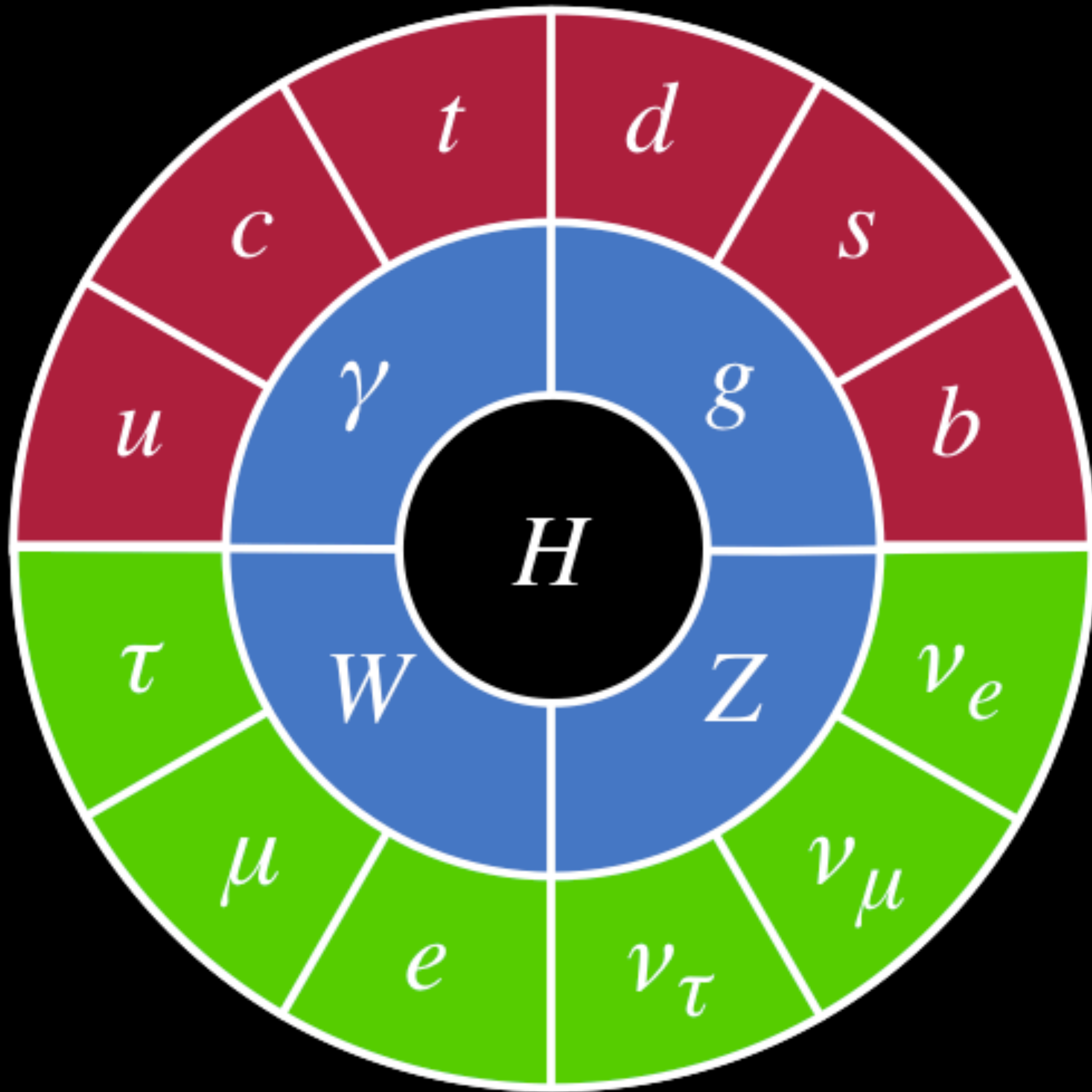
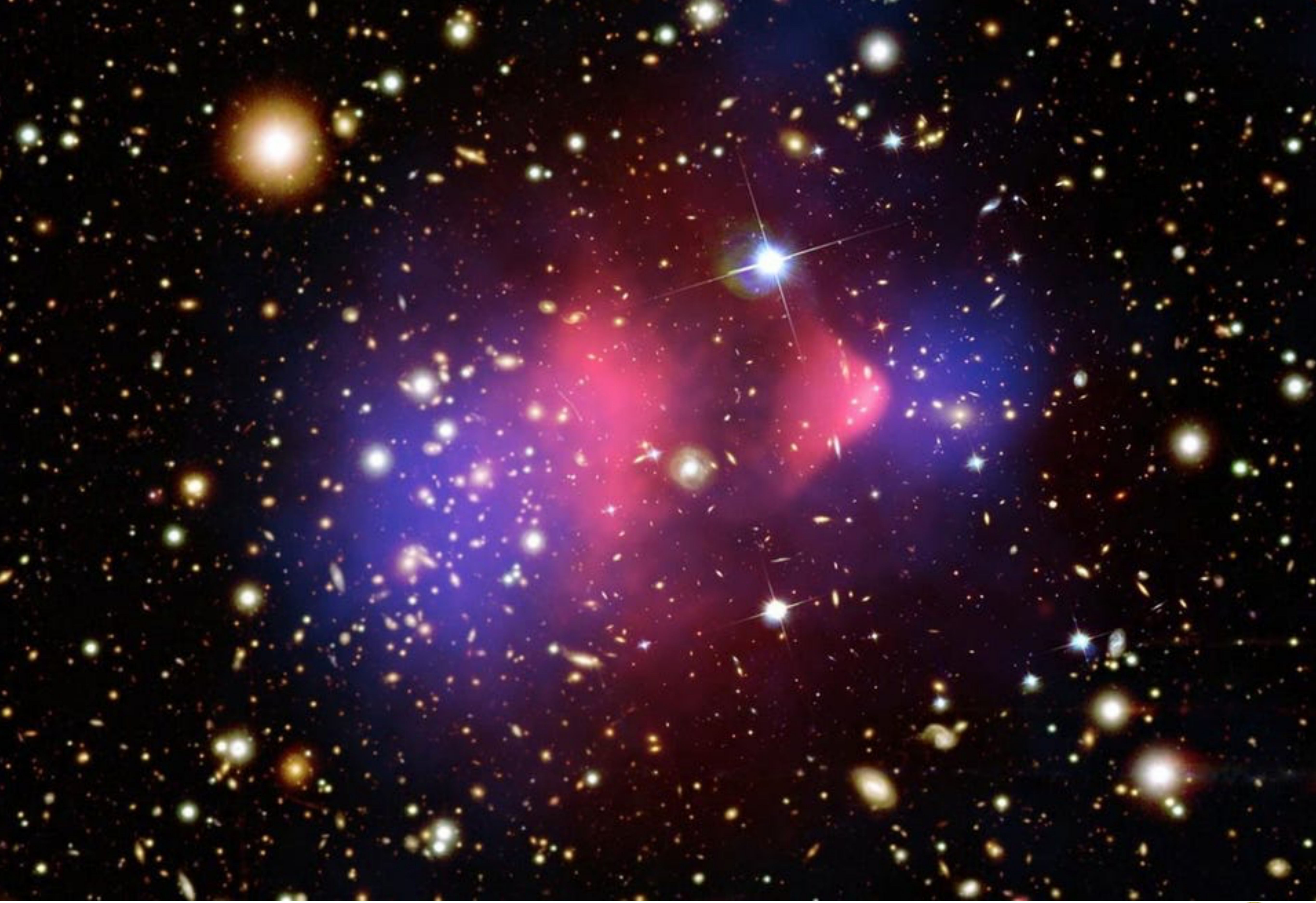
Quantum Computing for High Energy Physics

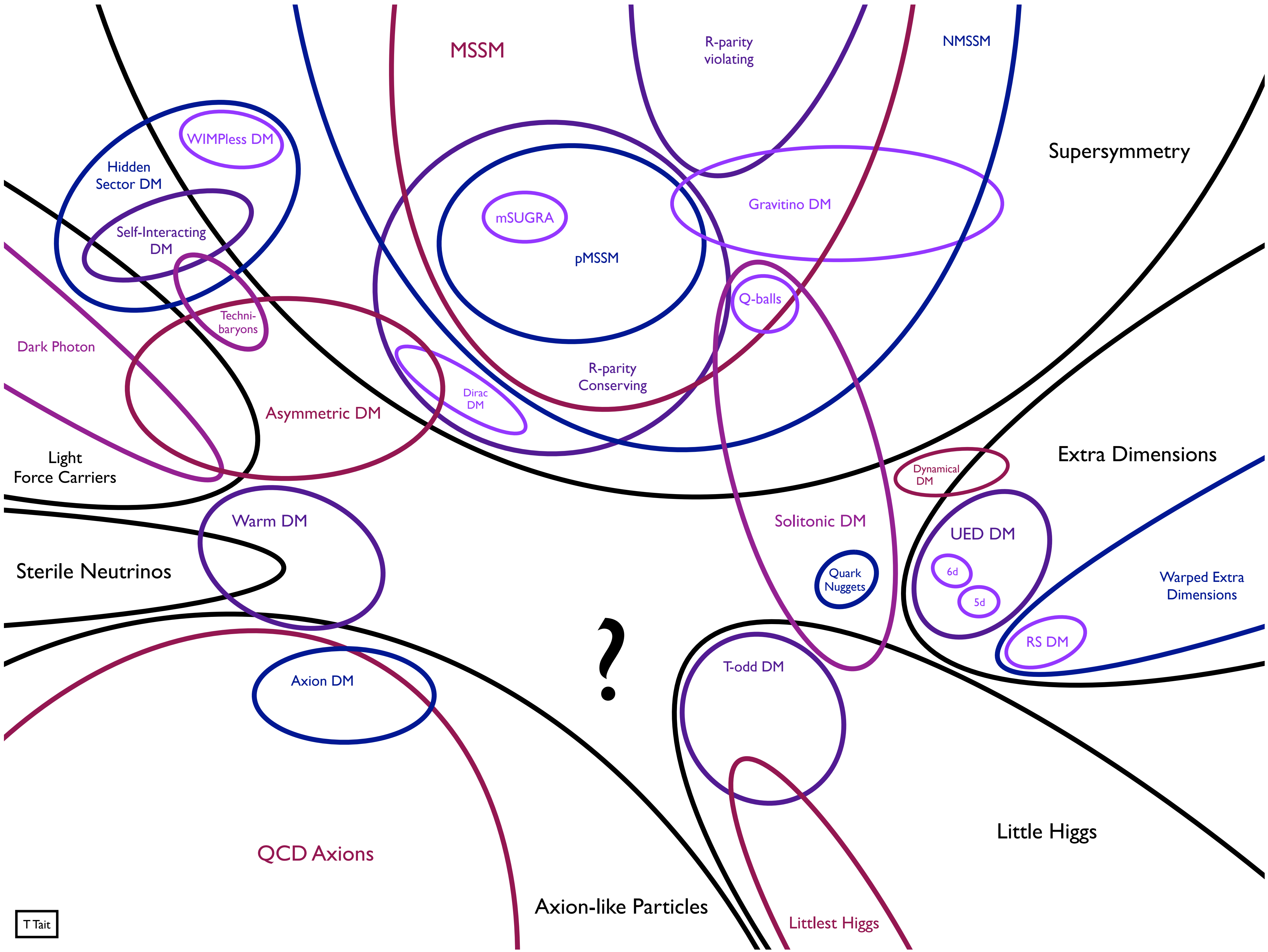
Christian Bauer



Walter Murch,

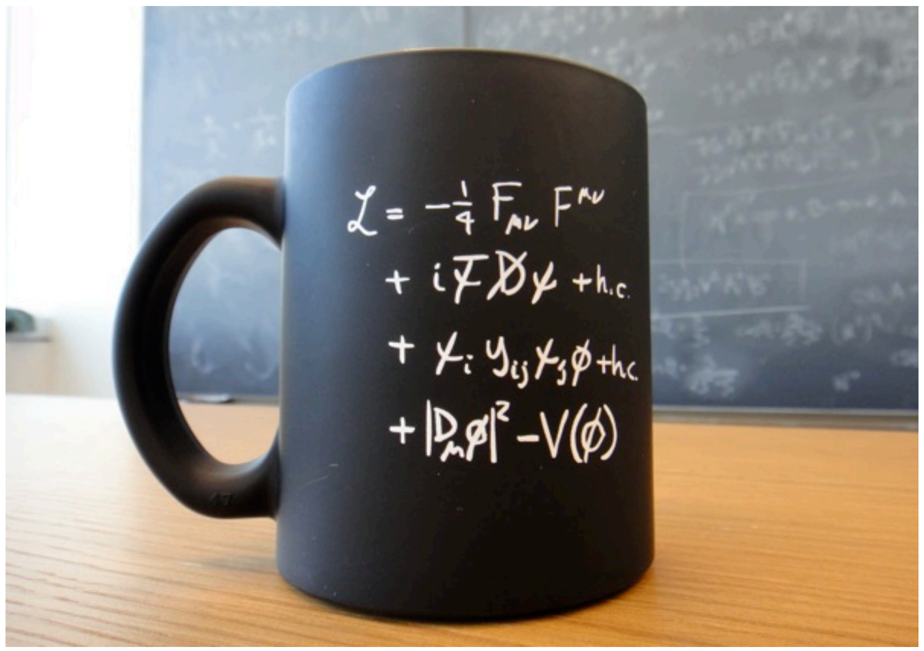


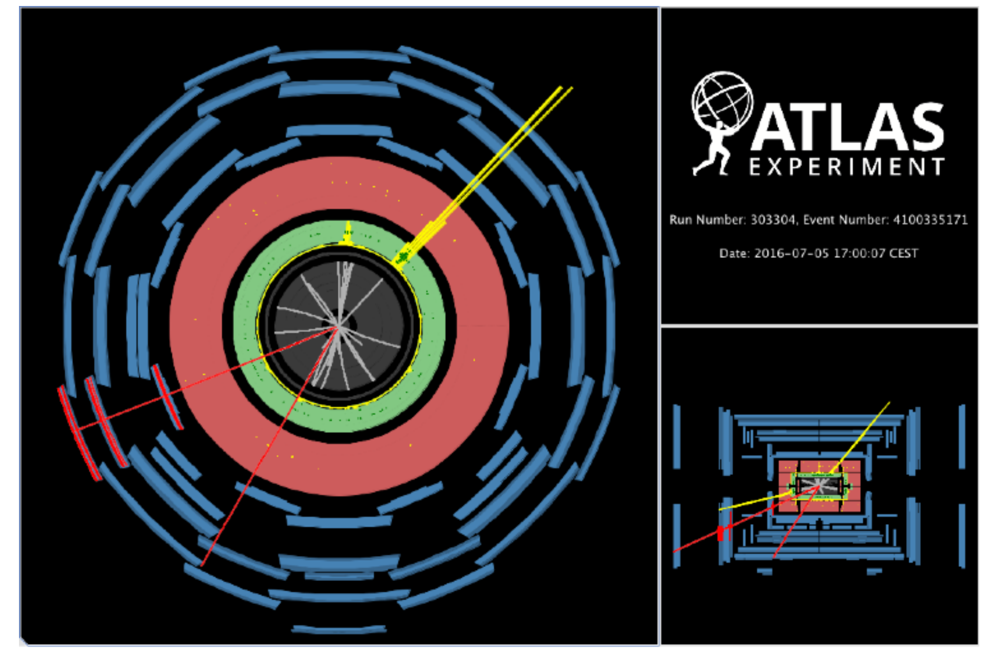
One can try to find possible and motivated extensions of the standard model…



…or look for deviations of experimental observations with SM expectations

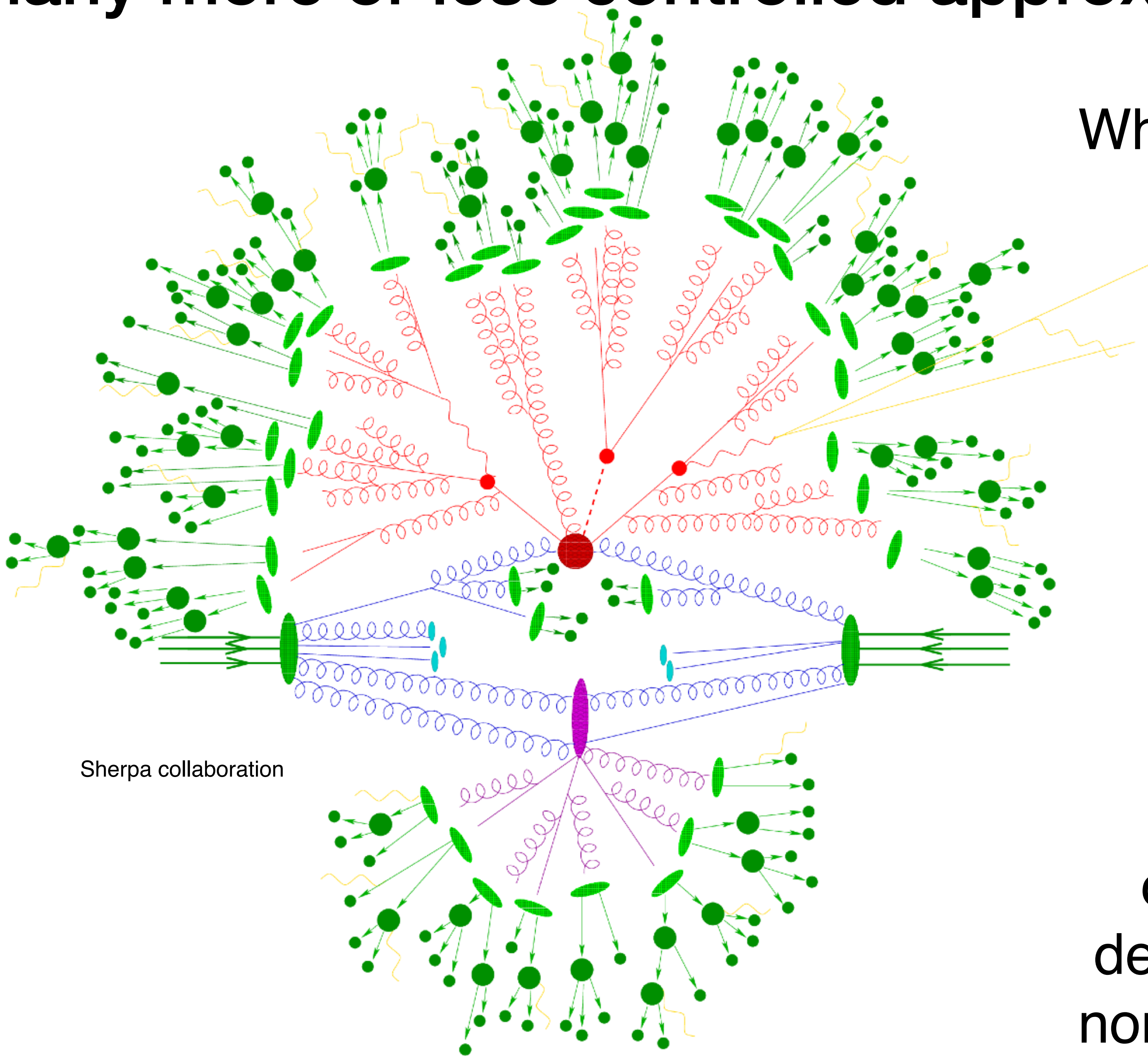
To truly understand if Standard Model describes data observed at LHC, need to connect theory and data





For this, need to be able to go from Lagrangian to fully exclusive events

Simulating a fully exclusive event is very difficult, and many more or less controlled approximations are needed



Sherpa collaboration

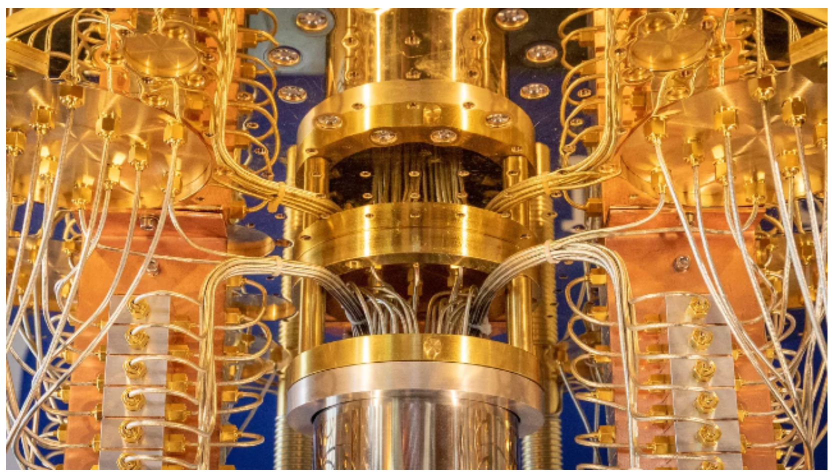
While the hard interaction only produces small number of particles, subsequent radiation produces lots more in final state.

Impossible to compute full results in perturbation theory

Need ways to perform calculations that allows to deal with this high multiplicity and non-perturbative physics

No known (classical) algorithm to do the required calculations in full generality

The issues with
classical techniques



How a quantum
computer can help

Recent work on
concrete problems

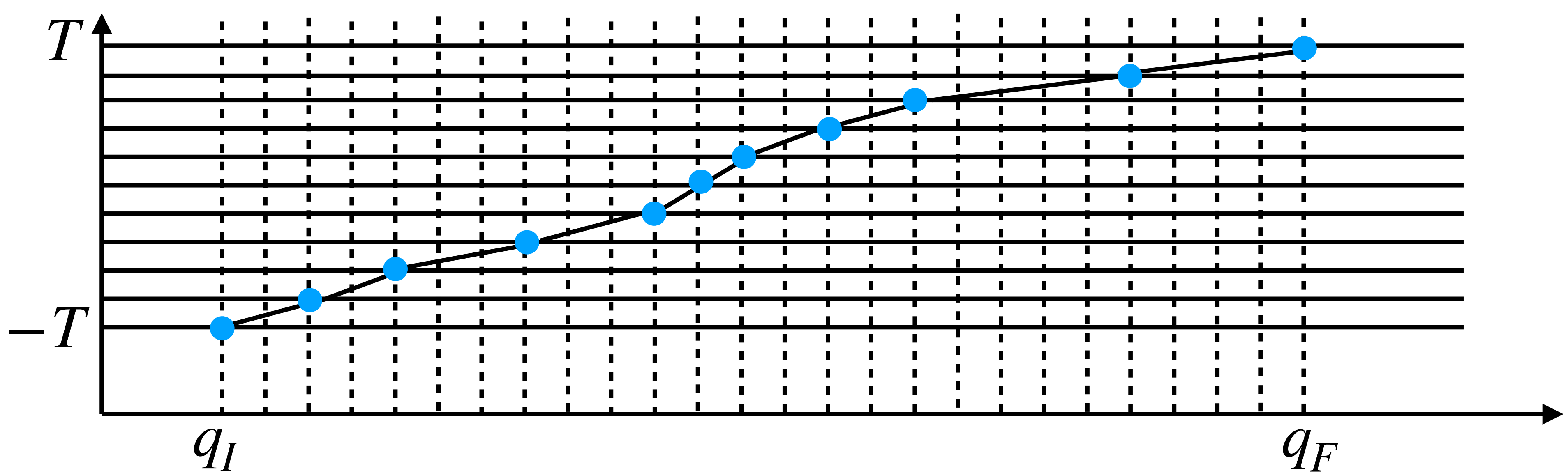
The issues with
classical techniques

The best known numerical technique for QFTs is Lattice Field Theory

Start from the path integral formulation

$$\langle q_F(T) \,|\, e^{-iH(2T)/\hbar} \,|\, q_I(-T) \rangle = \sum_{\text{paths}} \exp\left[iS\right]$$

Discretize paths by putting space time on hypersurface



$$\langle q_F(T) \,|\, e^{-iH(2T)/\hbar} \,|\, q_I(-T) \rangle = \sum_{\{x_i(t_j)\}} \exp\left[iS\right]$$

Sample over all possible values $\{x_i(t_j)\}$

The best known numerical technique for QFTs is Lattice Field Theory

$$\langle q_F(T) \,|\, e^{-iH(2T)/\hbar} \,|\, q_I(-T) \rangle = \sum_{\{x_i(t_j)\}} \exp\left[iS\right]$$

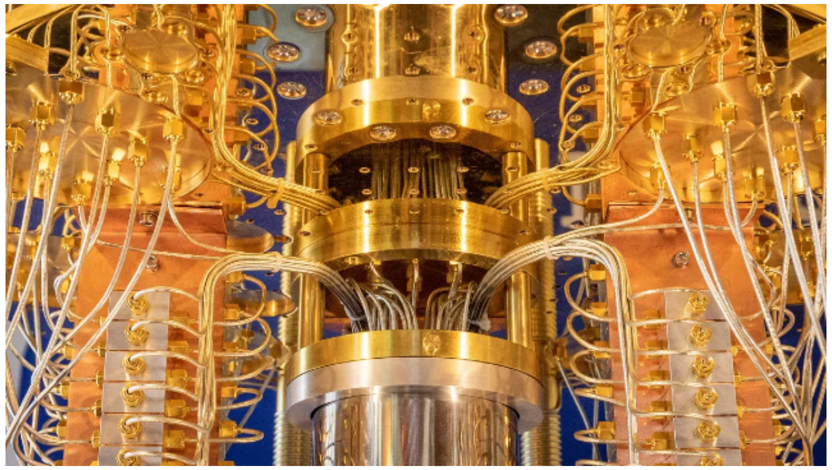
Complex exponent has rapidly oscillating phase,
can not integrate by MC method efficiently

Standard Lattice techniques use Wick rotation $t \to i\tau$

$$\langle q_F(\mathcal{T}) \,|\, e^{-H(2\mathcal{T})/\hbar} \,|\, q_I(-\mathcal{T}) \rangle = \sum_{x_i(t_j)} \exp\left[-S\right]$$

Can now be calculated with errors that scale statistically with $\sqrt{N_{\text{events}}}$,
independent of number of lattice points

But this does not allow real time simulation and inherently Minkowskian objects



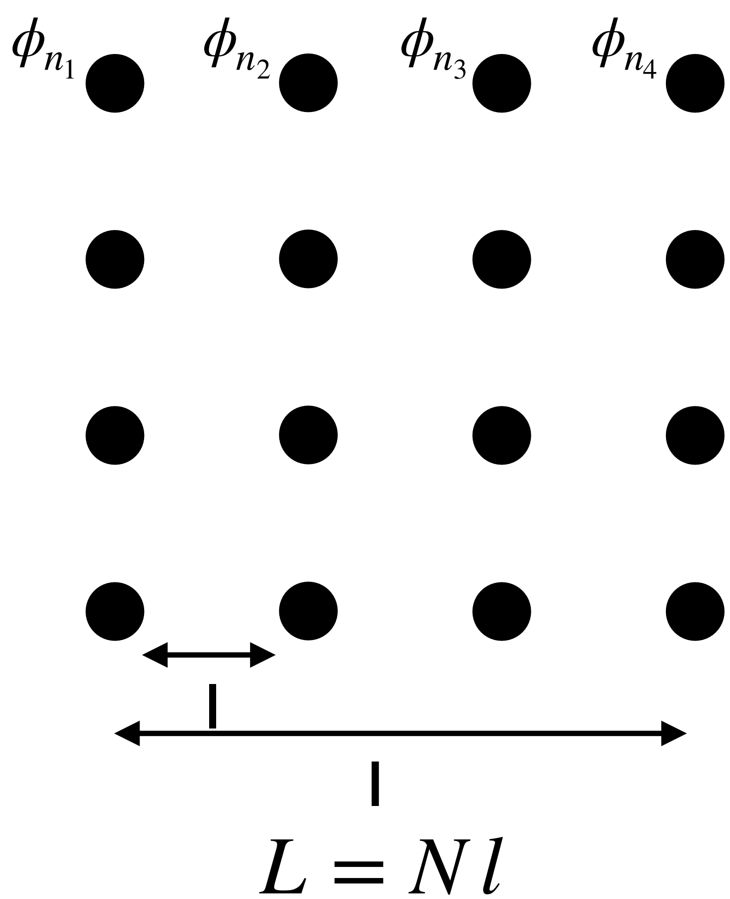
How a quantum computer can help

Can use directly the time evolution between initial and final state

$$\langle q_F(T) \,|\, e^{-iH(2T)/\hbar} \,|\, q_I(-T) \rangle$$

For full QFT, Hilbert space is infinite dimensional
(both position and field values are continuous)

Turn into finite dimensional Hilbert space by discretizing both spatial directions and field values



Hilbert space has dimension

$$\left(n_\phi\right)^{N^d}$$

n_ϕ : # of digitized field values
N : # of lattice points per dim
d : # of dimensions

Situation more complicated
for gauge theories

Can use directly the time evolution between initial and final state

$$\langle q_F(T) \,|\, e^{-iH(2T)/\hbar} \,|\, q_I(-T) \rangle$$

After discretization, states are vectors in the finite dimensional Hilbert space, while Hamiltonian is a matrix

Matrix element can now be computed through matrix multiplication

But $\left(n_\phi\right)^{N^d}$ is a very large number, so this is completely intractable using standard (classical) calculations

Quantum computers can do the calculation with resources (number of qubits and number of operations) that scale logarithmically in the size of Hilbert space

Let's try to estimate the resources we need to simulate physics at the LHC

Energy rage that can be described by lattice is given by

$$\frac{1}{Nl} \lesssim E \lesssim \frac{1}{l}$$

To simulate full energy range of LHC need

$$100\,\text{MeV} \lesssim E \lesssim 7\,\text{TeV}$$

This needs $\mathcal{O}(70{,}000^3) \sim 10^{14}$ lattice sites

Assume I need at least 5 bit digitization $\Rightarrow n_\phi = 2^5 = 32$

Dimension of Hilbert space is

$$32^{10^{14}} \sim \infty$$

Number of qubits and required

$$5 \times 10^{14}$$

HEP theory developments in quantum computing deal with 2 large classes of problems

1. Find good Lattice representations of the Hamiltonian of the gauge theories of the Standard Model

2. Develop techniques that allow to compute phenomenologically meaningful results with reasonable resource requirements

Recent work on concrete problems

The continuum Hamiltonian of QED is very simple, consisting of a magnetic and electric component

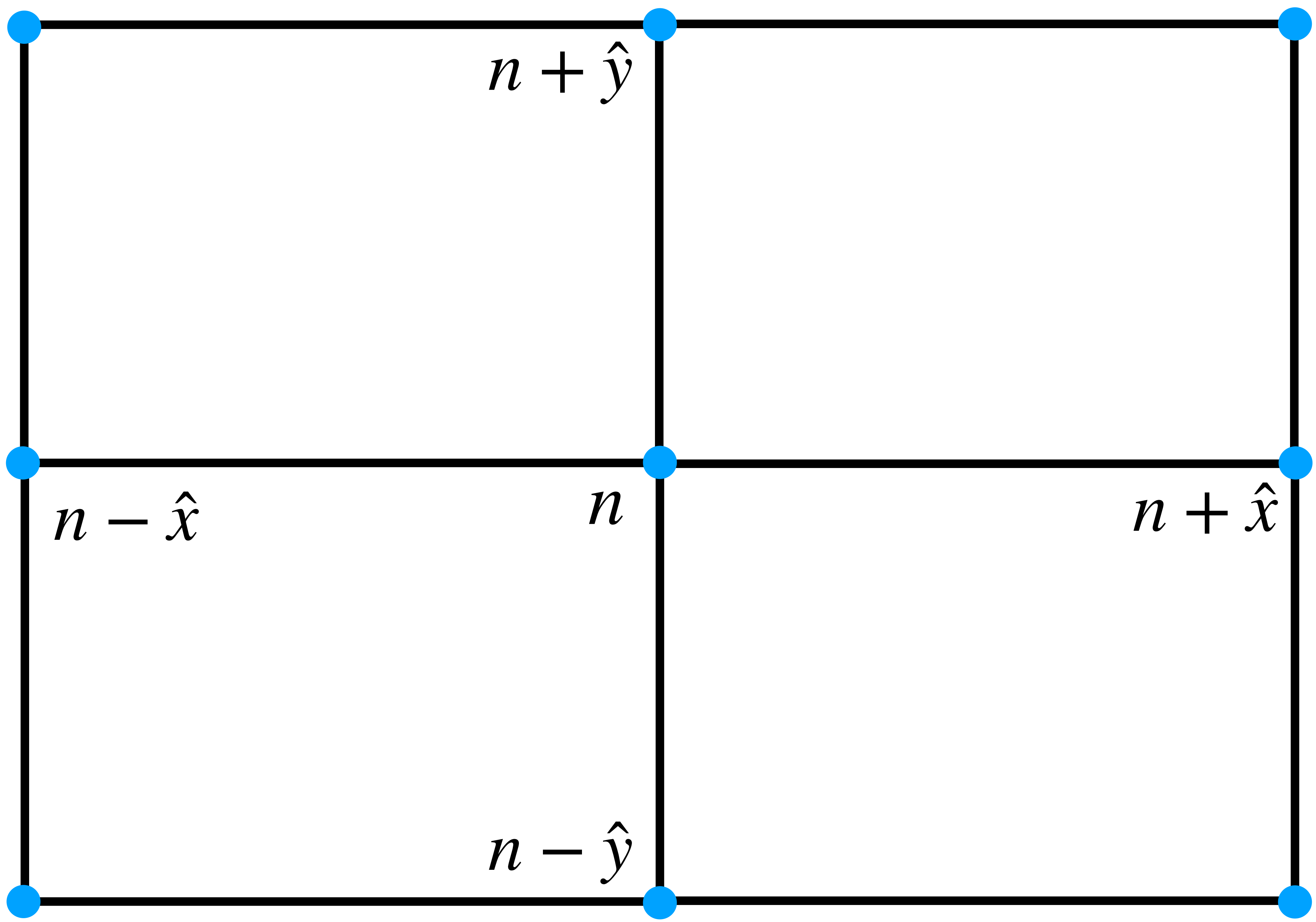
$$H = \int d^d x \left[E^2(x) + B^2(x)\right]$$

E and B have simple relations to the gauge field (working in $A_0 = 0$ gauge)

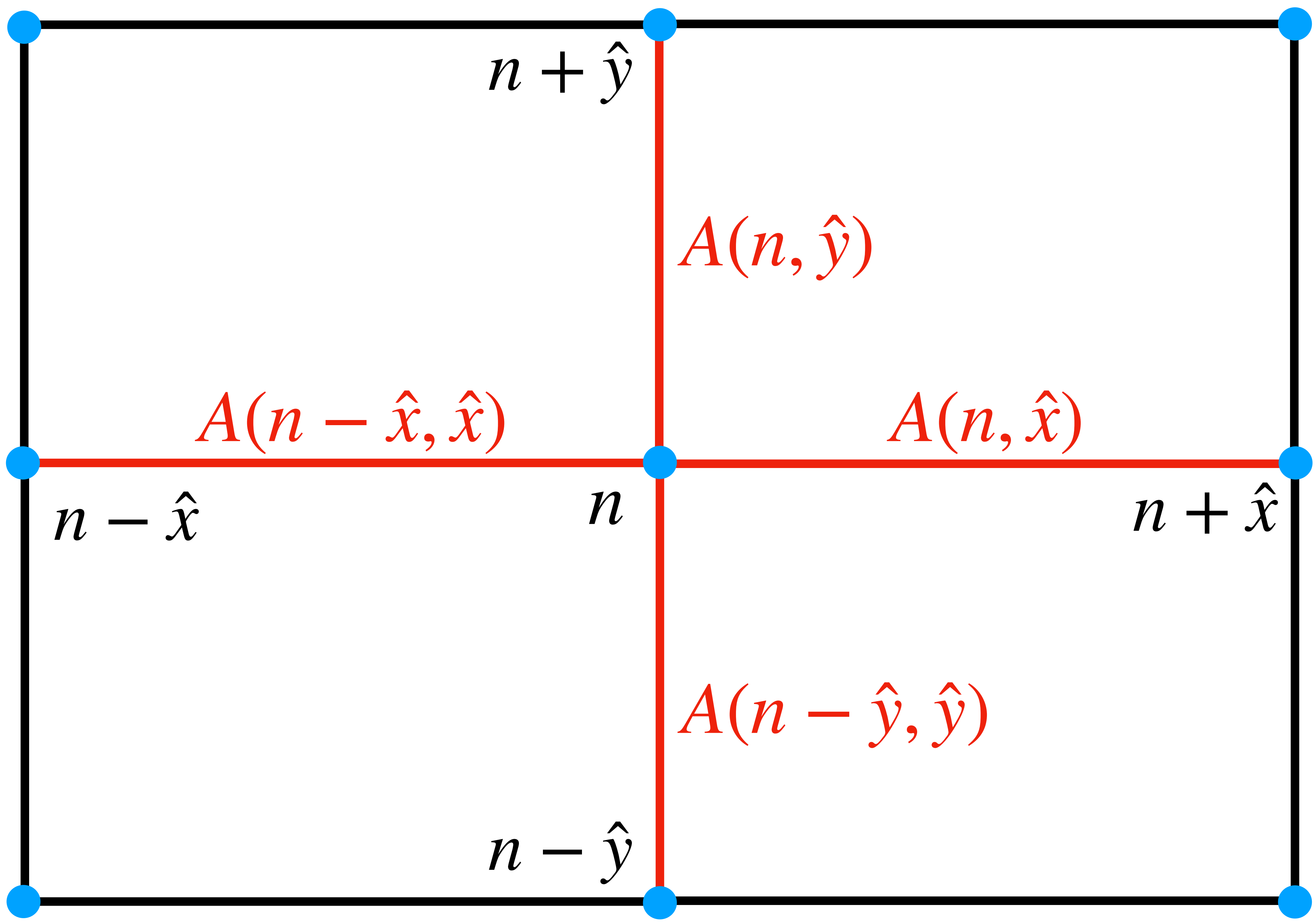
$$\overrightarrow{B}(x) = \overrightarrow{\nabla} \times \overrightarrow{A}(x)$$

$$\overrightarrow{E}(x) = -\partial \overrightarrow{A}(x)/\partial t$$

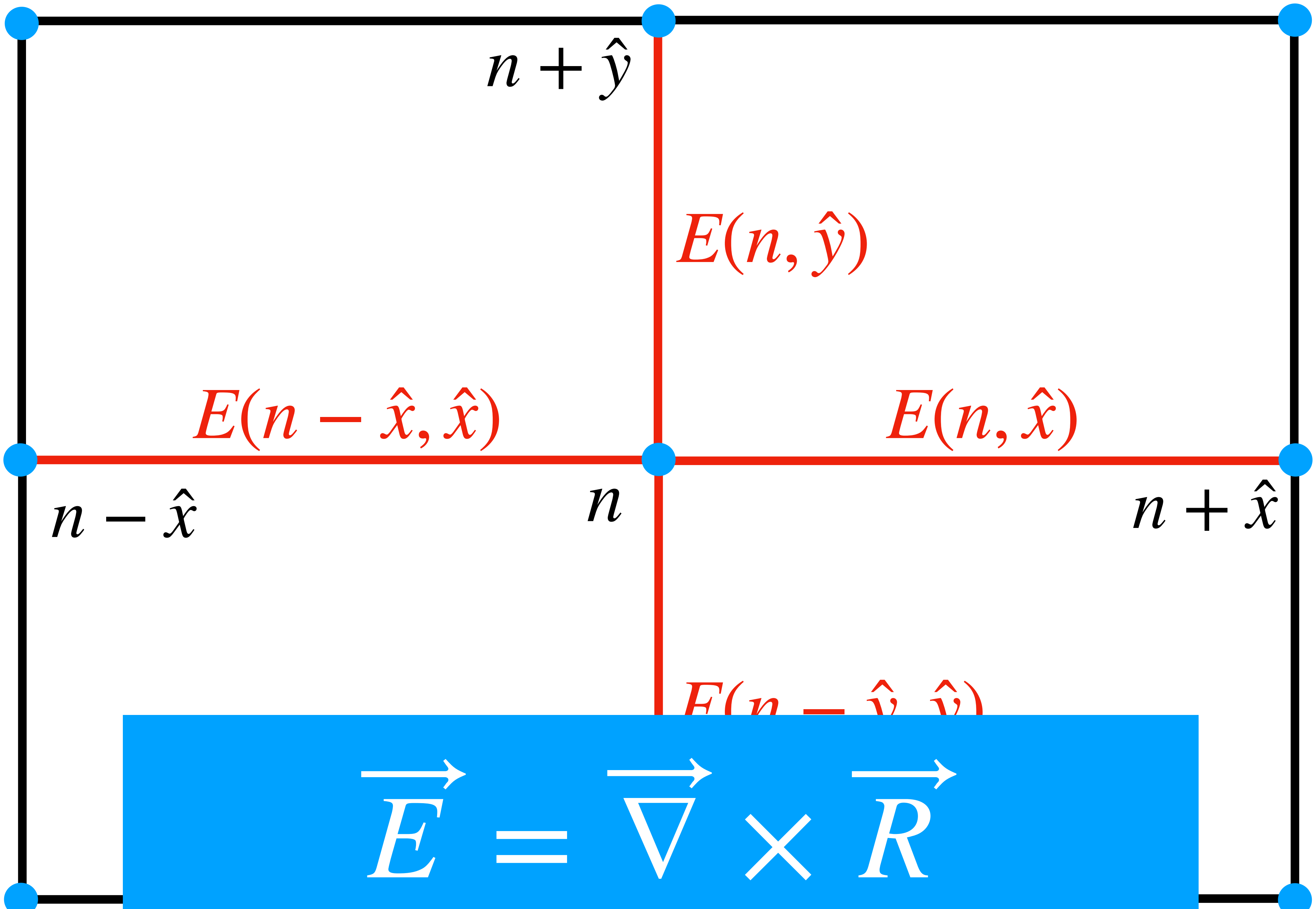
Consider a spacial lattice, similar to what we considered before for a scalar theory



Gauge fields are related to derivatives, so are related to differences of sites, and live on links

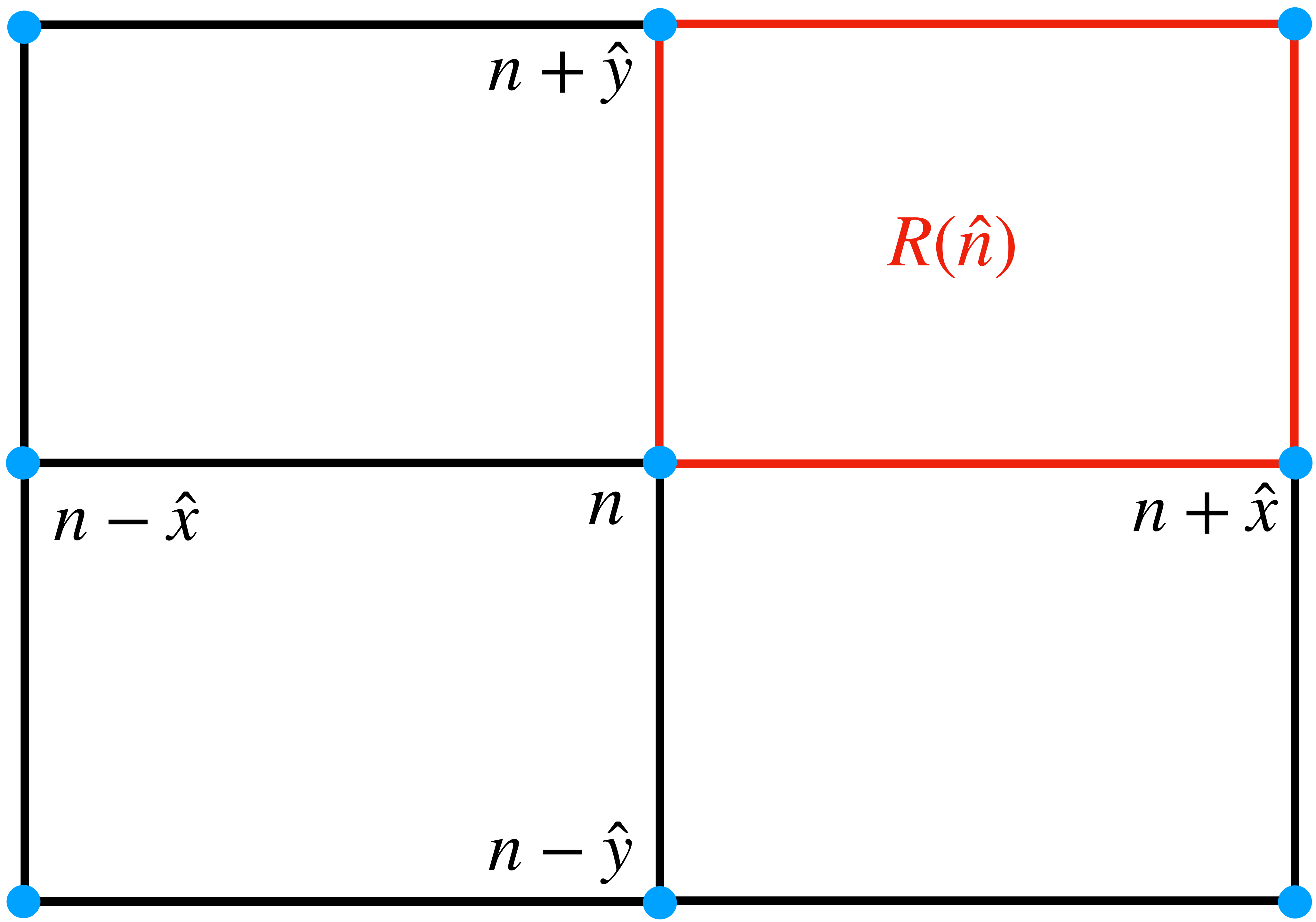


Gauss' law (part of gauge invariance) dictates that divergence of electric field vanishes (without charges)

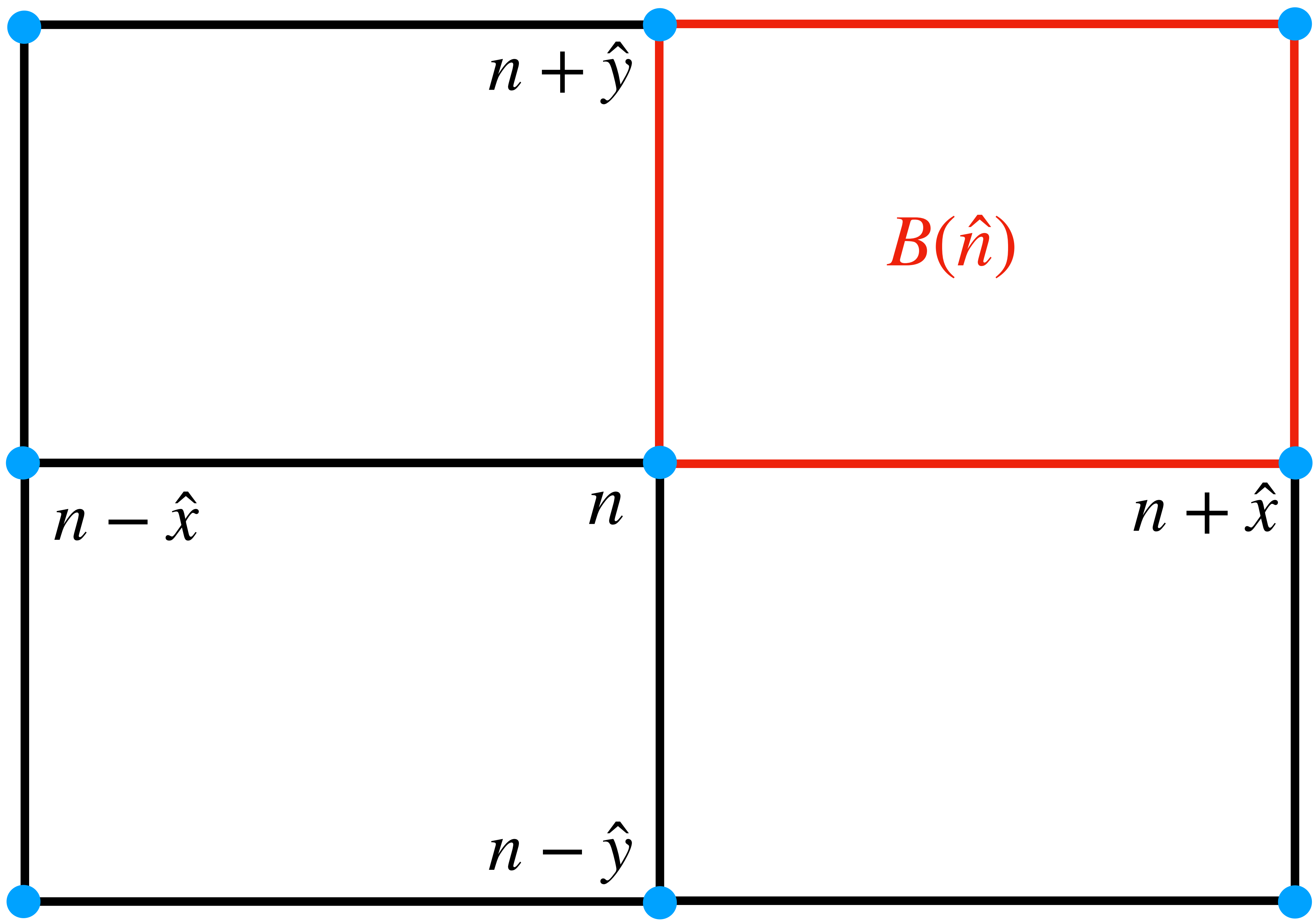


$$\vec{E} = \vec{\nabla} \times \vec{R}$$

Instead of electric field, can use a field R, which lives on a "plaquette". This automatically preserves gauge invariance



Conjugate variable is precisely the magnetic field that appears in the Hamiltonian



One can write Lattice version of Hamiltonian entirely in terms of rotors and magnetic fields

$$H = \sum_{p \in \mathrm{plaq}} \left[g^2 H_E[R_i] + \frac{1}{g^2} H_M[B_i] \right]$$

There is considerable interest in "compact" U(1) gauge theory, where
$-\pi < B_i < \pi$

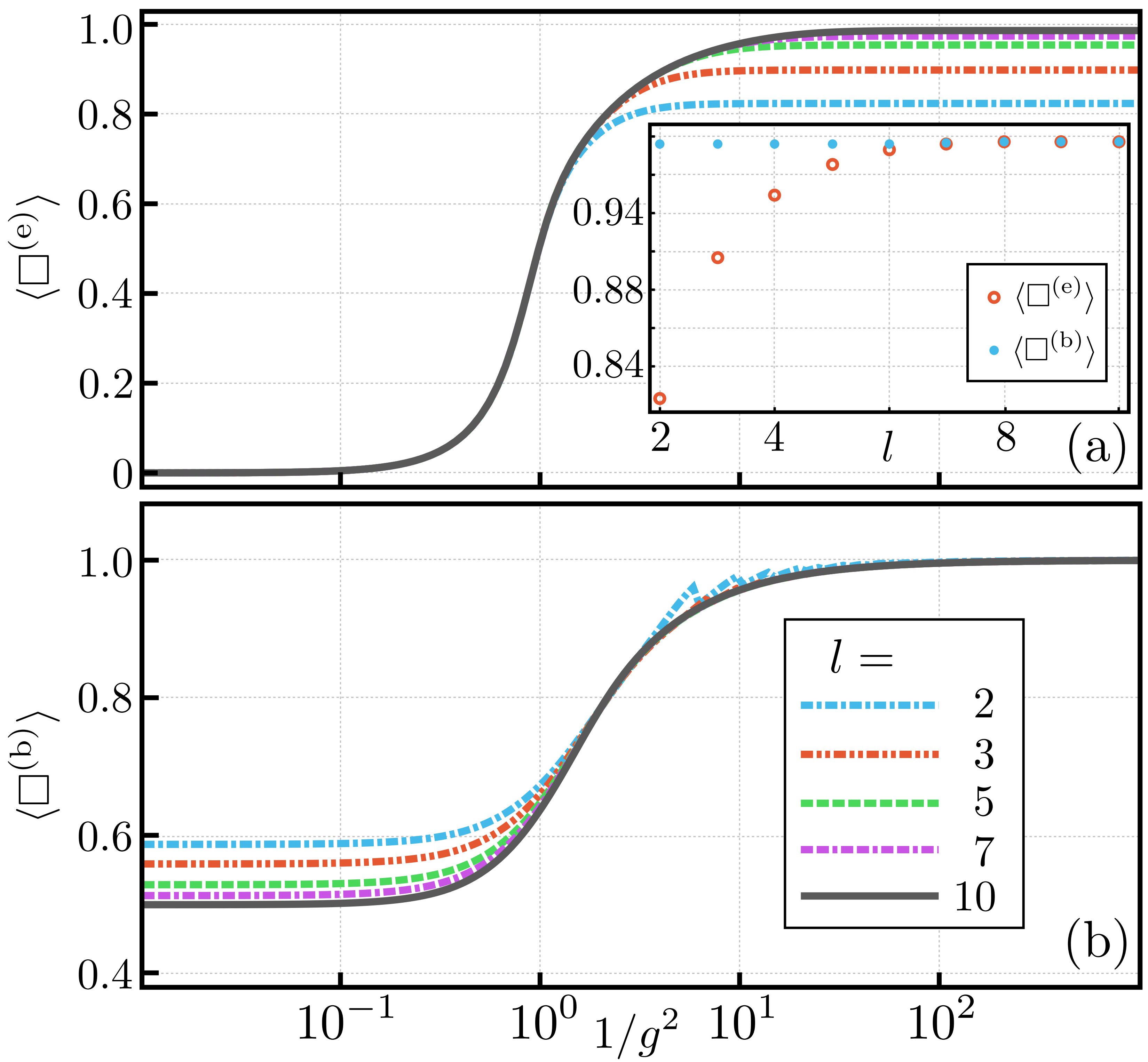
Since $[H_E, H_M] \neq 0$, H_E and H_B can not be diagonalized simultaneously

In limit $g \to \infty$ useful to work in electric basis, where H_E is diagonal

In limit $g \to 0$ useful to work in magnetic basis, where H_B is diagonal

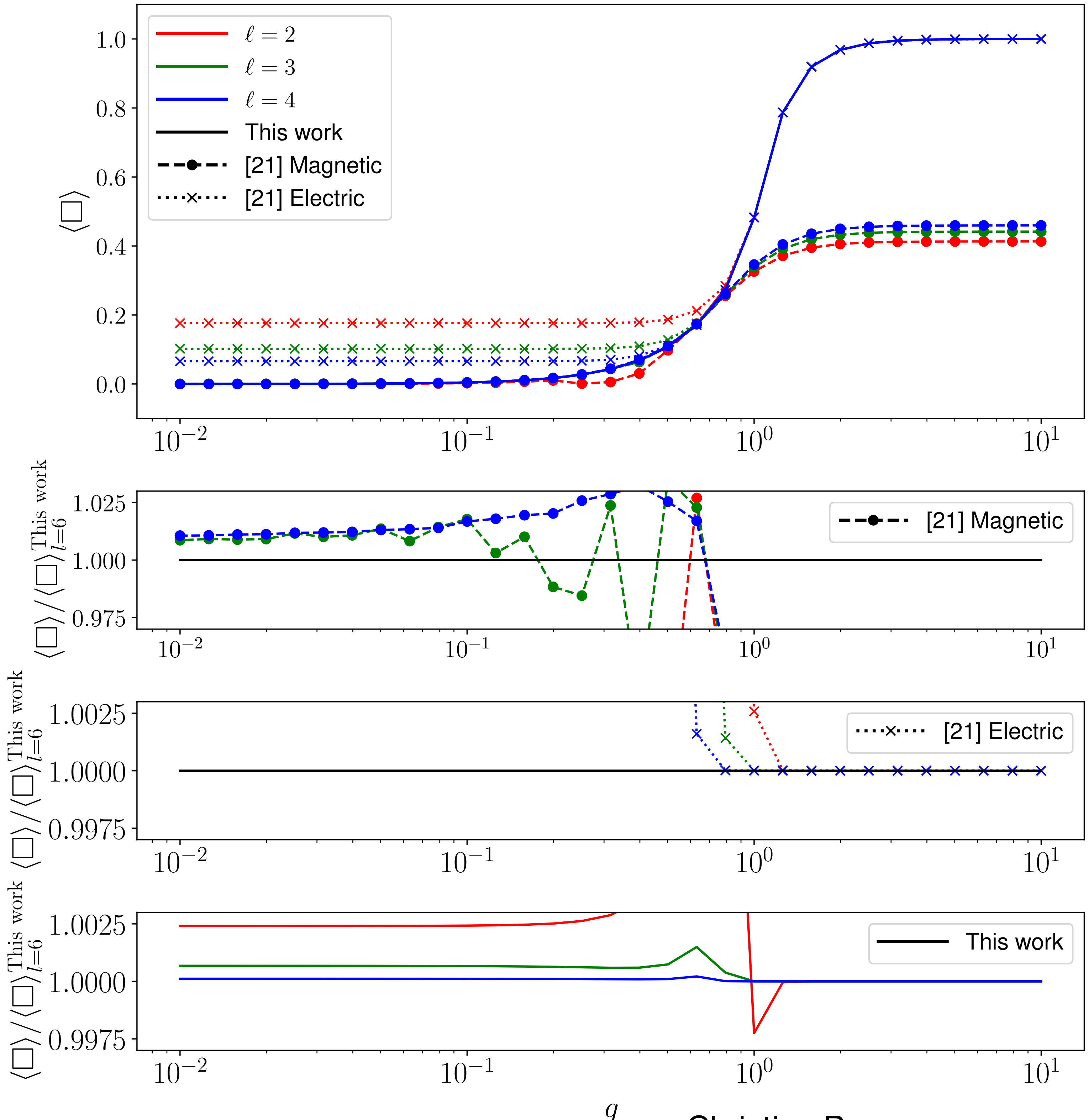
One can construct both magnetic and electric basis, and each work in the coupling limit they are designed for

Haase et al, 2006.14160



We developed a new representation of Hilbert space, that works in both limits of the coupling

CWB, Grabowska, 2111.08015



Does significantly better than the previous approach for all values of the coupling

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Dimension of Hilbert space is

$$32^{10^{14}} \sim \infty$$

Number of qubits and required

$$5 \times 10^{14}$$

Since biggest difficulty is with long distance physics,

Goal is to separate ingredients that are calculable in perturbation theory from those that really benefit from non-perturbative techniques

Effective Field Theories (SCET)

$$d\sigma = H \otimes J_1 \otimes \ldots \otimes J_n \otimes S$$

Most interesting object in above equation is the soft function S, which lives at the lowest energies

For 1TeV jets with 100GeV mass, find

$$\Lambda_S = (100\,\text{GeV})^2/(1000\,\text{GeV}) = 10\,\text{GeV}$$

Let's try to estimate the resources we need to simulate physics at the LHC

As I will argue later, can use effective field theories to limit required range to

$$100\,\mathrm{MeV} \lesssim E \lesssim 10\,\mathrm{GeV}$$

This needs $\mathcal{O}(100^3) \sim 10^6$ lattice sites

Dimension of Hilbert space is

$$32^{10^6} \sim \infty$$

Number of qubits required

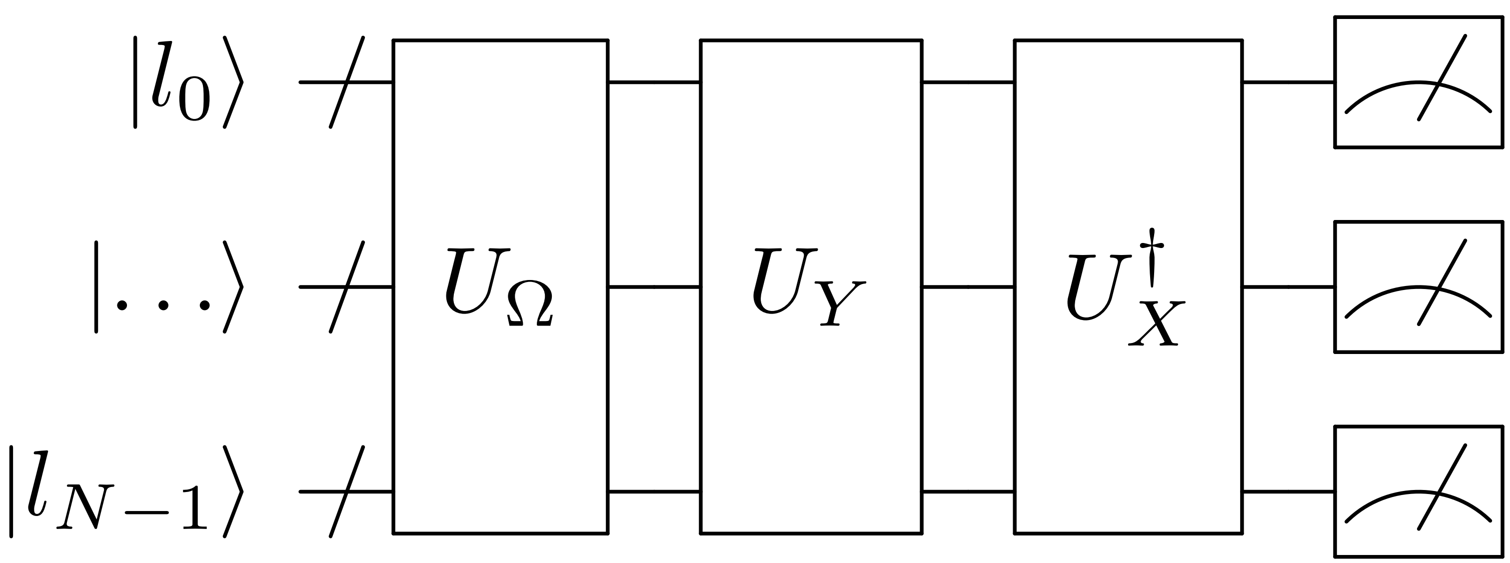
$$5 \times 10^6$$

Soft function is the expectation value of a "Wilson line" operator between initial and final state

$$S = \left| \langle X | T[Y_n Y_{\bar{n}}^\dagger] | \Omega \rangle \right|^2$$

Have worked out quantum circuit to create vacuum state $|\Omega\rangle$, circuit for $T[Y_n Y_{\bar{n}}^\dagger]$ and circuit to measure final state $|X\rangle$

CWB, Freytsis, Nachman, PRL 127 (2021), 212001



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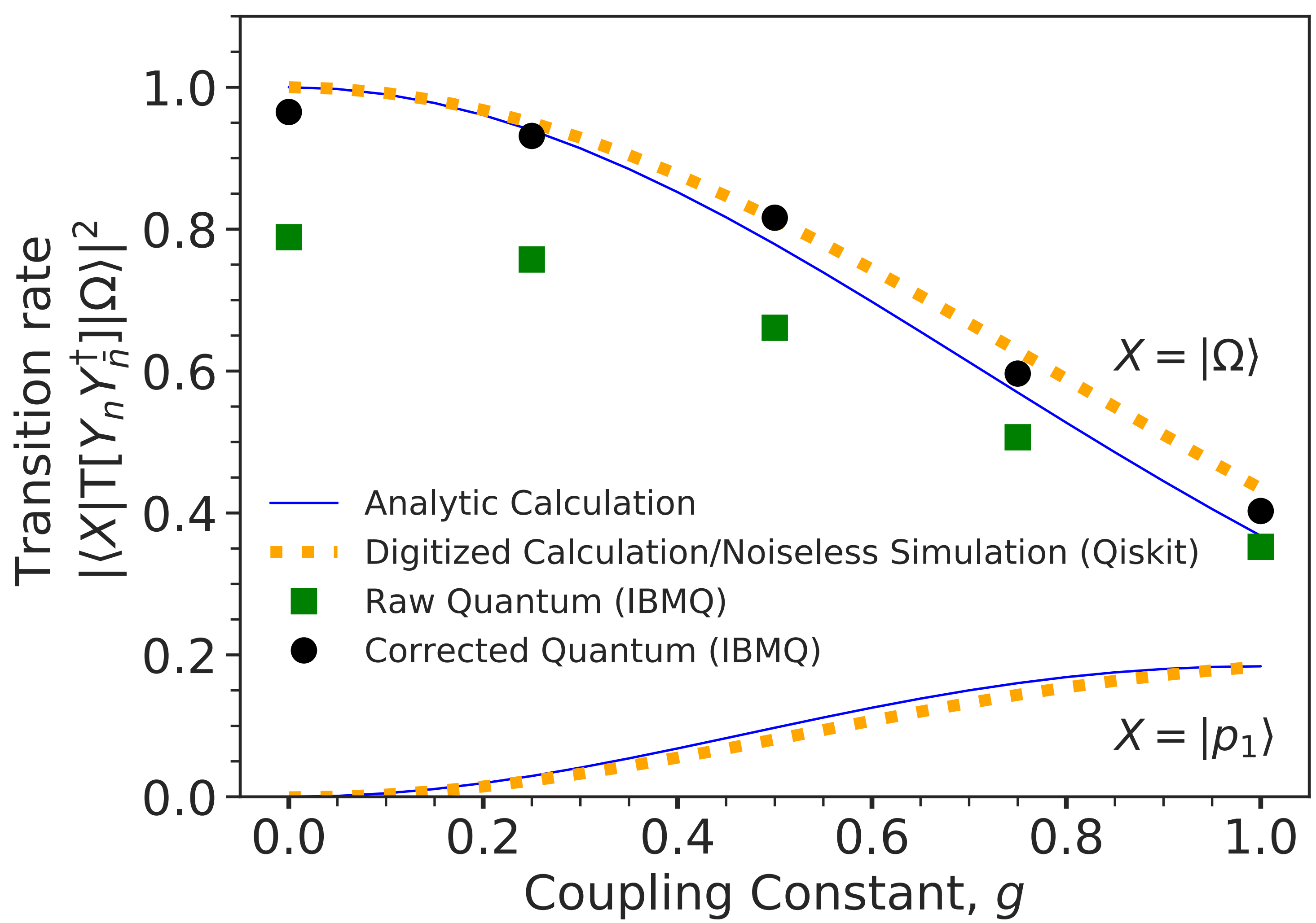
CWB, Freytsis, Nachman, PRL 127 (2021), 212001



Quantum computer gives a good description of the analytical result

- CWB, Freytsis, Nachman, PRL 127, 212001

Effective Field Theory treatment to allow quantum simulation of non-perturbative physics

- CWB, Grabowska 2111.08015
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Formulation of Field Theories suited for simulation on quantum devices

Quantum Computing in Physics Division

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