Ultraviolet Surprises in Gravity

BAPTS

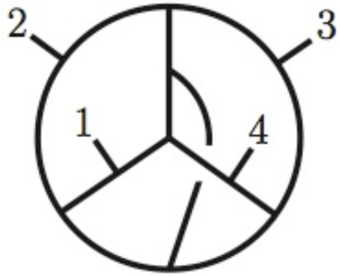
October 9

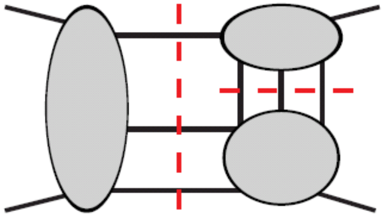
Zvi Bern, UCLA

ZB, Tristan Dennen, Scott Davies, Volodya Smirnov and Sasha Smirnov, arXiv:1309.2496

ZB, Tristan Dennen, Scott Davies, arXiv:1409.3089

ZB, Clifford Cheung, Huan-Hang Chi, Scott Davies, Lance Dixon, Josh Nohle. arXiv:1507.06118





UV in Gravity

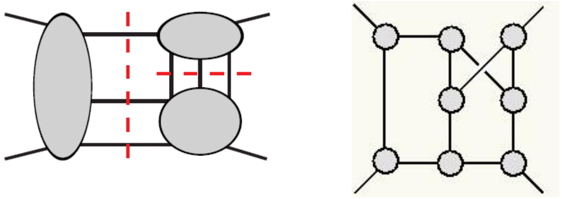
Most people in this audience believe that UV properties of quantum field theories of gravity are well understood, up to "minor" details.

The main purpose of my talk is to try to convince you that the UV behavior of gravity is both strange and surprisingly tame.

1. Examples of no UV divergence even when symmetry arguments suggest divergences.
2. When UV divergences are present in pure (super) gravity, properties are weird.

Our Basic Tools

We have powerful tools for computing scattering amplitudes in quantum gravity and for uncovering new structures:



- **Unitarity method.**

  ZB, Dixon, Dunbar, Kosower
  ZB, Carrasco, Johansson , Kosower

- **Advanced loop integration technology.**

  Chetyrkin, Kataev and Tkachov; A.V. Smirnov; V. A. Smirnov, Vladimirov; Marcus, Sagnotti; Czakon; etc

- **Duality between color and kinematics.**

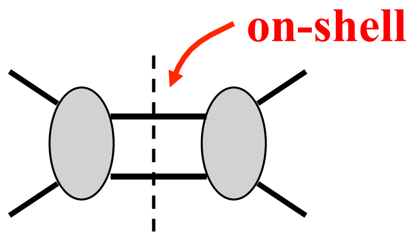
  ZB, Carrasco and Johansson

Many other tools and advances that I won't discuss here.

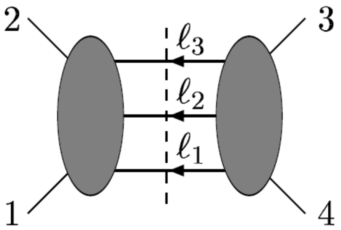
Bern, Dixon, Dunbar and Kosower (1998)

Unitarity Method: Rewrite of QFT

Two-particle cut:

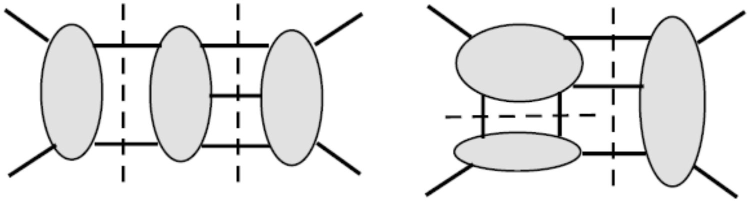


Three-particle cut:



Systematic assembly of complete amplitudes from cuts for any number of particles or loops.

Generalized unitarity as a practical tool:



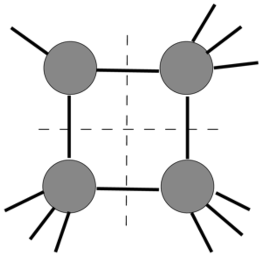
ZB, Dixon and Kosower; Britto, Cachazo and Feng; Forde; Ossala, Pittau, Papadopolous; Berger et al

Different cuts merged to give an expression with correct cuts in all channels.

Now a standard tool

complex momenta to solve cuts

Britto, Cachazo and Feng

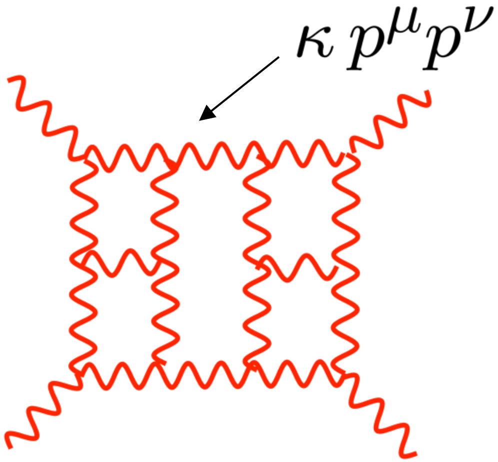


Method of maximal cuts is a powerful way of organizing this.

ZB, Carrasco, Johansson , Kosower

Non-Renormalizability of Gravity?

$$\kappa = \sqrt{32\pi G_N} \longleftarrow$$ Dimensionful coupling



Gravity:
$$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(\kappa p_j^\mu p_j^\nu)}{\text{propagators}}$$

Gauge theory:
$$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(g p_j^\nu)}{\text{propagators}}$$

- **Extra powers of loop momenta in numerator means integrals are badly behaved in the UV and must divergence at some loop order.**
- **Much more sophisticated power counting in supersymmetric theories but this is basic idea.**

- ***N*** **= 8 supergravity is best theory to look at.**
- **With more supersymmetry expect better UV properties.**
- **High symmetry implies simplicity.**

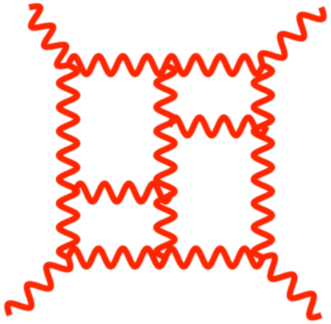
Status of UV Divergences

> **… it is not clear that general relativity, when combined with various other fields in supergravity theory, can not give a sensible quantum theory. Reports of the death of supergravity are an exaggeration. One year everyone believed that supergravity was finite. The next year the fashion changed and everyone said that supergravity was bound to have divergences even though none had actually been found.**
>
> ***— Stephen Hawking, 1994***

Today:

We finally found a divergence in a pure supergravity theory: $N = 4$ supergravity at four loops.

ZB, Davies, Dennen



But as we shall see, instead of answering Hawking's comment we only deepen the mystery surrounding UV behavior.

Where is First Potential $D = 4$ UV Divergence?

3 loops $N = 8$	Green, Schwarz, Brink (1982); Howe and Stelle (1989); Marcus and Sagnotti (1985)	✗
5 loops $N = 8$	Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998); Howe and Stelle (2003,2009)	✗
6 loops $N = 8$	Howe and Stelle (2003)	✗
7 loops $N = 8$	Grisaru and Siegel (1982); Bossard, Howe, Stelle (2009);Vanhove; Björnsson, Green (2010); Kiermaier, Elvang, Freedman(2010); Ramond, Kallosh (2010); Biesert et al (2010); Bossard, Howe, Stelle, Vanhove (2011)	?
3 loops $N = 4$	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops $N = 5$	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops $N = 4$	Vanhove and Tourkine (2012)	✓

ZB, Kosower, Carrasco, Dixon, Johansson, Roiban; ZB, Davies, Dennen, A. Smirnov, V. Smirnov; series of calculations.

← **Don't bet on divergence**

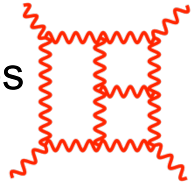
← **Weird structure. Quantum anomaly behind divergence.**

- **So far, every prediction of divergences in pure supergravity has either been wrong or missed crucial details.**
- **Conventional wisdom holds that it will diverge soon or later.**

Feynman Diagrams for Gravity

SUPPOSE WE WANT TO CHECK IF CONSENSUS OPINIONS ARE TRUE

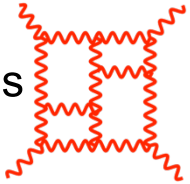
3 loops



~10^{20} TERMS

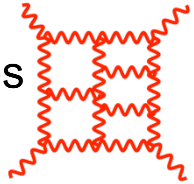
No surprise it has never been calculated via Feynman diagrams.

4 loops



~10^{26} TERMS

5 loops



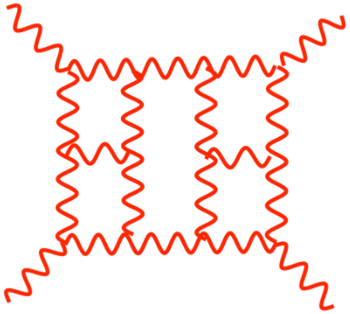
~10^{31} TERMS

More terms than atoms in your brain!

- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

Standard Feynman diagram methods are hopeless

New Structures?



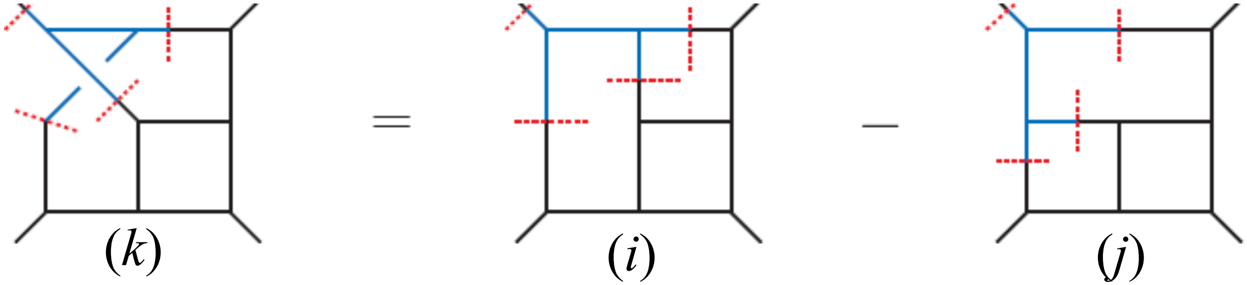
Might there be a new unaccounted structure in gravity theories that suggests the UV might be is tamer than conventional arguments suggest?

Yes!

Duality Between Color and Kinematics

ZB, Carrasco, Johansson (BCJ)

Conjecture: in gauge theory kinematic numerators exist with same algebraic properties a group theory color factors.



color factor

$$c_k = c_i - c_j$$

$$n_k = n_i - n_j$$

kinematic numerator

If you have a set of duality satisfying kinematic numerators.

$$n_i \sim k_1 \cdot l_1 \, k_3 \cdot l_2 \, \varepsilon_1 \cdot l_3 \, \varepsilon_2 \cdot k_3 \, \varepsilon_3 \cdot l_2 \, \varepsilon_4 \cdot k_3 + \ldots$$

gauge theory → gravity theory

simply take

color factor → kinematic numerator

Gravity loop integrands are trivial to obtain once we have gauge theory in a form where duality holds.

Gravity From Gauge Theory

$$
\begin{array}{lcc}
 & n & \tilde{n} \\
\mathcal{N}=8 \text{ sugra:} & (\mathcal{N}=4 \text{ sYM}) \times & (\mathcal{N}=4 \text{ sYM}) \\
\mathcal{N}=5 \text{ sugra:} & (\mathcal{N}=4 \text{ sYM}) \times & (\mathcal{N}=1 \text{ sYM}) \\
\mathcal{N}=4 \text{ sugra:} & (\mathcal{N}=4 \text{ sYM}) \times & (\mathcal{N}=0 \text{ sYM})
\end{array}
$$

- **A new structure that relates gravity theories to gauge theories.**
- **Impossibly hard quantum gravity calculations become doable!**

Some recent applications of BCJ duality and double copy structure:

- **Construction of nontrivial supergravities.**

  Anastasiou, Bornsten, Duff; Duff, Hughs, Nagy; Johansson and Ochirov; Carrasco, Chiodaroli, Günaydin and Roiban; Chiodaroli, Günaydin, Johansson, Roiban

- **Guidance for constructing string-theory loop amplitudes.**

  Mafra, Schlotterer and Steiberger; Mafra and Schlotterer

- **Recent applications to classical black hole solutions.**

  Monteiro, O'Connell and White

Predictions of Ultraviolet Divergences

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Björnsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Bossard, Howe, Stelle, Vanhove, etc

- **First quantized formulation of Berkovits' pure-spinor formalism.**

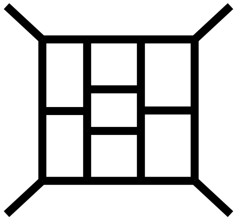
Bjornsson and Green

Key point: *all* supersymmetry cancellations are exposed.

Poor UV behavior, unless new types of cancellations between diagrams exist that are "not consequences of supersymmetry in any conventional sense"

Bjornsson and Green

- **$\mathcal{N} = 8$ sugra should diverge at 5 loops in $D = 24/5$.** ?
- **$\mathcal{N} = 8$ sugra should diverge at 7 loops in $D = 4$.** ?
- **$\mathcal{N} = 4$ sugra should diverge at 3 loops in $D = 4$.** ✗
- **$\mathcal{N} = 5$ sugra should diverge at 4 loops in $D = 4$.** ✗



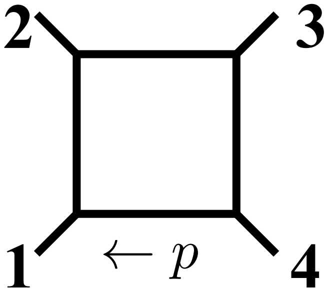
Consensus agreement from all methods

These new types of cancellations do exist: "enhanced cancellations".

ZB, Davies, Dennen

Enhanced UV Cancellations

Suppose diagrams in *all* possible covariant diagrammatic representations are UV divergent.



Pure gravity diagram necessarily is badly divergent

$$n_i \sim \prod_{i=1}^{4} p_\mu p_\nu \varepsilon_i^{\mu\nu}$$

Can't be moved to other diagrams

If sum over diagrams is UV finite by definition we have an "enhanced cancellation".

Pure Einstein gravity $$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{-g}R$$ $$\kappa^2 = 32\pi G_N$$

Despite divergent diagrams, pure gravity is one loop finite

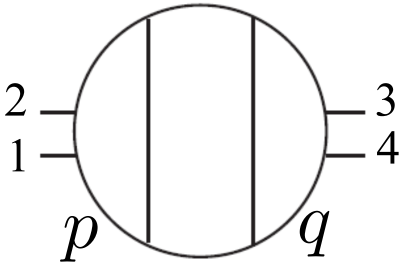
't Hooft and Veltman (1974)

Maximal Cut Power Counting

ZB, Davies, Dennen

Maximal cuts of diagrams poorly behaved:

$\mathcal{N} = 4$ sugra



already log divergent

$\mathcal{N} = 4$ sugra: pure YM x $\mathcal{N} = 4$ sYM

$$n_i \sim s^3 t A_4^{\rm tree} \, (p\cdot q)^2 \, \varepsilon_1 \cdot p \, \varepsilon_2 \cdot p \, \varepsilon_3 \cdot q \, \varepsilon_4 \cdot q + \ldots$$

This diagram is log divergent

$\mathcal{N} = 8$ sugra should diverge at 7 loops in $D = 4$. **Bet with David Gross**

$\mathcal{N} = 8$ sugra should diverge at 5 loops in $D = 24/5$ **Bet with Kelly Stelle**

$\mathcal{N} = 4$ sugra should diverge at 3 loops in $D = 4$
$\mathcal{N} = 5$ sugra should diverge at 4 loops in $D = 4$
} **Unfortunately no bets**

This result equivalent to the results of all other groups who have looked at the problem. Identify poorly behaved terms and count.

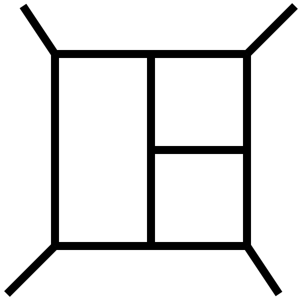
The fact that latter two cases are finite proves existence of "enhanced cancellation".

Three-Loop *N* = 4 Supergravity Construction

ZB, Davies, Dennen, Huang

$N = 4$ sugra : ($N = 4$ sYM) x ($N = 0$ YM)

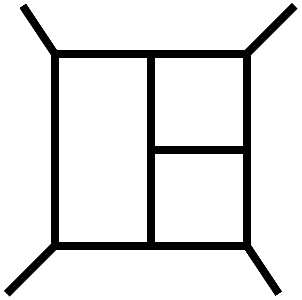
$N = 4$ sYM



$$l \cdot k \, s^2 t A_4^{\text{tree}}$$

BCJ representation

pure YM



$$(\varepsilon \cdot l)^4 \, l^4$$

Feynman representation

$c_i \rightarrow n_i$

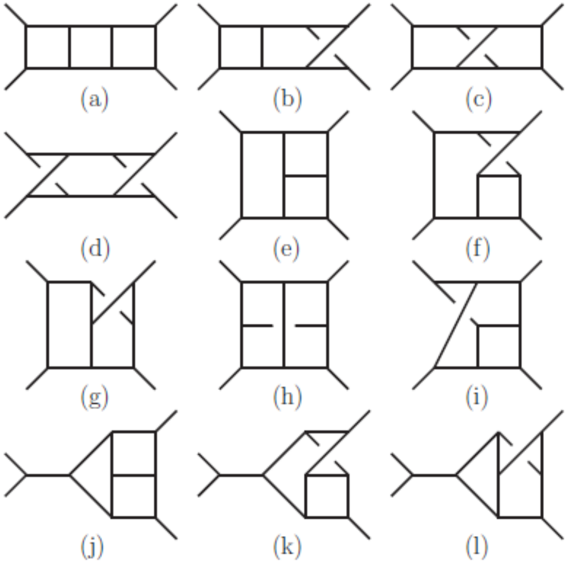
$N = 4$ sugra diagrams linearly divergent

$$\int (d^D l)^3 \frac{k^7 l^9}{l^{20}}$$

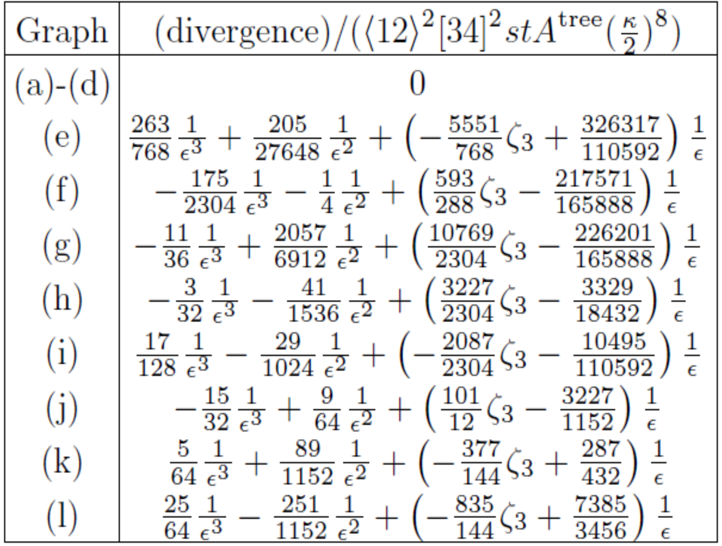
- **We trivially obtain $N = 4$ supergravity integrand.**
- **Integration to extract UV behavior straightforward using modern tools.**

Vladimirov; Marcus and Sagnotti

$N = 4$ Supergravity UV Cancellation



ZB, Davies, Dennen, Huang



Graph	(divergence)/$(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3}+\frac{205}{27648}\frac{1}{\epsilon^2}+\left(-\frac{5551}{768}\zeta_3+\frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3}-\frac{1}{4}\frac{1}{\epsilon^2}+\left(\frac{593}{288}\zeta_3-\frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3}+\frac{2057}{6912}\frac{1}{\epsilon^2}+\left(\frac{10769}{2304}\zeta_3-\frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3}-\frac{41}{1536}\frac{1}{\epsilon^2}+\left(\frac{3227}{2304}\zeta_3-\frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3}-\frac{29}{1024}\frac{1}{\epsilon^2}+\left(-\frac{2087}{2304}\zeta_3-\frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3}+\frac{9}{64}\frac{1}{\epsilon^2}+\left(\frac{101}{12}\zeta_3-\frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3}+\frac{89}{1152}\frac{1}{\epsilon^2}+\left(-\frac{377}{144}\zeta_3+\frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3}-\frac{251}{1152}\frac{1}{\epsilon^2}+\left(-\frac{835}{144}\zeta_3+\frac{7385}{3456}\right)\frac{1}{\epsilon}$

***All* three-loop divergences and subdivergences cancel completely!**

Still no symmetry explanation, despite valiant attempt.

Bossard, Howe, Stelle; ZB, Davies, Dennen

UV Cancellation is “enhanced”: Seems unlikely that a conventional symmetry explanation exists.

Some understanding from extrapolating from two-loop heterotic string amplitudes.

Tourkine and Vanhove

$N = 5$ Supergravity at Four Loops

ZB, Davies and Dennen

We also calculated four-loop divergence in $N = 5$ supergravity.

Industrial strength software needed: FIRE5 and C++

$N = 5$ sugra: ($N = 4$ sYM) x ($N = 1$ sYM)

Again crucial help from FIRE5 and (Smirnov)^2

$N = 4$ sYM

$N = 1$ sYM

Had we made susy cancellation manifest we would have expected log divergence

Straightforward but nontrivial following what we did in $N = 4$ sugra.

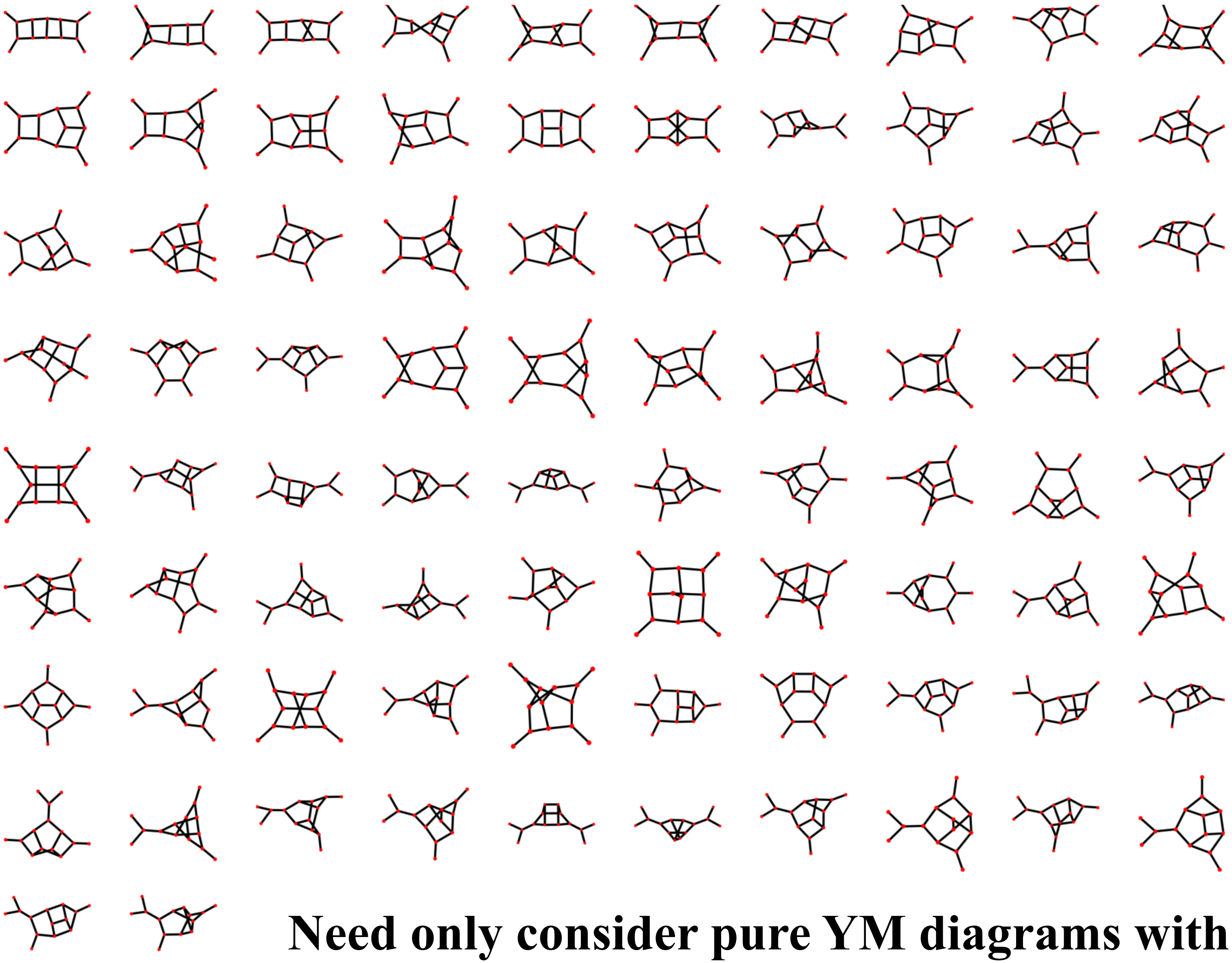
$N = 5$ supergravity has no D^2R^4 divergence at four loops.

Another example of an enhanced cancellation analogous to 7 loops in $N = 8$ sugra.

A pity we did not bet on this theory as well!

82 nonvanishing numerators in BCJ representation

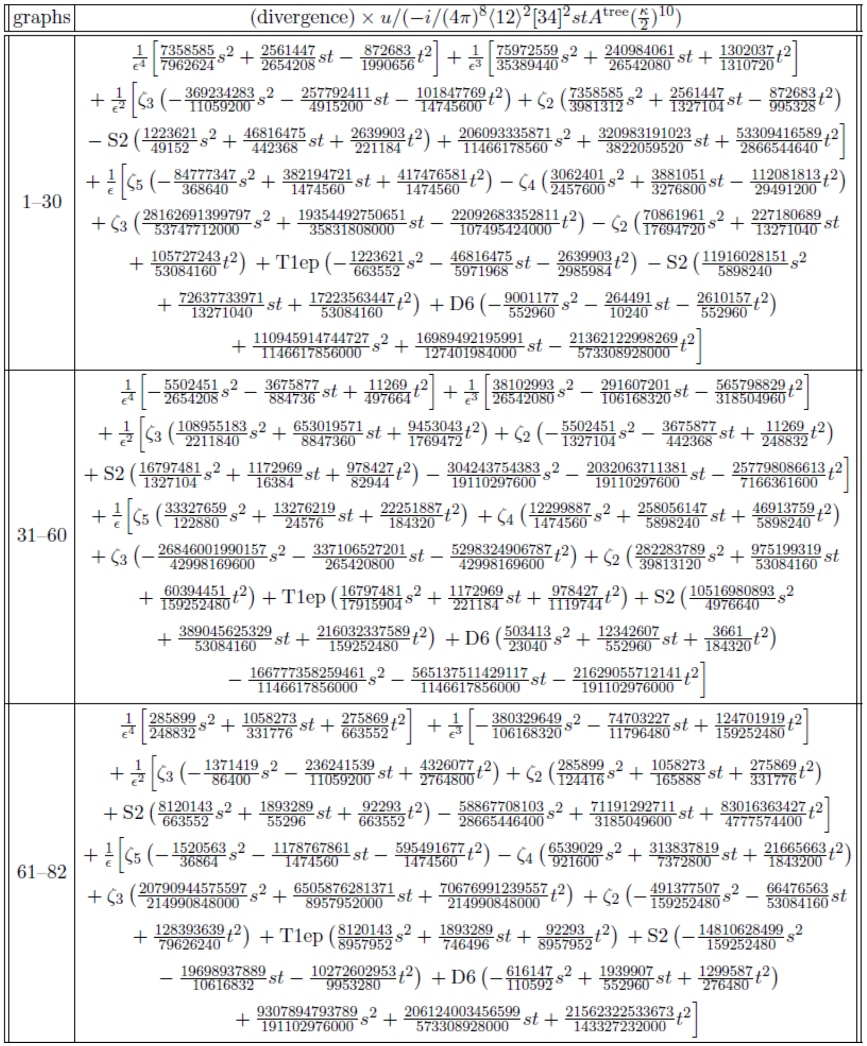
ZB, Carrasco, Dixon, Johansson, Roiban ($\mathcal{N} = 4$ sYM)



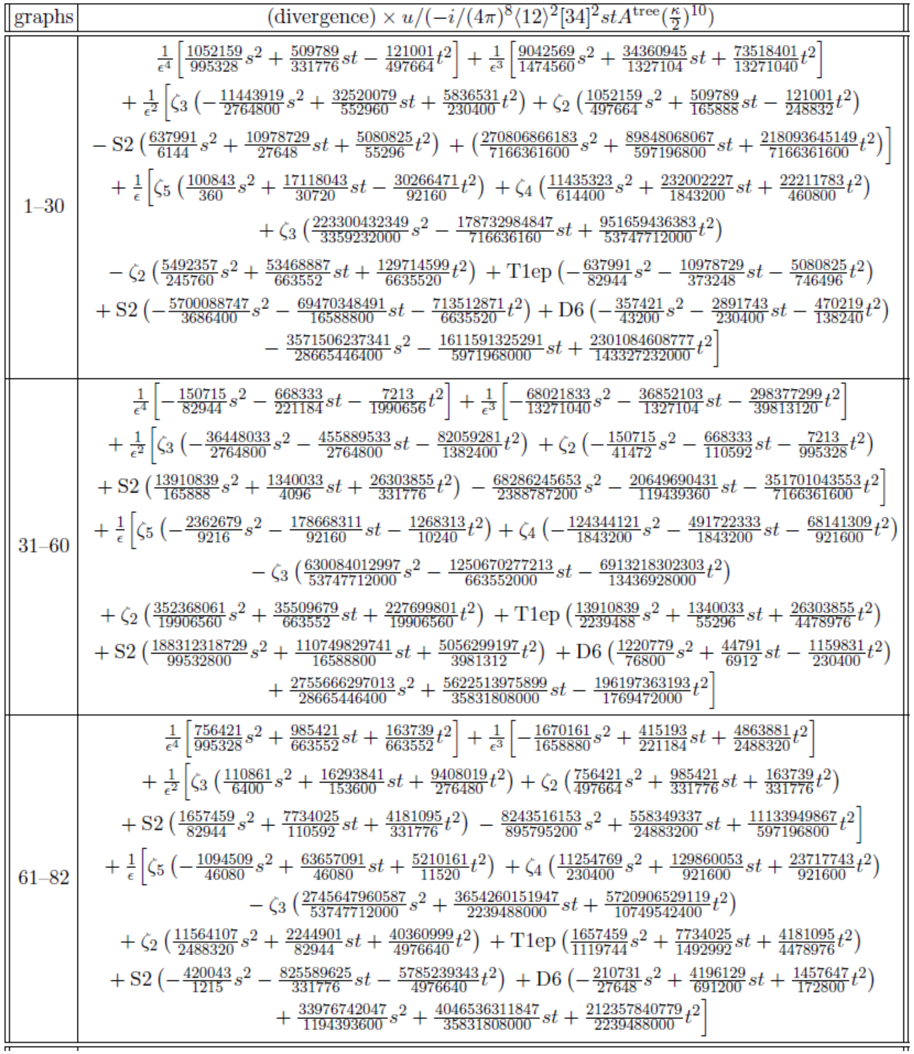
Need only consider pure YM diagrams with color factors that match these.

$N = 5$ supergravity at Four Loops

ZB, Davies and Dennen (2014)



graphs	$(\text{divergence})\times u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 stA^{\text{tree}}(\frac{\kappa}{2})^{10})$
1–30	$\frac{1}{\epsilon^4}[\frac{7358585}{7962624}s^2+\frac{2561447}{2654208}st-\frac{872683}{1990656}t^2]+\frac{1}{\epsilon^3}[\frac{75972559}{35389440}s^2+\frac{240984061}{26542080}st+\frac{1302037}{1310720}t^2]$ $+\frac{1}{\epsilon^2}[\zeta_3(-\frac{369234283}{11059200}s^2-\frac{257792411}{4915200}st-\frac{101847769}{14745600}t^2)+\zeta_2(\frac{7358585}{3981312}s^2+\frac{2561447}{1327104}st-\frac{872683}{995328}t^2)$ $-\mathrm{S2}(\frac{1223621}{49152}s^2+\frac{46816475}{442368}st+\frac{2639903}{221184}t^2)+\frac{206093335871}{11466178560}s^2+\frac{320983191023}{3822059520}st+\frac{53309416589}{2866544640}t^2]$ $+\frac{1}{\epsilon}[\zeta_5(-\frac{84777347}{368640}s^2+\frac{382194721}{1474560}st+\frac{417476581}{1474560}t^2)-\zeta_4(\frac{3062401}{2457600}s^2+\frac{3881051}{3276800}st-\frac{112081813}{29491200}t^2)$ $+\zeta_3(\frac{28162691399797}{53747712000}s^2+\frac{19354492750651}{35831808000}st-\frac{22092683352811}{107495424000}t^2)-\zeta_2(\frac{70861961}{17694720}s^2+\frac{227180689}{13271040}st$ $+\frac{105727243}{53084160}t^2)+\mathrm{T1ep}(-\frac{1223621}{663552}s^2-\frac{46816475}{5971968}st-\frac{2639903}{2985984}t^2)-\mathrm{S2}(\frac{11916028151}{5898240}s^2$ $+\frac{72637733971}{13271040}st+\frac{17223563447}{53084160}t^2)+\mathrm{D6}(-\frac{9001177}{552960}s^2-\frac{264491}{10240}st-\frac{2610157}{552960}t^2)$ $+\frac{110945914744727}{1146617856000}s^2+\frac{16989492195991}{127401984000}st-\frac{21362122998269}{573308928000}t^2]$
31–60	$\frac{1}{\epsilon^4}[-\frac{5502451}{2654208}s^2-\frac{3675877}{884736}st+\frac{11269}{497664}t^2]+\frac{1}{\epsilon^3}[\frac{38102993}{26542080}s^2-\frac{291607201}{106168320}st-\frac{565798829}{318504960}t^2]$ $+\frac{1}{\epsilon^2}[\zeta_3(\frac{108955183}{2211840}s^2+\frac{653019571}{8847360}st+\frac{9453043}{1769472}t^2)+\zeta_2(-\frac{5502451}{1327104}s^2-\frac{3675877}{442368}st+\frac{11269}{248832}t^2)$ $+\mathrm{S2}(\frac{16797481}{1327104}s^2+\frac{1172969}{16384}st+\frac{978427}{82944}t^2)-\frac{304243754383}{19110297600}s^2-\frac{2032063711381}{19110297600}st-\frac{257798086613}{7166361600}t^2]$ $+\frac{1}{\epsilon}[\zeta_5(\frac{33327659}{122880}s^2+\frac{13276219}{24576}st+\frac{22251887}{184320}t^2)+\zeta_4(\frac{12299887}{1474560}s^2+\frac{258056147}{5898240}st+\frac{46913759}{5898240}t^2)$ $+\zeta_3(-\frac{26846001990157}{42998169600}s^2-\frac{337106527201}{265420800}st-\frac{5298324906787}{42998169600}t^2)+\zeta_2(\frac{282283789}{39813120}s^2+\frac{975199319}{53084160}st$ $+\frac{60394451}{159252480}t^2)+\mathrm{T1ep}(\frac{16797481}{17915904}s^2+\frac{1172969}{221184}st+\frac{978427}{1119744}t^2)+\mathrm{S2}(\frac{10516980893}{4976640}s^2$ $+\frac{389045625329}{53084160}st+\frac{216032337589}{159252480}t^2)+\mathrm{D6}(\frac{503413}{23040}s^2+\frac{12342607}{552960}st+\frac{3661}{184320}t^2)$ $-\frac{166777358259461}{1146617856000}s^2-\frac{565137511429117}{1146617856000}st-\frac{21629055712141}{191102976000}t^2]$
61–82	$\frac{1}{\epsilon^4}[\frac{285899}{248832}s^2+\frac{1058273}{331776}st+\frac{275869}{663552}t^2]+\frac{1}{\epsilon^3}[-\frac{380329649}{106168320}s^2-\frac{74703227}{11796480}st+\frac{124701919}{159252480}t^2]$ $+\frac{1}{\epsilon^2}[\zeta_3(-\frac{1371419}{86400}s^2-\frac{236241539}{11059200}st+\frac{4326077}{2764800}t^2)+\zeta_2(\frac{285899}{124416}s^2+\frac{1058273}{165888}st+\frac{275869}{331776}t^2)$ $+\mathrm{S2}(\frac{8120143}{663552}s^2+\frac{1893289}{55296}st+\frac{92293}{663552}t^2)-\frac{58867708103}{28665446400}s^2+\frac{71191292711}{3185049600}st+\frac{83016363427}{4777574400}t^2]$ $+\frac{1}{\epsilon}[\zeta_5(-\frac{1520563}{36864}s^2-\frac{1178767861}{1474560}st-\frac{595491677}{1474560}t^2)-\zeta_4(\frac{6539029}{921600}s^2+\frac{313837819}{7372800}st+\frac{21665663}{1843200}t^2)$ $+\zeta_3(\frac{20790944575597}{214990848000}s^2+\frac{6505876281371}{8957952000}st+\frac{70676991239557}{214990848000}t^2)+\zeta_2(-\frac{491377507}{159252480}s^2-\frac{66476563}{53084160}st$ $+\frac{128393639}{79626240}t^2)+\mathrm{T1ep}(\frac{8120143}{8957952}s^2+\frac{1893289}{746496}st+\frac{92293}{8957952}t^2)+\mathrm{S2}(-\frac{14810628499}{159252480}s^2$ $-\frac{19698937889}{10616832}st-\frac{10272602953}{9953280}t^2)+\mathrm{D6}(-\frac{616147}{110592}s^2+\frac{1939907}{552960}st+\frac{1299587}{276480}t^2)$ $+\frac{9307894793789}{191102976000}s^2+\frac{206124003456599}{573308928000}st+\frac{21562322533673}{143327232000}t^2]$



graphs	$(\text{divergence})\times u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 stA^{\text{tree}}(\frac{\kappa}{2})^{10})$
1–30	$\frac{1}{\epsilon^4}[\frac{1052159}{995328}s^2+\frac{509789}{331776}st-\frac{121001}{497664}t^2]+\frac{1}{\epsilon^3}[\frac{9042569}{1474560}s^2+\frac{34360945}{1327104}st+\frac{73518401}{13271040}t^2]$ $+\frac{1}{\epsilon^2}[\zeta_3(-\frac{11443919}{2764800}s^2+\frac{32520079}{552960}st+\frac{5836531}{230400}t^2)+\zeta_2(\frac{1052159}{497664}s^2+\frac{509789}{165888}st-\frac{121001}{248832}t^2)$ $-\mathrm{S2}(\frac{637991}{6144}s^2+\frac{10978729}{27648}st+\frac{5080825}{55296}t^2)+(\frac{270806866183}{7166361600}s^2+\frac{89848068067}{597196800}st+\frac{218093645149}{7166361600}t^2)]$ $+\frac{1}{\epsilon}[\zeta_5(\frac{100843}{360}s^2+\frac{17118043}{30720}st-\frac{30266471}{92160}t^2)+\zeta_4(\frac{11435323}{614400}s^2+\frac{232002227}{1843200}st+\frac{22211783}{460800}t^2)$ $+\zeta_3(\frac{223300432349}{3359232000}s^2-\frac{178732984847}{716636160}st+\frac{951659436383}{53747712000}t^2)$ $-\zeta_2(\frac{5492357}{245760}s^2+\frac{53468887}{663552}st+\frac{129714599}{6635520}t^2)+\mathrm{T1ep}(-\frac{637991}{82944}s^2-\frac{10978729}{373248}st-\frac{5080825}{746496}t^2)$ $+\mathrm{S2}(-\frac{5700088747}{3686400}s^2-\frac{69470348491}{16588800}st-\frac{713512871}{6635520}t^2)+\mathrm{D6}(-\frac{357421}{43200}s^2-\frac{2891743}{230400}st-\frac{470219}{138240}t^2)$ $-\frac{3571506237341}{28665446400}s^2-\frac{1611591325291}{5971968000}st+\frac{2301084608777}{143327232000}t^2]$
31–60	$\frac{1}{\epsilon^4}[-\frac{150715}{82944}s^2-\frac{668333}{221184}st-\frac{7213}{1990656}t^2]+\frac{1}{\epsilon^3}[-\frac{68021833}{13271040}s^2-\frac{36852103}{1327104}st-\frac{298377299}{39813120}t^2]$ $+\frac{1}{\epsilon^2}[\zeta_3(-\frac{36448033}{2764800}s^2-\frac{455889533}{2764800}st-\frac{82059281}{1382400}t^2)+\zeta_2(-\frac{150715}{41472}s^2-\frac{668333}{110592}st-\frac{7213}{995328}t^2)$ $+\mathrm{S2}(\frac{13910839}{165888}s^2+\frac{1340033}{4096}st+\frac{26303855}{331776}t^2)-\frac{68286245653}{2388787200}s^2-\frac{20649690431}{119439360}st-\frac{351701043553}{7166361600}t^2]$ $+\frac{1}{\epsilon}[\zeta_5(-\frac{2362679}{9216}s^2-\frac{178668311}{92160}st-\frac{1268313}{10240}t^2)+\zeta_4(-\frac{124344121}{1843200}s^2-\frac{491722333}{1843200}st-\frac{68141309}{921600}t^2)$ $-\zeta_3(\frac{630084012997}{53747712000}s^2-\frac{1250670277213}{663552000}st-\frac{6913218302303}{13436928000}t^2)$ $+\zeta_2(\frac{352368061}{19906560}s^2+\frac{35509679}{663552}st+\frac{227699801}{19906560}t^2)+\mathrm{T1ep}(\frac{13910839}{2239488}s^2+\frac{1340033}{55296}st+\frac{26303855}{4478976}t^2)$ $+\mathrm{S2}(\frac{188312318729}{99532800}s^2+\frac{110749829741}{16588800}st+\frac{5056299197}{3981312}t^2)+\mathrm{D6}(\frac{1220779}{76800}s^2+\frac{44791}{6912}st-\frac{1159831}{230400}t^2)$ $+\frac{2755666297013}{28665446400}s^2+\frac{5622513975899}{35831808000}st-\frac{196197363193}{1769472000}t^2]$
61–82	$\frac{1}{\epsilon^4}[\frac{756421}{995328}s^2+\frac{985421}{663552}st+\frac{163739}{663552}t^2]+\frac{1}{\epsilon^3}[-\frac{1670161}{1658880}s^2+\frac{415193}{221184}st+\frac{4863881}{2488320}t^2]$ $+\frac{1}{\epsilon^2}[\zeta_3(\frac{110861}{6400}s^2+\frac{16293841}{153600}st+\frac{9408019}{276480}t^2)+\zeta_2(\frac{756421}{497664}s^2+\frac{985421}{331776}st+\frac{163739}{331776}t^2)$ $+\mathrm{S2}(\frac{1657459}{82944}s^2+\frac{7734025}{110592}st+\frac{4181095}{331776}t^2)-\frac{8243516153}{895795200}s^2+\frac{558349337}{24883200}st+\frac{11133949867}{597196800}t^2]$ $+\frac{1}{\epsilon}[\zeta_5(-\frac{1094509}{46080}s^2+\frac{63657091}{46080}st+\frac{5210161}{11520}t^2)+\zeta_4(\frac{11254769}{230400}s^2+\frac{129860053}{921600}st+\frac{23717743}{921600}t^2)$ $-\zeta_3(\frac{2745647960587}{53747712000}s^2+\frac{3654260151947}{2239488000}st+\frac{5720906529119}{10749542400}t^2)$ $+\zeta_2(\frac{11564107}{2488320}s^2+\frac{2244901}{82944}st+\frac{40360999}{4976640}t^2)+\mathrm{T1ep}(\frac{1657459}{1119744}s^2+\frac{7734025}{1492992}st+\frac{4181095}{4478976}t^2)$ $+\mathrm{S2}(-\frac{420043}{1215}s^2-\frac{825589625}{331776}st-\frac{5785239343}{4976640}t^2)+\mathrm{D6}(-\frac{210731}{27648}s^2+\frac{4196129}{691200}st+\frac{1457647}{172800}t^2)$ $+\frac{33976742047}{1194393600}s^2+\frac{4046536311847}{35831808000}st+\frac{212357840779}{2239488000}t^2]$

Adds up to zero: no divergence. Enhanced cancellations!

Four-loop $\mathcal{N} = 4$ Supergravity Divergences

ZB, Davies, Dennen, Smirnov, Smirnov

We also calculated four-loop divergence in $\mathcal{N} = 4$ supergravity.

Industrial strength software needed: FIRE5 and C++

$\mathcal{N} = 4$ sugra: ($\mathcal{N} = 4$ sYM) x ($\mathcal{N} = 0$ YM)

$\mathcal{N} = 4$ sYM

$$(l \cdot k)^2 s^2 t A_4^{\text{tree}}$$

BCJ representation

pure YM

$$(\varepsilon \cdot l)^4 l^6$$

Feynman representation

$\mathcal{N} = 4$ sugra diagrams quadratically divergent

$$\int (d^D l)^4 \frac{k^8 l^{12}}{(l^2)^{13}}$$

$D^2 R^4$ counterterm

82 nonvanishing diagram types using $\mathcal{N} = 4$ sYM BCJ form.

The 4 loop Divergence of *N* = 4 Supergravity

ZB, Davies, Dennen, A.V. Smirnov, V.A. Smirnov

4 loops similar to 3 loops except we need industrial strength software: FIRE5 + special purpose C++ code.

dim. reg. UV pole

$$\boxed{\mathcal{M}^{4\text{-loop}}\Big|_{\text{div.}} = \frac{1}{(4\pi)^8}\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^{10}\frac{1}{144}(1-264\zeta_3)\,\mathcal{T}}$$

kinematic factor

$D = 4 - 2\epsilon$

It diverges but it has strange properties:

- **Contributions to helicity configurations that vanish were it not for a quantum anomaly in *U*(1) subgroup of duality symmetry.**
- **These helicity configuration have vanishing integrands in *D* = 4. Divergence is 0/0. Anomaly-like behavior not found in *N* ≥ 5 sugra.**

Carrasco, Kallosh, Tseytlin and Roiban

Motivates closer examination of divergences.
Want simpler example: Pure Einstein gravity is simpler.

Pure Einstein Gravity

Standard argument for 1 loop finiteness of pure gravity:

’t Hooft and Veltman (1974)

$\cancel{R^2}$ $\cancel{R^2_{\mu\nu}}$

Divergences vanish by equation of motion and can be eliminated by field redefinition.

$\cancel{R^2_{\mu\nu\rho\sigma}}$

In *D* = 4 topologically trivial space, Gauss-Bonnet theorem eliminates Riemann square term.

$$\int d^4x\,\sqrt{-g}(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma}) = 32\pi^2\chi$$

Euler Characteristic.

Pure gravity divergence with nontrivial topology:

Capper and Duff; Tsao ; Critchley; Gibbons, Hawking, Perry Goroff and Sagnotti, etc

$$\mathcal{L}^{\rm GB} = -\frac{1}{(4\pi)^2}\frac{1}{360\epsilon}\Big(4\cdot 53 + 1 + 91 - 180\Big)(R^2 - 4R_{\mu\nu} + R^2_{\mu\nu\rho\sigma})$$

graviton **scalar** **antisym. tensor** **3 form tensor** **Gauss-Bonnet**

Related to “trace anomaly”.

Gauss-Bonnet one-loop divergence is “evanescent”

The Trace Anomaly

Capper and Duff (1974); Tsao (1977); Critchley (1978); Gibbons, Hawking, Perry (1978);
Duff and van Nieuwenhuizen (1980); Siegel (1980); Grisaru, Nielsen, Siegel, Zanon (1984);
Goroff and Sagnotti (1986); Bornsen and van de Ven (2009); Etc.

The Gauss-Bonnet counterterm exactly corresponds to trace anomaly.

$$D = 4 - 2\epsilon$$

$$\mathcal{L}^{\text{GB}} = -\frac{1}{(4\pi)^2}\frac{1}{360\epsilon}\Big(\underset{\substack{\uparrow\\ \textbf{graviton}}}{4\cdot 53} + \underset{\substack{\uparrow\\ \textbf{scalar}}}{1} + \underset{\substack{\uparrow\\ \textbf{2 form}}}{91} - \underset{\substack{\uparrow\\ \textbf{3 form}}}{180}\Big)\underset{\substack{\uparrow\\ \textbf{Gauss-Bonnet}}}{(R^2 - 4R_{\mu\nu} + R^2_{\mu\nu\rho\sigma})}$$

$$T^\mu{}_\mu = -\frac{1}{(4\pi)^2}\frac{2}{360}\Big(4\cdot 53 + 1 + 91 - 180\Big)(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma})$$

Duff and van Nieuwenhuizen (1980);

Referred to as trace, conformal, trace or Weyl anomaly.

Quantum Inequivalence?

$$D = 4 - 2\epsilon$$
$$D \to 4$$

$$T^{\mu}{}_{\mu} = -\frac{1}{(4\pi)^2}\frac{2}{360}\Big(\underset{\text{graviton}}{4\cdot 53} + \underset{\text{scalar}}{1} + \underset{\text{2 form}}{91} - \underset{\text{3 form}}{180}\Big)\underset{\text{Gauss-Bonnet}}{(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma})}$$

two form dual to scalar

$$\partial_\mu \phi \leftrightarrow \varepsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma}$$

three form not dynamical

$$\Lambda^{1/2} \leftrightarrow \varepsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma} \qquad D = 4$$

- **Quantum *in*equivalence under duality transformations.**
  Duff and van Nieuwenhuizen (1980)
- **Quantum equivalence under duality. Gauge artifact.**
  Siegel (1980)
- **Quantum equivalence of effective action (ignoring trace anomaly).**
  Fradkin and Tseytlin (1984)
- **Quantum equivalence of susy 1 loop effective action (with Siegel's argument for higher loops)**
  Grisaru, Nielsen, Siegel, Zanon (1984)
- **Quantum *in*equivalence and boundary modes.**
  Finn Larsen and Pedro Lisbao (2015)

What is physical significance?

Scattering amplitudes good to look at. Cross sections physical.
One loop not really good enough because anyway evanescent.

Two Loop Pure Gravity

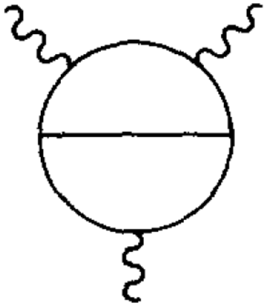
By two loops there is a valid *R*^{3} counterterm and corresponding divergence.

Goroff and Sagnotti (1986); Van de Ven (1992)

$D = 4 - 2\epsilon$

Divergence in pure Einstein gravity:

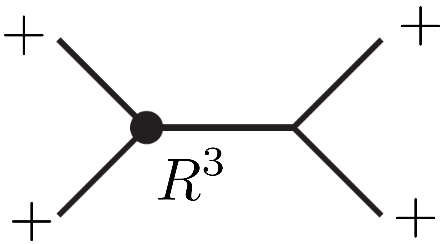
$$\mathcal{L}^{R^3} = \frac{209}{2880} \frac{1}{(4\pi)^4} \frac{1}{2\epsilon} R^{\alpha\beta}{}_{\gamma\delta} R^{\gamma\delta}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta}$$



- **The Goroff and Sagnotti result is correct in all details.**
- **On surface nothing weird going on (not evanescent).**

However, as we shall see the UV divergences in pure gravity is subtle and weird, once you probe carefully.

Two Loop Identical Helicity Amplitude

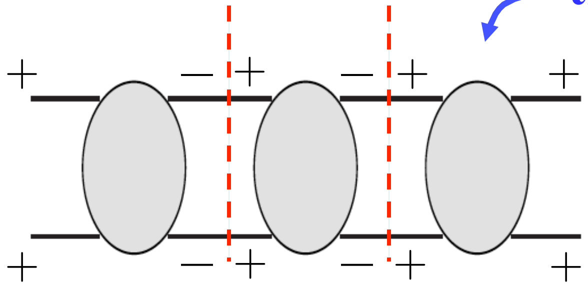


Pure gravity identical helicity amplitude sensitive to Goroff and Sagnotti divergence.

$$D = 4 - 2\epsilon$$

$$\mathcal{M}^{R^3}\Big|_{\text{div.}} = \frac{209}{24\epsilon}\mathcal{K} \qquad \mathcal{K} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\, stu \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2$$

Curious feature:



tree amplitude vanishes

- Integrand vanishes for four-dimensional loop momenta.
- Nonvanishing because of ϵ-dimensional loop momenta.

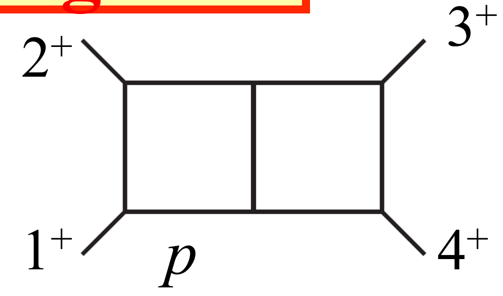
Bardeen and Cangemi pointed out nonvanishing of identical helicity is connected to an anomaly in self-dual sector.

A surprise:

Divergence is *not* generic but appears tied to anomaly-like behavior.

Full Two-Loop Integrand

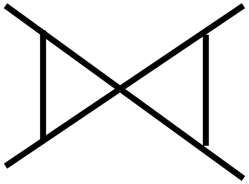
Using spinor helicity very compact:

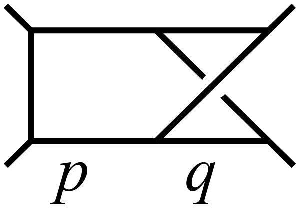


$$n = \frac{D_s(D_s-3)}{2}(\lambda_p^2\lambda_q^2 + \lambda_p^2\lambda_{p+q}^2 + \lambda_q^2\lambda_{p+q}^2)^2 - \frac{D_s(D_s-6)}{2}\lambda_p^2\lambda_q^2\lambda_{p+q}^2(\lambda_p^2 + \lambda_q^2 + \lambda_{p+q}^2)$$
$$+ 12D_s((\lambda_p\cdot\lambda_q)^2 - \lambda_p^2\lambda_q^2)(\lambda_p^2\lambda_q^2 + \lambda_p^2\lambda_{p+q}^2 + \lambda_q^2\lambda_{p+q}^2) + 144((\lambda_p\cdot\lambda_q)^2 - \lambda_p^2\lambda_q^2)^2\,,$$

Bow-tie and nonplanar contributions similar:

$$p_i = p_i^{(4)} + \lambda_i$$

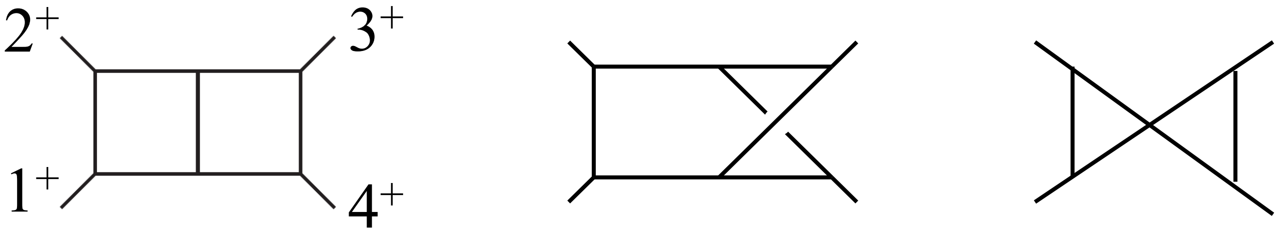




- **Integrand vanishes for *D* = 4 loop momenta:** λ^8
- **Upon integration ultraviolet divergent.**
- **Nonplanar is integrand is identical to planar. Reflection of BCJ duality.**

Two-Loop Bare Divergence

ZB, Cheung, Chi, Davies, Dixon and Nohle



$$209 = 11 \cdot 19$$

$$3431 = 47 \cdot 73$$

Integrating we obtain:

$$\mathcal{M}_4^{2\text{-loop}}\Big|_{\text{bare div.}} = -\frac{1}{\epsilon}\,\frac{3431}{5400}\,\mathcal{K} \qquad \mathcal{K} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\, stu \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2$$

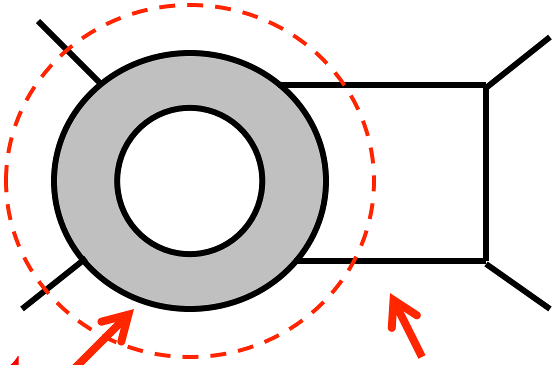
Not the same as the Goroff and Sagnotti result

However, Goroff and Sagnotti subtracted subdivergences integral by integral.

Subdivergences? What subdivergences?
There are no one-loop divergences. Right?

Subdivergences?

The integrand has subdivergences



Representative diagram.

Gauss-Bonnet subdivergence

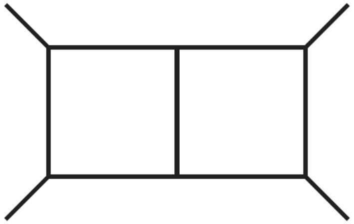
$D = 4$, **no subdivergences**
$D \neq 4$, **subdivergences!**

A strange phenomenon: no one loop divergences, yet there are one-loop subdivergences!

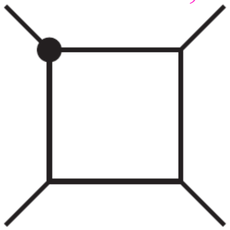
- **To match the G&S result we need to subtract subdivergences.**
- **We use counterterm method, allowing us to track pieces.**

Pure Gravity Divergence

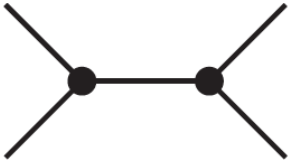
ZB, Cheung, Chi, Davies, Dixon and Nohle



2 loop bare



single GB subdivergence



double GB subdivergence

$$D = 4 - 2\epsilon$$

$$209 = 19 \times 11$$
$$3431 = 47 \times 73$$
$$5618 = 2 \times 53^2$$

$$\left.\begin{aligned}
\mathcal{M}_4^{\text{2-loop}}\Big|_{\text{div.}} &= -\frac{1}{\epsilon}\,\frac{3431}{5400}\,\mathcal{K} \\
\mathcal{M}_4^{\text{1-loop GB}}\Big|_{\text{div.}} &= \frac{1}{\epsilon}\,\frac{689}{675}\,\mathcal{K} \\
\mathcal{M}_4^{\text{tree GB}^2}\Big|_{\text{div.}} &= \frac{1}{\epsilon}\,\frac{5618}{675}\,\mathcal{K}
\end{aligned}\right\} \qquad \mathcal{M}_4^{\text{total}}\Big|_{\text{div.}} = \frac{1}{\epsilon}\,\boxed{\frac{209}{24}}\,\mathcal{K}$$

Goroff and Sagnotti divergence reproduced

Surprise: Evanescent Gauss-Bonnet (GB) operator crucial part of UV structure. Link to conformal anomaly.

Meaning of Divergence?

What does the divergence mean?

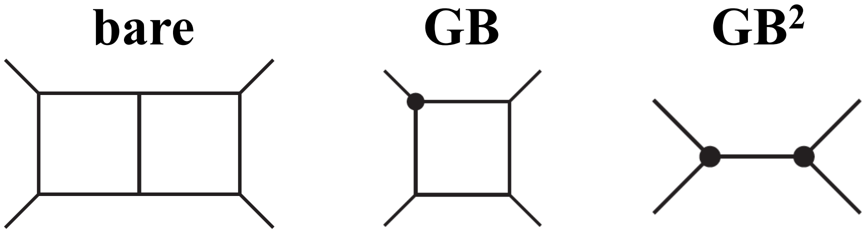
$$\Lambda^{1/2} \leftrightarrow \varepsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma}$$

Adding n_3 3-form field offers good way to understand this:

- On the one hand, no degrees of freedom in $D = 4$, so no change in divergence expected.
- On the other hand, the trace anomaly is affected, so expect change in divergence.
- Note that 3 form proposed as way to dynamically neutralize cosmological constant.

Brown and Teitelboim; Bousso and Polchinski

	$1/\epsilon$
bare	$-\frac{3431}{5400} - \frac{199 n_3}{30} + 6 n_3^2$
GB	$\frac{4\cdot 53 - 180 n_3}{360} \cdot \frac{2\cdot(13+180 n_3)}{15}$
GB^2	$24\left(\frac{4\cdot 53 - 180 n_3}{360}\right)^2$
total	$\frac{209}{24} - \frac{15}{2} n_3$



← Divergence depends on nondynamical 3-form fields!

Divergences Differ Under Dualities

ZB, Cheung, Chi, Davies, Dixon and Nohle

Single scalar or anti-symmetric tensor coupled to gravity.

$$\mathcal{L}_{gd} = \left(\frac{2}{\kappa^2} R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right) \qquad \mathcal{L}_{ga} = \left(\frac{2}{\kappa^2} R + \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho}\right)$$

Related by *D* = 4 duality transformation

$$\partial_\mu\phi = \frac{i}{\sqrt{2}}\varepsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma}$$

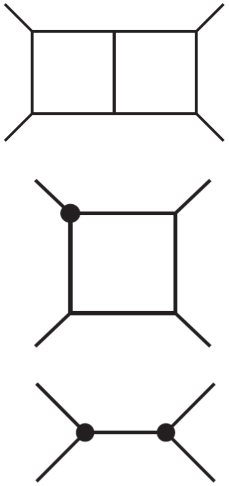
Coefficient of $1/\epsilon$:

graviton + n_0 scalars

bare	$-\frac{3431}{5400} - \frac{277 n_0}{10800} + \frac{n_0^2}{5400}$
GB	$\frac{4\cdot53+n_0}{360}\cdot\frac{2\cdot(13-n_0)}{15}$
GB^2	$24\left(\frac{4\cdot53+n_0}{360}\right)^2$
total	$\frac{209}{24} - \frac{1}{48} n_0$

graviton + n_2 2-forms

bare	$-\frac{3431}{5400} + \frac{8543 n_2}{10800} + \frac{8281 n_2^2}{5400}$
GB	$\frac{4\cdot53+91n_2}{360}\cdot\frac{2\cdot(13-91n_2)}{15}$
GB^2	$24\left(\frac{4\cdot53+91n_2}{360}\right)^2$
RHH	$5n_2$
total	$\frac{209}{24} + \frac{299}{48} n_2$



UV divergences altered by duality transformations

UV result suggests that theories are quantum mechanically *in*equivalent as proposed by Duff and van Nieuwenhuizen.

But wait: what about finite parts?

Scattering Amplitudes

Pure Gravity:

IR singularities subtracted and independent of 3 form

$$\mathcal{M}_G^{(2)}(1^+,2^+,3^+,4^+) = \mathcal{N}\Big[\boxed{\frac{1}{\epsilon}\frac{209}{24}stu} + \frac{117617}{21600}stu + \left(\frac{1}{10}stu - \frac{1}{60}s^3\right)\log\left(\frac{-s}{\mu^2}\right) + \frac{1}{120}\left(s^2+t^2+u^2\right)s\log^2\left(\frac{-s}{\mu^2}\right) + \text{perms}\Big]$$

Gravity + 3 Form:

divergences different. logarithms identical!

$$\mathcal{M}_{G3}^{(2)}(1^+,2^+,3^+,4^+) = \mathcal{N}\Big[\boxed{\frac{1}{\epsilon}\frac{29}{24}stu} + \frac{411617}{21600}stu + \left(\frac{1}{10}stu - \frac{1}{60}s^3\right)\log\left(\frac{-s}{\mu^2}\right) + \frac{1}{120}\left(s^2+t^2+u^2\right)s\log^2\left(\frac{-s}{\mu^2}\right) + \text{perms}\Big]$$

- **Value of divergence not physical. Absorb into counterterm.**
- **3 form is a Cheshire Cat field: scattering unaffected.**

Similar results comparing scalar and two-forms.

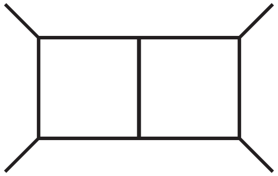
Results consistent with quantum equivalence under duality.
Firmly in quantum equivalence camp.

Renormalization Scale Dependence

Simple rule for tracking renormalization scale:

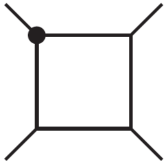
$$D = 4 - 2\epsilon$$

bare



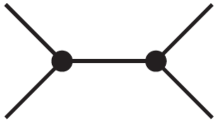
$$(\mu^2)^{2\epsilon}\frac{c_2}{\epsilon} = \frac{c_2}{\epsilon} + 2c_2 \ln \mu^2 + \mathcal{O}(\epsilon)$$

1 counterterm



$$(\mu^2)^{\epsilon}\frac{c_1}{\epsilon} = \frac{c_1}{\epsilon} + c_1 \ln \mu^2 + \mathcal{O}(\epsilon)$$

GB²



$$(\mu^2)^{0\epsilon}\frac{c_0}{\epsilon} = \frac{c_0}{\epsilon}$$

Total: $(c_2 + c_1 + c_0)\frac{1}{\epsilon} + (2c_2 + c_1) \ln \mu^2$

Divergences and Duality

ZB, Cheung, Chi, Davies, Dixon and Nohle

As probe add n_3 3-form fields to theory.

- **No dynamical degrees of freedom.**
- **Field strength dual to cosmological constant.**

$$\Lambda^{1/2} \leftrightarrow \varepsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma}$$

Brown and Teitelboim; Bousso and Polchinski

$$\mathcal{M}_4 = \left[\frac{1}{\epsilon}\left(\frac{209}{24} - \frac{15}{2} n_3\right) - \boxed{\frac{1}{4}} \ln \mu^2\right] \mathcal{K} + \text{finite}$$

divergence

number of 3 forms

independent of 3 forms

Renormalization scale

Weird that renorm. scale and UV divergence not linked!
Happens because of evanescent Gauss-Bonnet subdivergence.

- **Divergence sensitive to nondynamical 3 forms.**
- **3 forms have no physical effect in scattering amplitudes!**
- **3 form is "Cheshire Cat Field".**

A simple two-loop formula

ZB, Cheung, Chi, Davies, Dixon and Nohle

Focus on renormalization scale dependence *not* divergences!

Looking at various theories, we wind up with a simple 2 loop formula:

$$\mathcal{M}_4^{(2)}\Big|_{\ln \mu^2} = -\mathcal{K}\,\frac{N_b - N_f}{8}\,\ln \mu^2$$

N_b is number of bosonic states.
N_f is number of fermionic states.

- **Vanishes at two loops in susy theory, as expected.**
- **Unless lnμ^2 dependence vanishes, theory should still be considered nonrenormalizable.**
- **It would be very interesting to understand higher loops.**

UV properties of gravity subtle and interesting!

Summary

1. Gravity integrands from gauge theory. Very powerful tool!
2. Standard view of gravity UV much too naive:
   - — New phenomenon: "Enhanced" UV cancellations in gravity.
   - — Known pure (super)gravity divergences are anomaly-like: 0/0 behavior.
   - — Gravity leading divergences can depend on evanescent fields and operators and on duality transformations.
   - — Renormalized scattering amplitudes independent of duality transformations.
3. Better to focus on renormalization scale dependence rather than divergences. Not the same!

UV structure of gravity is tamer than expected.
Behavior of gravity under duality transformations surprising.

Expect many more surprises as we probe gravity theories using modern perturbative tools.