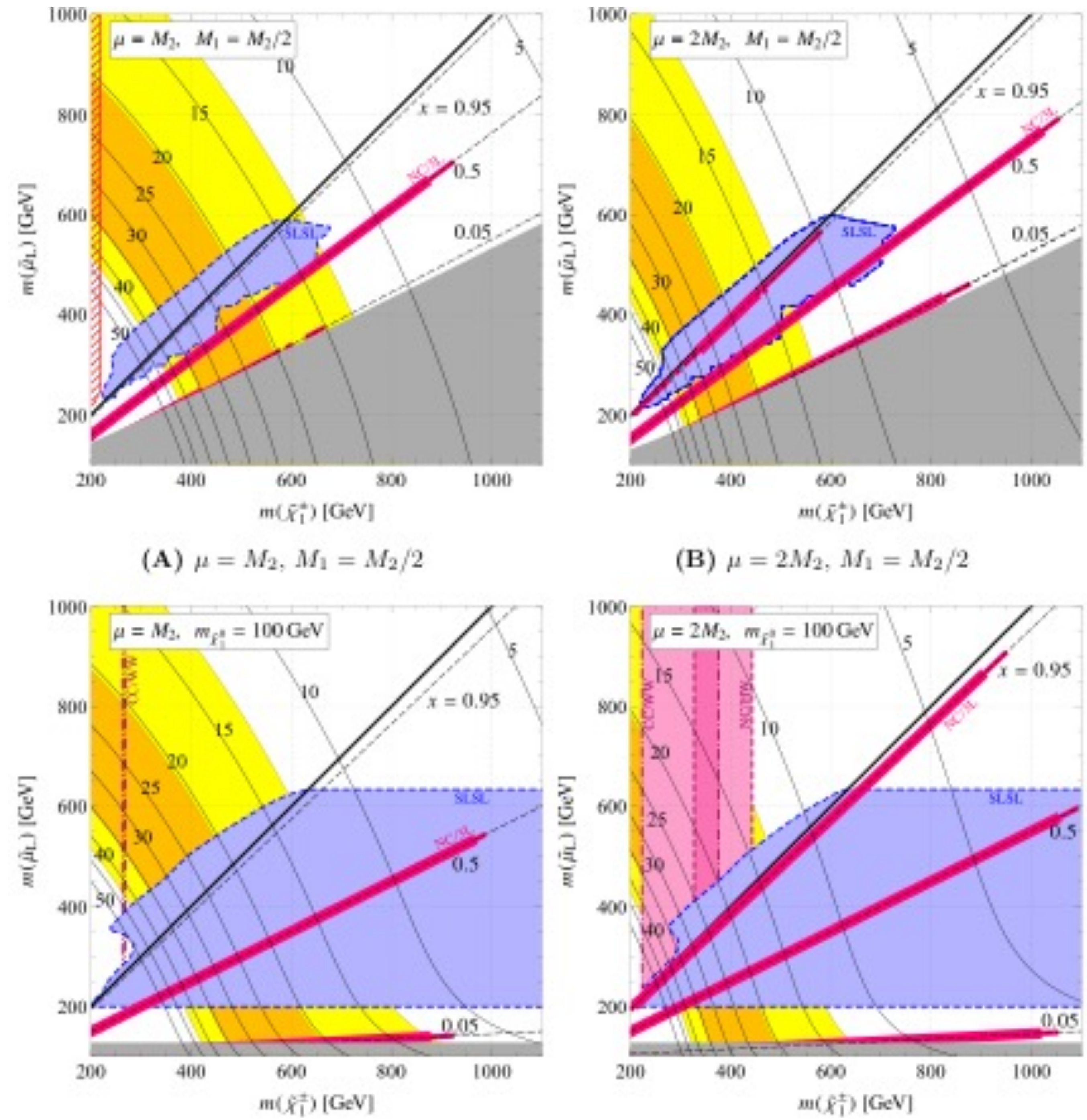
What could BSM theory look like?

- Supersymmetry
- Leptoquarks
- Light scalar
- Two higgs doublets
- …

SUSY at LHC

JHEP04 (2020) 165





(A) $\mu = M_2,\ M_1 = M_2/2$ **(B)** $\mu = 2M_2,\ M_1 = M_2/2$

(C) $\mu = M_2,\ m_{\tilde{\chi}_1^0} = 100\,\mathrm{GeV}$ **(D)** $\mu = 2M_2,\ m_{\tilde{\chi}_1^0} = 100\,\mathrm{GeV}$

Figure 1. LHC Run 2 bounds on the chargino-dominated SUSY scenario for the muon $g-2$ anomaly. Four parameter spaces with $\tan\beta = 40$, eq. (2.8), are considered. The black contours show $a_\mu^{\rm SUSY} \times 10^{10}$, but lines corresponding to > 50 are omitted; $a_\mu^{\rm SUSY} = (27.8 \pm 7.4) \times 10^{-10}$ is satisfied in the orange-filled (yellow-filled) regions at the 1σ (2σ) level. The thick black line corresponds to $m_{\tilde{\mu}_L} = m_{\tilde{\chi}_1^\pm}$. The gray-filled region, where the LSP is $\tilde{\nu}$, and the red-hatched region in (A), which corresponds to a compressed spectrum (see the text), are not studied. The LHC constraint from the CC/WW (NC/HW) analysis is shown by the red-filled regions with the dash-dotted (dashed) boundaries. The blue-filled regions are excluded by the SLSL analysis. The constraints from the NC/3L analysis are investigated on the model points with $x = 0.05$, 0.5, and 0.95 (see eq. (3.25)), where the exclusion ranges are shown by the magenta lines.

Leptoquarks

JHEP06 (2020) 089

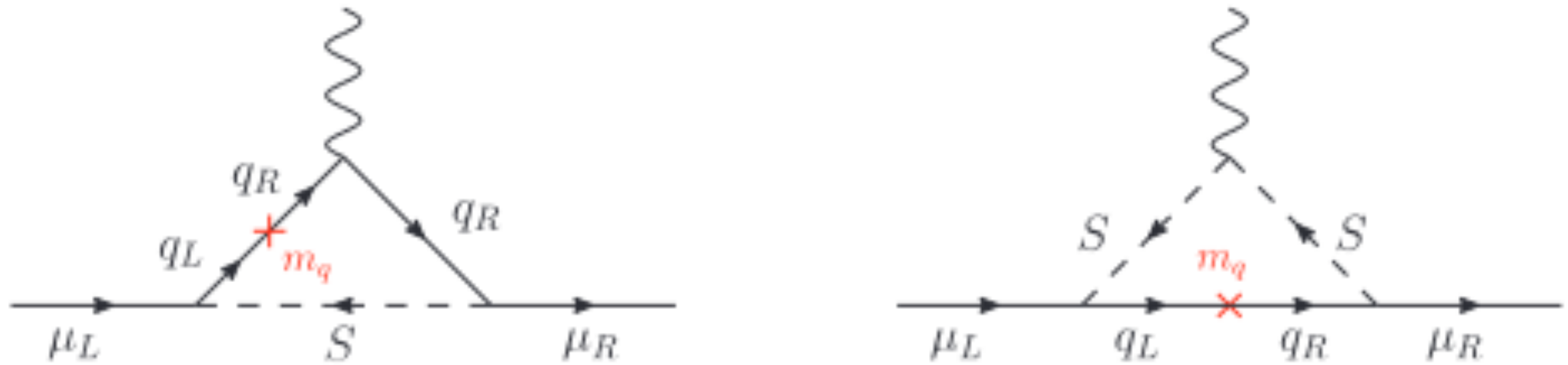
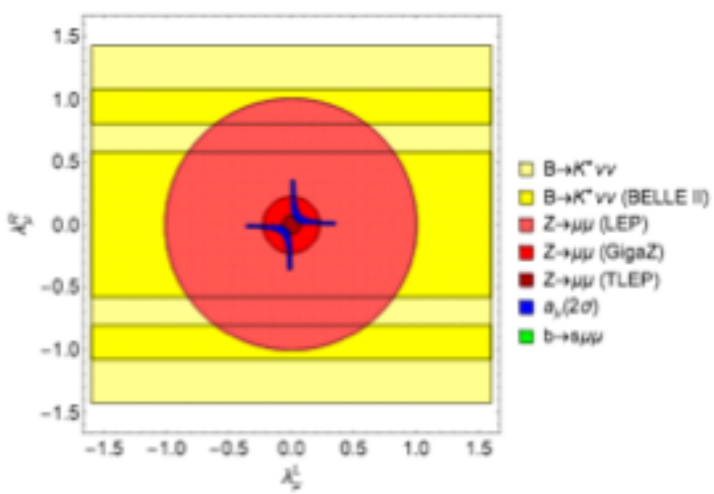
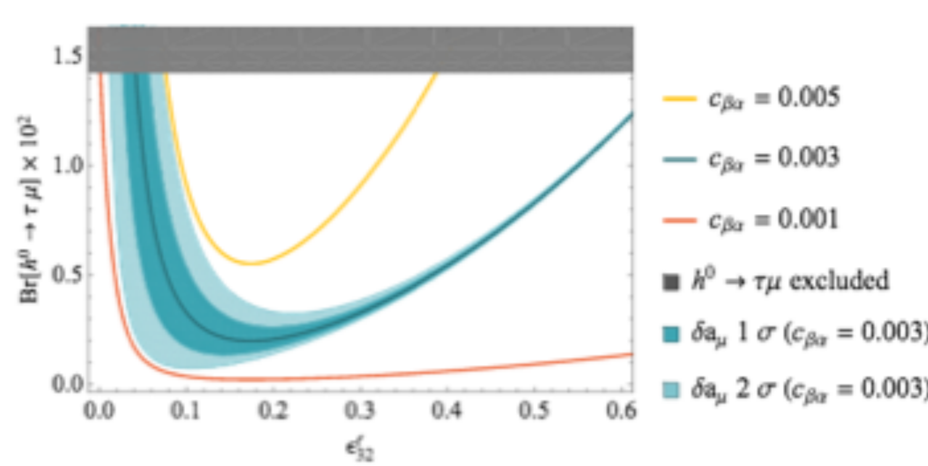


Figure 1: The chirality-enhanced one-loop contributions to muon dipoles ($\propto m_q/m$) due to a presence of scalar S that couples to both left- and right-chiral muons, where S is either R_2 or S_1 and $q \in \{u, c, t\}$.





arXiv:1905.03789

Electron g-2 and Light scalar

arXiv:1806.10252v2

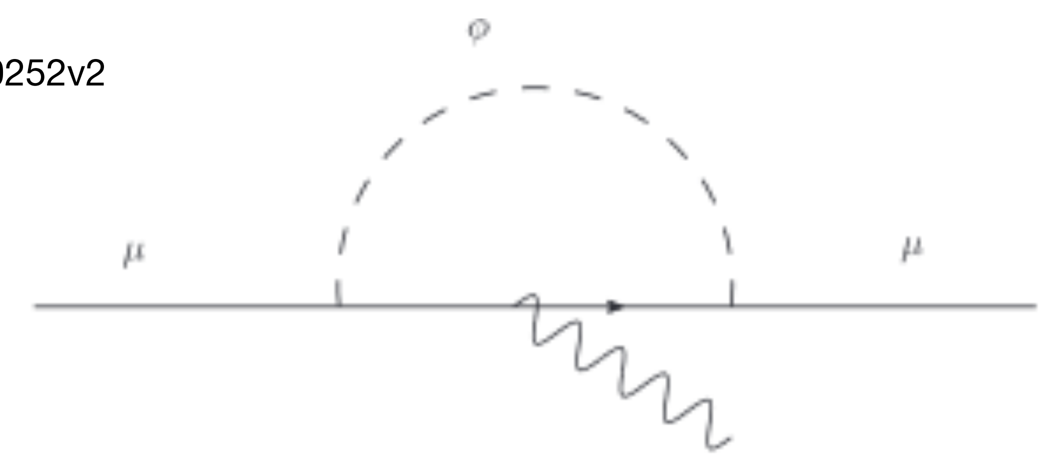


FIG. 1: One-loop ϕ contribution to g_μ 2.

- New value for α leads to 2.4-σ discrepancy for a_e with *opposite* sign to a_μ
- Newer value knocks that down to 1.6, *same* sign

$2\, m\mu \; < m\varphi <$ few GeV

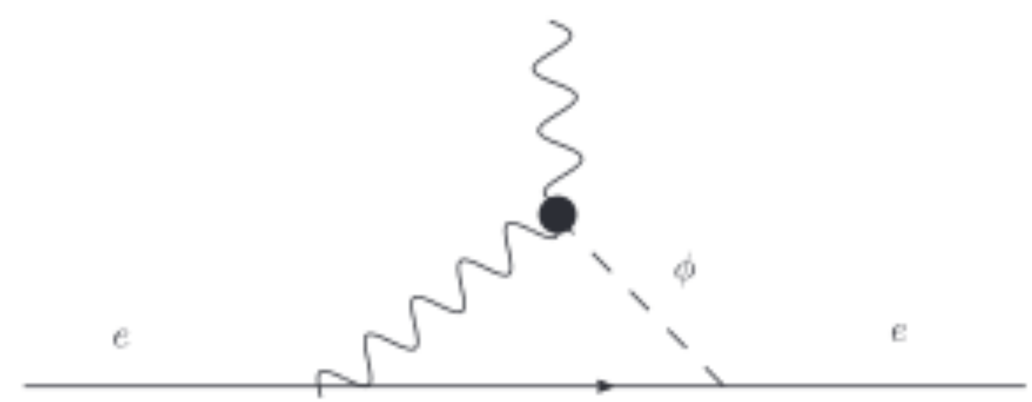


FIG. 2: E ective two-loop Barr-Zee diagram contribution to g_e 2, with fermion loops integrated out. The dot (•) represents light and heavy fermion loops that contribute to κ .

Outlook

- SM is a remarkable success
- Muon g-2 best chance for new physics at the moment
- Fermilab Muon g-2 experiment E989 to announce 1st results very soon