Effective field theories from scattering amplitudes

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with Clifford Cheung, Karol Kampf, Jiri Novotny, Chia-Hsien Shen

(related with Nima Arkani-Hamed, Laurentiu Rodina)

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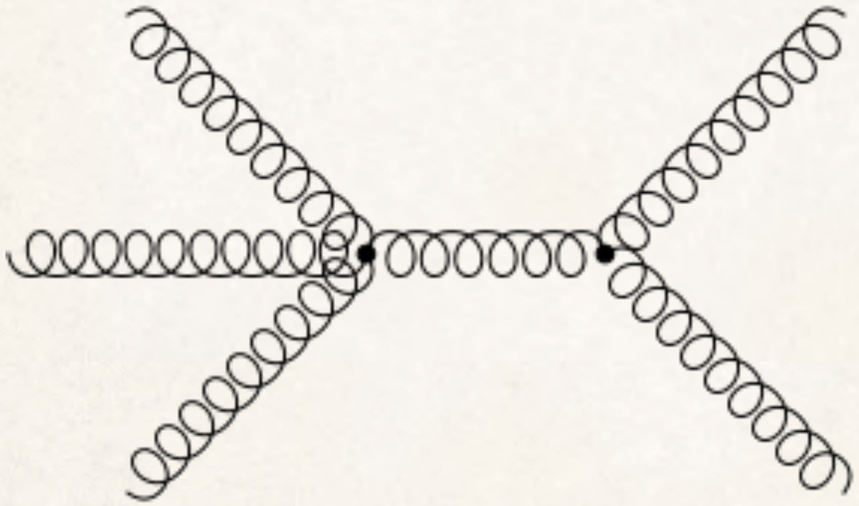
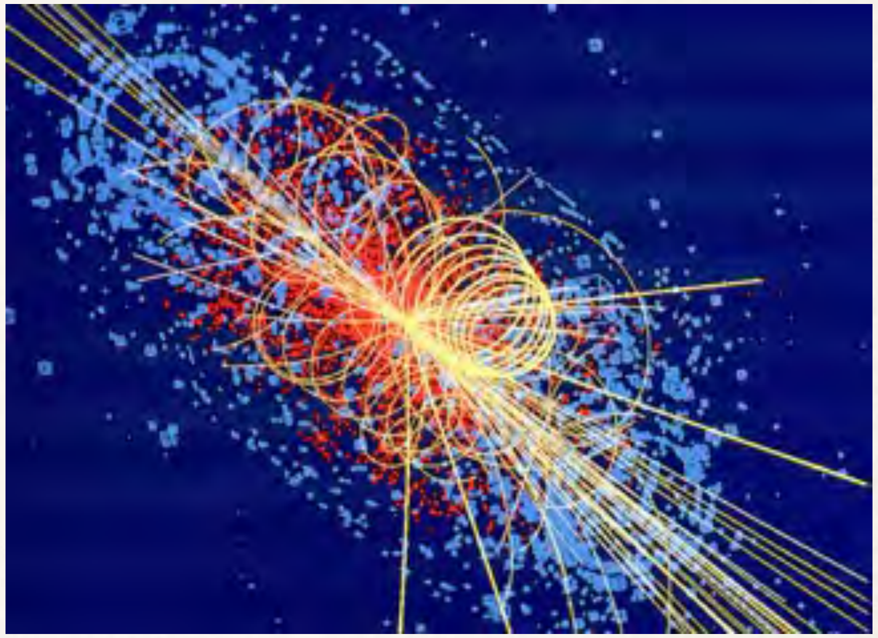
Scattering amplitudes

Predictions of outcomes of particle interactions





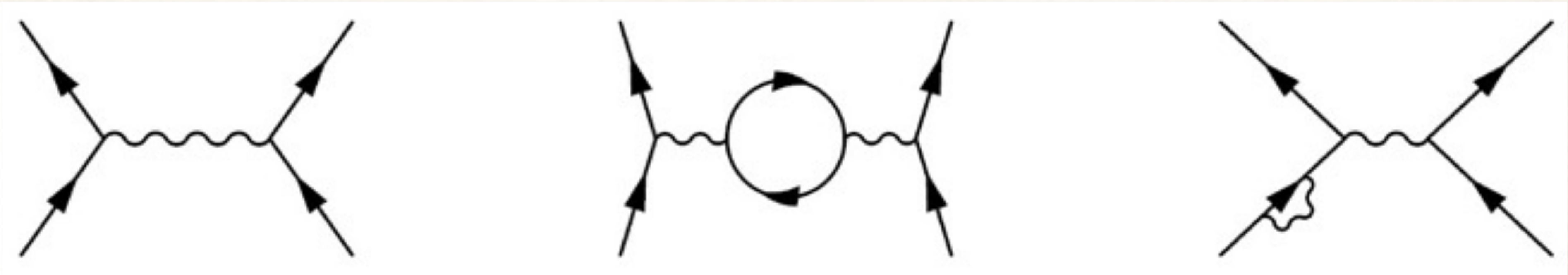
Other motivation: probes to study QFT

Feynman diagrams

- Fields, Lagrangian, Path integral

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\not{D}\psi - m\overline{\psi}\psi \qquad \int \mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\overline{\psi}\; e^{iS(A,\psi,\overline{\psi},J)}$$

- Feynman diagrams: pictures of particle interactions
  Perturbative expansion: trees, loops



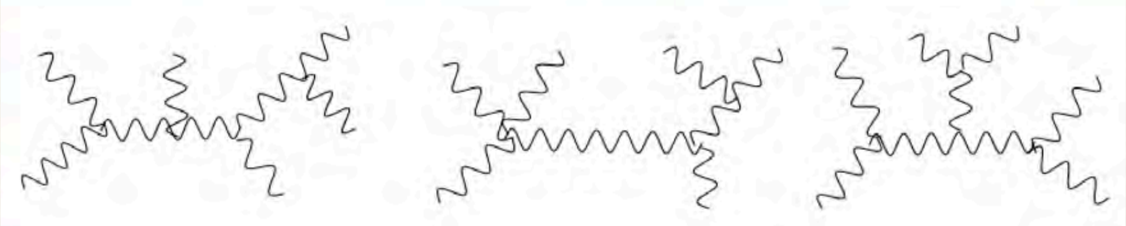
Unexpected simplicity

- Tree-level amplitudes of gluons in massless QCD

1985: calculation of 6pt amplitude for SSC

Brute force calculation: 100 pages like that

$(k_1 \cdot k_4)(\epsilon_2 \cdot k_1)(\epsilon_1 \cdot \epsilon_3)(\epsilon_4 \cdot \epsilon_5)$



Ughhh….

Unexpected simplicity

(Parke, Taylor 1985)

- Tree-level on-shell amplitudes of gluons in massless QCD

Surprisingly simple answer

$$\mathcal{M}_6 = \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle}$$

Parke-Taylor formula
for helicities (++++- -)

Spinor-helicity variables for massless particles

$$p^2 = m^2 = 0$$

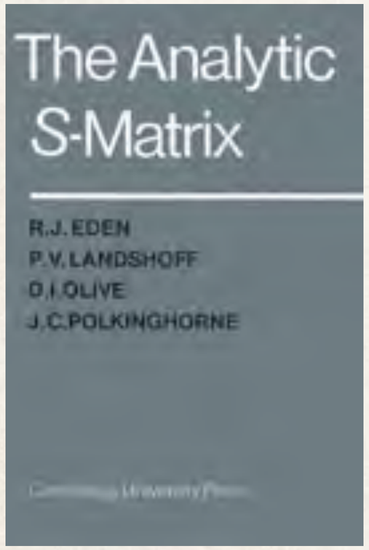
$$p^\mu = \sigma^\mu_{a\dot{a}} \lambda_a \tilde{\lambda}_{\dot{a}} \qquad \langle 12 \rangle = \epsilon_{ab} \lambda_a^{(1)} \lambda_b^{(2)}$$

New methods for amplitudes

- Problem with Feynman diagrams: not gauge invariant

  Huge cancellations among diagrams $\epsilon^\mu \to \epsilon^\mu + \alpha p^\mu$

- New methods:
  - Work with on-shell gauge invariant quantities
  - Amplitude is as a **single object**, not a sum of Feynman diagrams



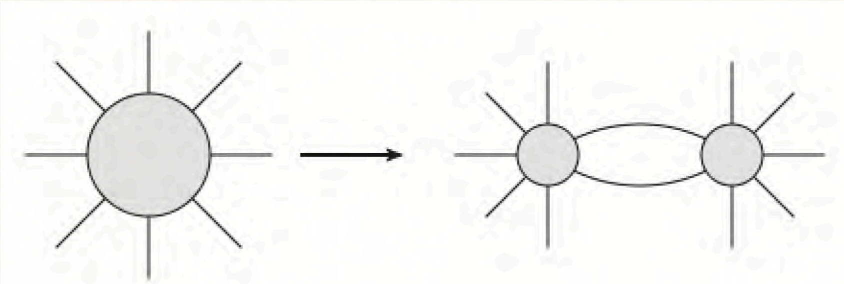
Revival of the S-matrix program from 1960s

Amplitude is a **unique** object satisfying certain constraints

Import difference: we work in the perturbation theory

Constraints

- Example of perturbative constraints:



Unitarity cut for loop integrands

- Final amplitudes: we do not know the full set of constraints
- In planar N=4 SYM we have a remarkable control of the perturbation theory (which is the full result in this case)

Hexagon bootstrap

(Dixon et al)

- complete set of constraints
- explicit results up to 6-loops

Amplituhedron

(Arkani-Hamed, JT)

- for integrands and trees
- constraints come from geometry

Tree-level amplitudes

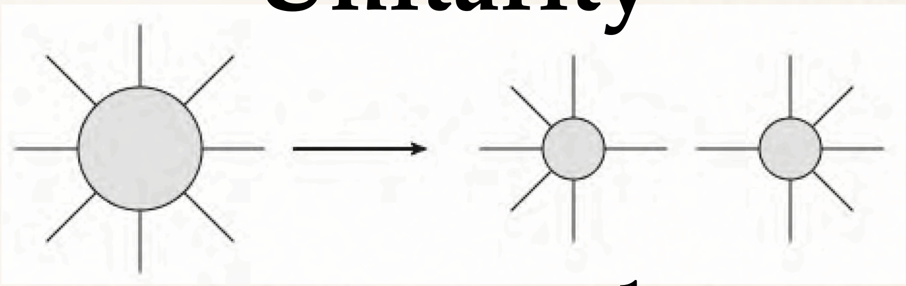
- Two important physical constraints

Locality

- Point-like interaction
- Only poles are

$$\frac{1}{P^2} \to \infty$$

Unitarity



$$\mathcal{M} \xrightarrow[P^2=0]{} \mathcal{M}_L \frac{1}{P^2} \mathcal{M}_R$$

- Amplitude: unique gauge invariant function which is local and factorizes properly on all channels

On-shell constructible theories

For example: Yang-Mills, GR, Standard Model,….

Recursion relations

(Britto, Cachazo, Feng, Witten, 2005)

- "Integrate" the relation $\mathcal{M} \xrightarrow[P^2=0]{} \mathcal{M}_L \frac{1}{P^2} \mathcal{M}_R$
- Write an amplitude using products of lower point amplitudes — we have to shift momenta

$$p_1 \to p_1 + zq$$
$$p_2 \to p_2 - zq$$
$$q^2 = (p_1 \cdot q) = (p_2 \cdot q) = 0$$

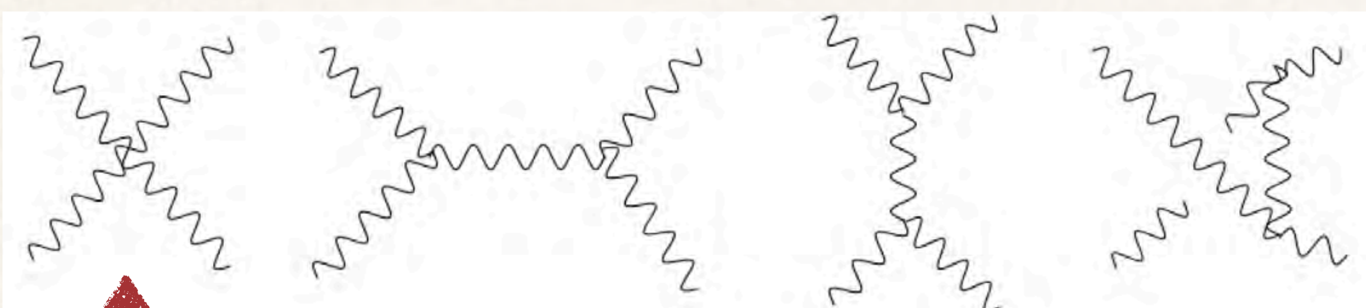
Cauchy formula for shifted amplitude

$$\int \frac{dz}{z} \mathcal{M}(z) = 0$$

$$\boxed{\mathcal{M} = \sum_P \mathcal{M}_L(z_P) \frac{1}{P^2} \mathcal{M}_R(z_P)}$$

Problem with contact terms?

- Our statement: factorizations fix everything
- Four point amplitude in Yang-Mills theory



Gauge invariance!

Contact term: not detected by factorization

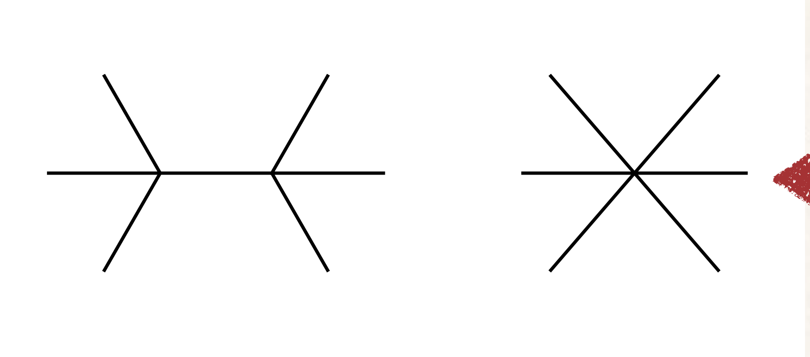
- Lagrangian perspective

$$\mathcal{L} \sim (\partial A)^2 + gA^2\partial A + \tilde{g}A^4 \xrightarrow{\tilde{g}\sim g^2} F^2$$

Scalar EFTs

- What if there is no gauge invariance like in scalar EFT

$$\mathcal{L} = (\partial\phi)^2 + \lambda_4(\partial\phi)^4 + \lambda_6(\partial\phi)^6 + \ldots$$



Contact term not fixed

$$\mathcal{M}_6 = \frac{\lambda_4^2(\ldots)}{s_{123}} + \cdots + \lambda_6(\ldots)$$

- Not fixed by the behavior on poles or gauge invariance: nothing unique about this amplitude

Non-linear sigma model

(Weinberg 1966)

- Famous example of an effective field theory

SU(N) non-linear sigma model

Low energy QCD

- Lagrangian: infinite tower of terms

$$\mathcal{L} \sim (\partial\phi)^2 + \frac{1}{F^2}\phi^2(\partial\phi)^2 + c_6\phi^4(\partial\phi)^2 + \ldots \longrightarrow \partial_\mu U \partial^\mu U^\dagger$$

$$U = \exp\left(\frac{i\phi}{F}\right)$$

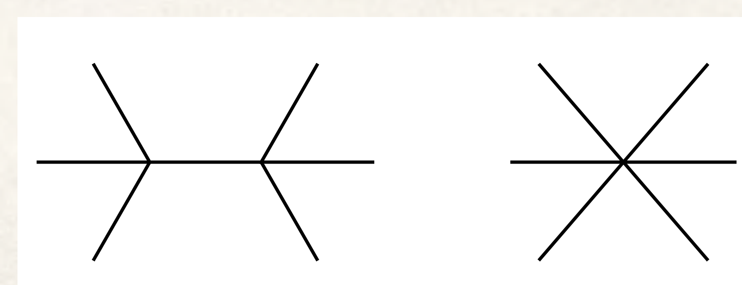
non-linearly realized shift symmetry

$$\phi \to \phi + a$$

fixes all coefficients

- Amplitudes: vanishing soft-limit

$$\lim_{p\to 0}\mathcal{M}(p) = 0$$



Requires cancelation between diagrams

Back to our example

- Write a generic Lagrangian with power counting

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \lambda_4(\partial\phi)^4 + \lambda_6(\partial\phi)^6 + \lambda_8(\partial\phi)^8 + \dots$$

- Soft limit vanishing is trivial because the Lagrangian is derivatively coupled: does not fix coefficients

- Impose more: double vanishing in the soft limit

$$\lim_{p\to 0} \mathcal{M}(p) = \mathcal{O}(p^2)$$

Back to our example

- Write a generic Lagrangian with power counting

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \lambda_4(\partial\phi)^4 + 4\lambda_4^2(\partial\phi)^6 + 20\lambda_4^3(\partial\phi)^8 + \dots$$

- Soft limit vanishing is trivial because the Lagrangian is derivatively coupled: does not fix coefficients

- Impose more: double vanishing in the soft limit

$$\lim_{p\to 0} \mathcal{M}(p) = \mathcal{O}(p^2)$$

What is this theory?

Result: DBI action

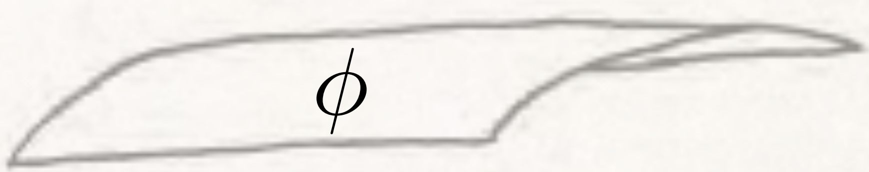
(Dirac, Born, Infeld 1934)

- This is an expansion of

$$\mathcal{L} = \frac{1}{8\lambda_4}\left(1 - \sqrt{1 - 8\lambda_4(\partial\phi)^2}\right)$$ DBI action

- Important role in string theory, inflationary models…

- Scalar field on the 4d-brane



- Symmetry: reminiscence of 5d Lorentz symmetry

$$\phi \to \phi + a + (\theta \cdot x) - (\theta \cdot \phi)\partial\phi$$

Soft limit behavior

- Soft limit: important property at low energy

Yang-Mills, gravity: soft factors fixed by gauge invariance

Scalars: vanishing

- Generalization: $$\mathcal{M}(tp_i) \xrightarrow[t\to 0]{} \mathcal{O}(t^\sigma)$$

Denote it as

$$\mathcal{M} = \mathcal{O}(p^\sigma)$$

- Additional constraint: unique answer for amplitude

Standard approach:

symmetries of the theory

↓

properties of amplitudes

Our approach:

uniquely fixed amplitudes

↓

special theories

Next case

- So far we reconstructed a known theory

- Let us go further with power-counting

$$\mathcal{L} = (\partial\phi)^2 + \lambda_4(\partial^6\phi^4) + \lambda_6(\partial^8\phi^6) + \ldots$$

$\downarrow$

- Calculate amplitudes
- Impose $\mathcal{M} = \mathcal{O}(p^2)$

No higher terms are needed

$$\lambda_6 = \lambda_8 = \cdots = 0$$

- Two solutions identified with Galileons

$$\phi \to \phi + a + (b \cdot x)$$

Relevant for cosmological models

Special Galileon

(Cheung, Kampf, Novotny, JT, 2014) (Cachazo, He, Yuan, 2014)

- Surprise: the behavior in the soft limit is even stronger for a particular choice of the coefficients

$$\mathcal{M}_n = \mathcal{O}(p^3)$$

- No symmetry explanation at that time

Effective Field Theories from Soft Limits

Clifford Cheung, Karol Kampf, Jiri Novotny, Jaroslav Trnka

(Submitted on 12 Dec 2014)

- One month later found

A Hidden Symmetry of the Galileon

Kurt Hinterbichler, Austin Joyce

(Submitted on 29 Jan 2015)

Special Galileon

$$\phi \to s_{\mu\nu}x^\mu x^\nu + \frac{\lambda_4}{12} s^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)$$

Soft limit recursion

(Cheung, Kampf, Novotny, Shen, JT 2015)

- From the Cauchy formula derive recursion relations

$$\mathcal{M} = -\sum_{z_P} F(z_P) \frac{\mathcal{M}(z_P)\,\mathcal{M}_R(z_P)}{P^2}$$

where $F(z)$ is the modification incorporating soft limit

- New representation of amplitudes in these theories

- Future study: properties of individual terms

→ Searching for new mathematical structures

Exceptional theories

- Similar uniqueness story with Yang-Mills, GR

$$\mathcal{M} = \sum_{\Gamma} \frac{N_\Gamma}{P_1^2 P_2^2 \dots P_m^2} \quad \text{impose } \mathcal{M} = 0 \text{ for } \epsilon_k^\mu \to \alpha p_k^\mu$$

(Arkani-Hamed, Rodina, JT, 2016)

soft limit -> gauge invariance

kinematically similar, physics different

- All these theories play also important role in other context: scattering equations, CHY formula

(Cachazo, He, Yuan, 2013)

- The uniqueness there is related to the string theory

Final remarks

- **Uniqueness** crucial for the progress in planar N=4 SYM
- We studied tree-level amplitudes in larger class of theories, perhaps we can get more: different limits of amplitudes, more spins (Kampf, Novotny, JT, in progress)
- It can lead to new powerful tools and theoretical constructions like hexagon bootstrap, Amplituhedron

Thank you for your attention