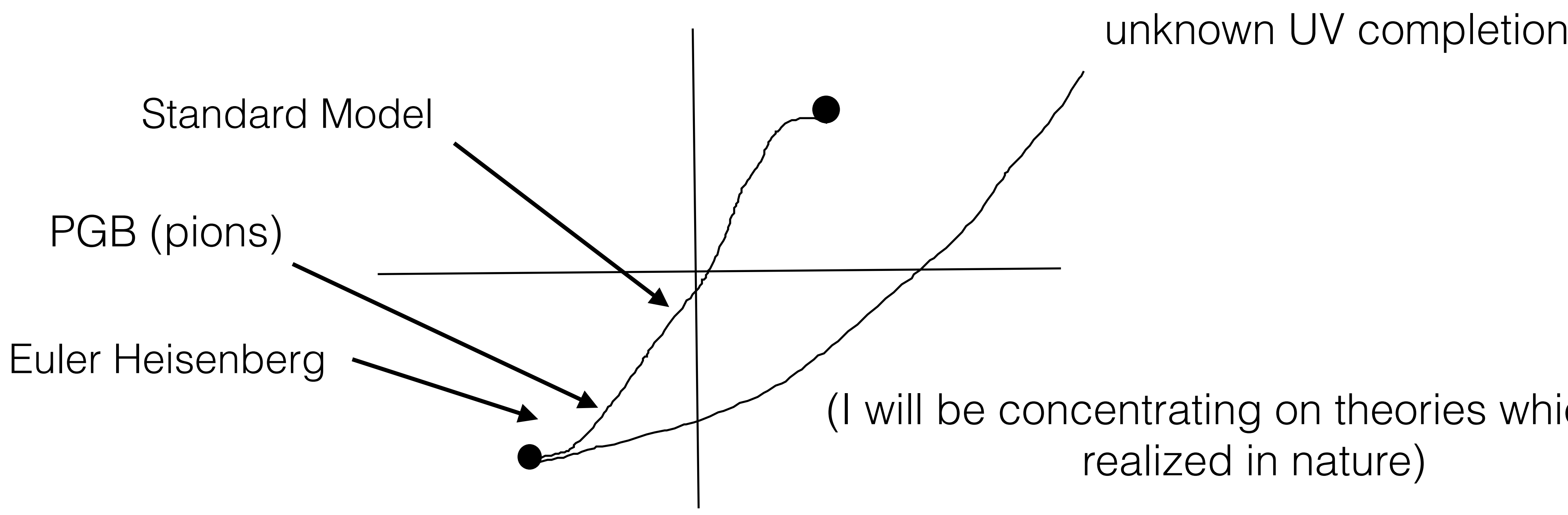
The Field Theoretic Challenge of non-Fermi Liquids

Ira Rothstein (CMU)

Based on work with Prashant Shrivastava

(Also, Tristan McKinney, Anton Kapustin)

QFT= CFT+EFT



(I will be concentrating on theories which are realized in nature)

IR trivial fixed point

Gapped, or derivatively coupled zero modes (Goldstones)

To get something interesting we should consider non-trivial vacua breaking space-time symmetries

HQET

$$\psi(x) = \sum_{v^\mu} e^{imv\cdot x} h_v(x) \qquad S = \sum_v S[h_v(x), A(x)]$$

Residual Momentum order Λ

Vacuum $\quad | \, b(v') \rangle \quad$ Superselection rule $\quad v \quad$ fixed

Ground state picks out one particular four velocity, breaks Lorentz down in Translations

Lorentz Invariance must be realized non-linearly without the Goldstone Crutch

When Space-time Symmetries are Broken

there is no longer a 1-1 map between Goldstone modes and broken generators

$$\langle 0 \mid [Q(t), O(0)] \mid 0\rangle \neq 0$$

implies

$$\sum_n [\langle 0 \mid j_0(0) \mid n\rangle\langle n \mid O(0) \mid 0\rangle e^{iE_n t} - \langle 0 \mid O(0) \mid n\rangle\langle n \mid j_0(0) \mid 0\rangle\rangle e^{-iE_n t}]\delta^d(p_n) \neq 0$$

There exist a state with zero energy as the momentum approaches zero, but this says nothing about the spectral weight. e.g. could be saturated by free electron-hole pair in a metal.

Impose that the charges obey the Lorentz Algebra

$$L = \sum_v \bar{h}_v (iv \cdot D) h_v + c_1 \bar{h} \frac{D_\perp^2}{2m} h_v + ...$$

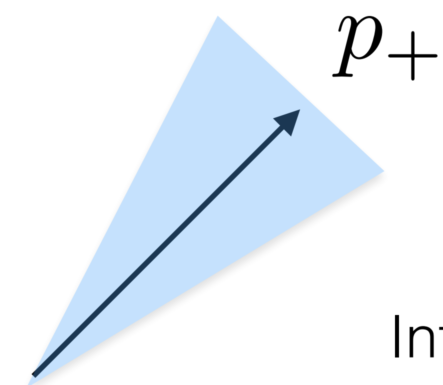
$$[H, K_i] = -iP_i \longrightarrow \bar{h}_v \vec{D}_\perp h_v - c_1 \bar{h}_v \vec{D}_\perp h_v = 0$$

constraint trivially saturated by $c_1 = 1$

As we shall see there will be more interesting examples where the Algebra generates highly non-trivial constraints on the theory unless we have a propagating Goldstone mode.

Soft Collinear EFT (SCET)

Integrate out all mode outside cone of specified jet directions.



Integrate out modes outside cone

$$k^2 \sim \Lambda^2 \sim k_+ k_- - k_\perp^2$$

$k^\mu \sim (Q, \Lambda^2/Q, \Lambda)$ Collinear

$k^\mu \sim (\Lambda, \Lambda, \Lambda)$ Soft

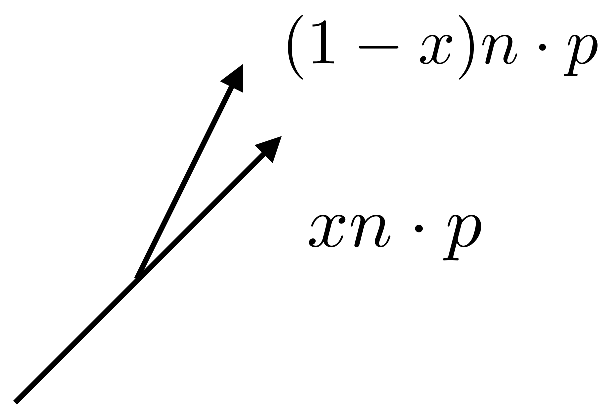
Decompose fields: $$A \equiv \sum_{n_i} A_{n_i} + A_s \qquad A_n(x) = \sum_{n \cdot p} e^{-in\cdot p \bar{n}\cdot x} A_{n,n\cdot p}(x)$$

Ground state: $$| \, 0\rangle = | \, p_n\rangle \otimes | \, p_{n'}\rangle \otimes \ldots.$$

Parton number not fixed only total collinear momentum of each jet.
Factorization of Hilbert spaces

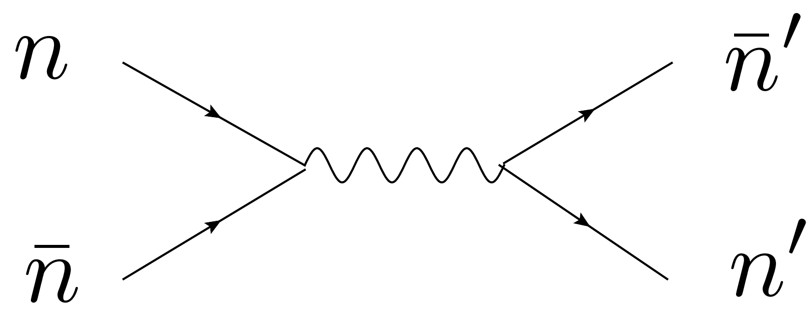
$$T_{\mu\nu} = T^{n}_{\mu\nu} + T^{n'}_{\mu\nu} + T^{s}_{\mu\nu} + O(1/Q)$$

Key distinction from HQET is that there exists interactions which change the field labels

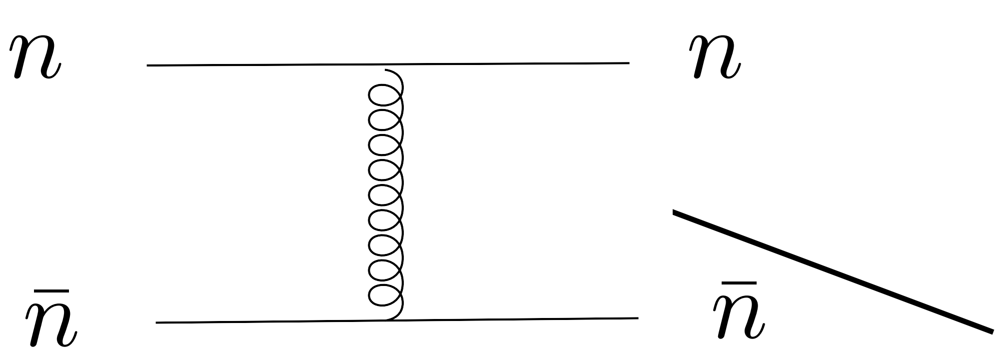


However, these interactions are all within the same sector of Hilbert space

Operators which involves different collinear directions are power suppressed



$$\sim \frac{1}{Q^2}\bar{\xi}_n \Gamma \xi_{\bar{n}} \bar{\xi}_{n'} \Gamma \xi_{\bar{n}'}$$

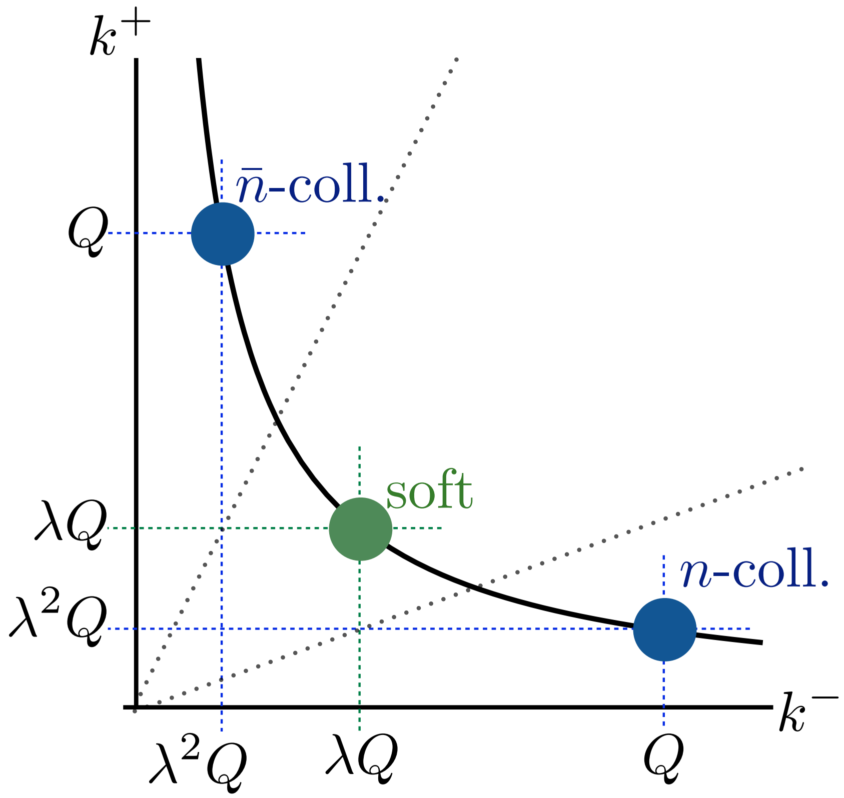


Except for special kinematic configurations leads to leading order interaction

(Glaubers )

We will see that something similar happens in Fermi liquids though for disparate reasons

This theory has non-canonical running due to Sudakov double Logs: one loop vertex correction



$$I_S = \int [d^n k] \frac{1}{(k^2 - M^2)} \frac{1}{(-n \cdot k + i\epsilon)} \frac{1}{(-\bar{n} \cdot k + i\epsilon)}$$

$$I_n = \int [d^n k] \frac{1}{(k^2 - M^2)} \frac{1}{(k^2 - n \cdot k \, \bar{n} \cdot p_1 + i\epsilon)} \frac{1}{(-\bar{n} \cdot k + i\epsilon)}$$

not regulated by dim reg.
need to introduce rapidity regulator

$$I_S = g^2 C_F \left[-\frac{e^{\gamma_E \epsilon} \Gamma(\epsilon) \left(\frac{\mu}{M}\right)^{2\epsilon}}{4\pi^2 \eta} + \frac{1}{4\pi^2} \left(\frac{\ln(\frac{\mu}{\nu})}{\epsilon} + \ln^2(\frac{\mu}{M}) - 2\ln(\frac{\mu}{M}) \ln(\frac{\nu}{M}) + \frac{1}{2\epsilon^2} \right) - \frac{1}{96} \right]$$

$$I_n = g^2 C_F \left[\frac{e^{\gamma_E \epsilon} \Gamma(\epsilon) \left(\frac{\mu}{M}\right)^{2\epsilon}}{8\pi^2 \eta} + \frac{1}{4\pi^2} \left(\ln(\frac{\mu}{M}) \ln(\frac{\nu}{\bar{n} \cdot p_1}) + \ln(\frac{\mu}{M}) + \frac{1}{2\epsilon} \left(1 + \ln(\frac{\nu}{\bar{n} \cdot p_1}) \right) + \frac{1}{2} \right) - \frac{1}{48}$$

Rapidity divergences cancel in sum: Something remarkably similar will actually be related to non-Fermi liquid behavior

As in HQET the SCET vacuum spontaneously breaks Lorentz, but the algebra can be trivially satisfied by equation coefficients in the action

In general how do we know whether or not the constraints from space-time symmetry algebra can be obeyed without a Goldstone boson in the spectrum?

Naively might think that, since GB's are derivatively coupled, it should always be possible to realize the symmetry in the IR w/o the need for Goldstone!!

However, the assumption of derivatively coupled Goldstone can fail.

Counter-examples: Relativistic Dilaton

Rotational Goldstone for Nematic fluids (Oganesyan et. al. 2008),

What are the generalized criteria for non-derivatively coupled Goldstones?

Vishwanath and Watanabe (2014)

$$[P, X] \neq 0 \qquad X \mid 0\rangle \neq 0$$

$$[L_i, P_j] = i\epsilon_{ijk} P_k \qquad L_i \mid 0\rangle \neq 0$$

(nematic fluid, Oganesyan et al (01))

$$H_{int} = \pi^a [Q^a, H_0]$$

Derivative coupling ?

$$\langle e(k)\pi(q) \mid \pi^a [Q^a, H_0] \mid e(k')\rangle = \langle e(k) \mid Q^a \mid e(k')\rangle (E(k) - E(k')) \sim \langle e(k) \mid Q^a \mid e(k')\rangle \vec{q} \cdot \frac{\partial E}{\partial k}$$

However: $\langle e(k) \mid [Q^a, P] \mid e(k')\rangle \propto q \langle e(k) \mid Q^a \mid e(k')\rangle_{q \to 0} \neq 0$

Singular !

Can't rule out possibility that space-time Goldstones are relevant in the IR and play a role in symmetry realization

Further Clue: Even if a symmetry is broken it need not lead to a Goldstone as they may be redundant or gapped. If we can eliminate all GB's in this way then we've answered our question!

Inverse Higgs Constraint and the Space-time Coset

$$G \to H$$

Unbroken translations

$$U^{-1}\partial_\mu U = E_\mu^A(\bar{P}_A + \nabla_A \pi^a X^a + A_A^i T^i) \quad X \in L[G/H]$$

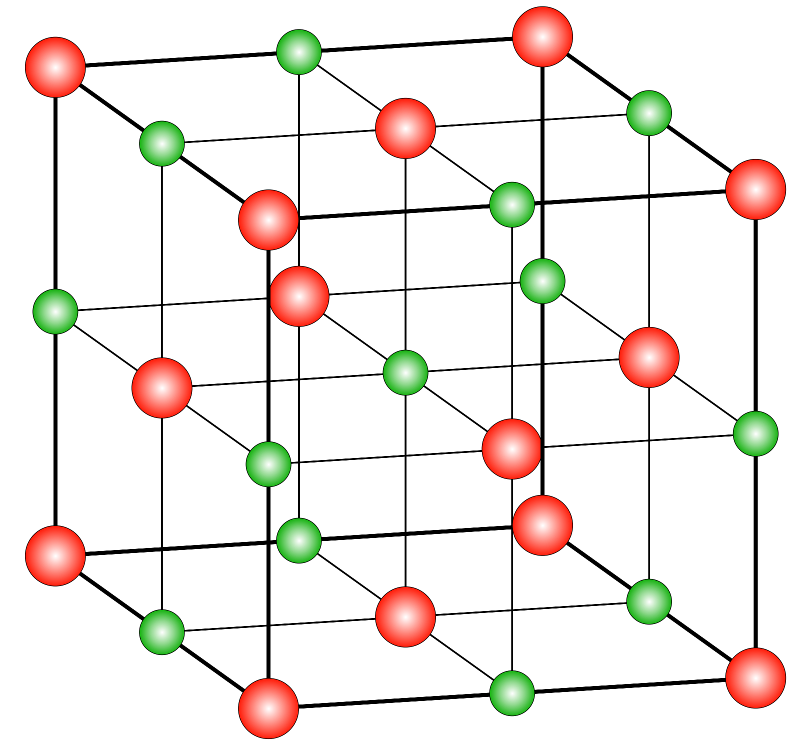
$$T \in L[H]$$

Building Blocks which transforms under G as

$$\nabla_A \pi^a \to h_B^A(\pi, g) h_b^a(\pi, g) \nabla_A \pi^b$$

If a covariant derivative contains a term linear in a Goldstone field then we can eliminate that Goldstone in favor of another

Classic Example: crystal lattice



$$(H, \vec{T}, \vec{L}, \vec{K}, M : \vec{Q}, \vec{R}) \in G$$

Internal (rigid) symmetry)

$$(H, M : \vec{Q} + \vec{T}, \vec{R} + \vec{L}) \in H$$

Naively 9 Goldstone Bosons: Physically only phonons are left over. Missing “Angulons” (Rotations) and “Framons” (Boosts)

$$\nabla_0 \pi^i = \eta^i + \dot{\pi}^i + \ldots.$$

$$\nabla^j \pi^i = i\theta^k \epsilon^{ijk} + \partial^i \pi^j$$

Choose: $\nabla \pi = 0$ or $L = C_1 (\nabla \pi)^2$ (Not true in general)

Below Scale of Gap identical physics

General Conditions for IH

$$[X, \bar{P}^{\mu}] \sim X$$

For crystal breaking pattern

$$[K_i, H] \sim P_i \qquad\qquad [L_i, \bar{P}_i] \sim P_i$$

Framon Angulon

What happens if we don't break translations or rotations (Framid: Nicolis et al), e.g. Helium3?

Not only should Framon exist (in He 3) it should be non-derivatively coupled!

Returning now to our question

In general how do we know whether or not the constraints from space-time symmetry algebra can be obeyed without a Goldstone boson in the spectrum?

Answer

We know it can be removed if the symmetry breaking pattern allows it, otherwise ?

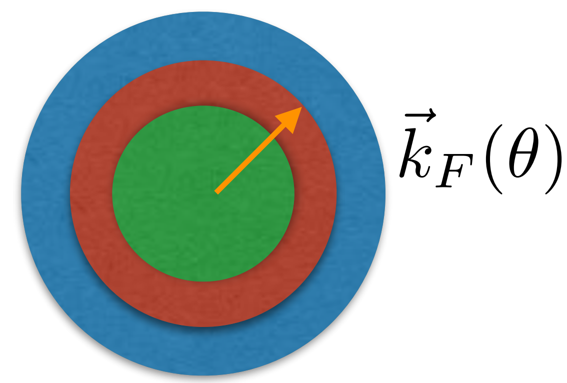
Note: SCET and HQET there is no IH at play since ground state does not break translational invariance.

What does all this have to do with (non) Fermi Liquid theory?

As we shall see the Coset construction will allow us to derive the complete EFT of Fermi liquid theory without any recourse to scaling arguments regarding the Fermi surface. It all follows from symmetries. The existence of the framon will play a crucial role.

This line of reasoning will make clear how to AVOID Fermi liquid behavior

Fermi Liquids



Sharp Analogy with SCET: Integrating out fluctuations around ``fixed'' large momenta

$$\psi(x) = \sum_{\theta} e^{i\vec{k}_F(\theta)\cdot x}\psi_\theta(x)$$

Conjugate to small residual momentum

Fermi Surface Breaks Boost Invariance (but not rotations necessarily, He3) how is this symmetry realized?

Assume weak coupling in the UV and ``electrons" are relevant degrees of freedom

$$E_f > \alpha^2 m$$

rough criteria

Coulomb interaction assumed to be screened and high energy phonons integrated out leaving only local interactions.

Check self consistency by calculating quasi-particle width aposteriori

$$\lim_{E \to 0} \frac{\Gamma(E)}{E} \to 0$$

Canonical Fermi Liquid

$$\Gamma \sim E^2$$

Most General Action

$$S_0 = \int d^d k dt \ \psi^\dagger(k,t)[(i\partial_0 - \epsilon(k_\perp))]\psi(-k,t)$$

Assumption of Spherical Symmetry

$$S_1 = \int \prod_{i=1,4} d^d k_i dt \ g(k_{||}^i)\psi^\dagger(k_1,t)\psi(k_2,t)\psi^\dagger(k_3,t)\psi(k_4,t)(2\pi)^d \delta(\sum_i k_i)$$

$$k_\perp \sim \lambda \qquad k_{||} \sim 1$$

Lacks boost invariance: symmetry breaking pattern apparently necessitates a framon

Coset Construction

$$U = e^{iP\cdot x + i\vec{K}\cdot\vec{\eta}}$$

$$E_0^0 = 1 \quad E_i^j = \delta_i^j, \quad E_0^i = \eta^i, E_i^0 = 0. \qquad A_i = \eta_i, A_0 = -\frac{1}{2}\vec{\eta}^2$$

Note: framid clearly not derivatively coupled $\qquad [K_i, P_J] = i\delta_{ij}M$

Form H invariant Action

$$\partial_i \to \partial_i + m\eta_i$$

$$\epsilon(k) \to \epsilon(k+\eta) \qquad g(k_i) \to g(k_i+\eta)$$

$$L_0 = i\psi^\dagger[(E^{-1})_0^\mu \partial_\mu + A_0^A T^A]\psi$$

$$S_0 = \int d^d x dt\ \psi^\dagger \left[i(\partial_0 - \eta^i \partial_i) + \frac{1}{2} m\vec{\eta}^2 + \varepsilon(i\partial_i + m\eta_i) \right] \psi$$

$$S_{int} = \int dt \prod_i dk_i g(k_i + \eta_i)\psi^\dagger(k_1,t)\psi(k_2,t)\psi^\dagger(k_3,t)\psi(k_4,t)\delta^d(\sum k_i)$$

This action realizes all the symmetries, predicts non-derivatively coupled Goldstone

$$L_{int} = \psi^\dagger_{\vec{k}_f(\theta)} \psi_{\vec{k}_F(\theta)} \vec{k}_f(\theta) \cdot \vec{\eta}$$

Consider d=2+1

$$\eta \sim \lambda, k_\eta \sim \lambda^2 \qquad\qquad \psi \sim \lambda^{1/2}, k_\psi \sim \lambda$$

Framon does not transfer momentum to fermions!
Must multipole expand the field to have
consistent EFT power counting.

$$L = \psi^\dagger(x)\psi(x)\eta(0) + \ldots.$$

Foramen non-dynamical field acts a
Lagrange multiplier enforcing
constraints of boost invariance

$$O_i^B = \Big(\int \frac{d^dp}{(2\pi)^d}\, \psi_p^\dagger(p_i - m\frac{\partial\varepsilon_p}{\partial p_i})\psi_p - \frac{m}{2}\int \prod_{a=1}^{4} \frac{d^dp_a}{(2\pi)^{3d}}\delta^{(d)}(p_1+p_2-p_3-p_4)\Big(\sum_i \frac{\partial g(p_a)}{\partial p_{i,a}}\Big)\psi_{p_4}^\dagger\psi_{p_3}^\dagger\psi_{p_2}\psi_{p_1}\Big)$$

highly non-trivial
operator constraint

$$O_B = 0$$

Consider one particle matrix element

$$\langle k \mid O_B \mid k\rangle = 0$$

$$k_i = m\frac{\partial \varepsilon_k}{\partial k_i} + 2m\frac{\partial}{\partial k_i}\int \frac{d^2p}{(2\pi)^2}\ \Theta(p_F - p)g(p,k) + 2m\int \frac{d^2p}{(2\pi)^2}\ g(p,k)\delta(\varepsilon_F - \varepsilon)\frac{\partial \varepsilon_p}{\partial p_i}$$

Implement rotational symmetry

$$g(\theta) = \sum_l g_l P_l(\cos\theta)$$

$$\frac{k_F}{m} = v_F + \frac{2p_F}{(2\pi)^2}\int d\theta\ \ \cos\theta \sum_l g_l P_l(\cos\theta).$$

$$\boxed{\frac{m^\star}{m} = 1 + \frac{1}{3}\frac{2m^\star}{(2\pi)^2}g_1}$$

``Landau Relation''

At this point this does not hold to all orders.

Operator constraint has much more information.
In particular from it we may conclude that the
only possible marginal interactions correspond to
special kinematic configurations

$$\frac{d}{d\mu}O_B = 0$$

$$O_i^B = \Big(\int \frac{d^d p}{(2\pi)^d}\, \psi_p^\dagger (p_i - m\frac{\partial \varepsilon_p}{\partial p_i})\psi_p - \frac{m}{2}\int \prod_{a=1}^{4} \frac{d^d p_a}{(2\pi)^{3d}} \delta^{(d)}(p_1+p_2-p_3-p_4)\Big(\sum_i \frac{\partial g(p_a)}{\partial p_{i,a}}\Big)\psi_{p_4}^\dagger \psi_{p_3}^\dagger \psi_{p_2}\psi_{p}$$

Forward scattering: $L_{FS} = g_{FS}(\theta)\psi_{\vec{k}}^\dagger \psi_{\vec{k}} \psi_{\vec{p}}^\dagger \psi_{\vec{p}}$

Back to Back (BCS): $L_{BCS} = g_{BCS}(\theta)\psi_{\vec{k}}^\dagger \psi_{-\vec{k}} \psi_{\vec{p}}^\dagger \psi_{-\vec{p}}$

BCS does not contribute to Landau
relation and FS has vanishing beta
function!

Consequence the constraint,
corrections to fermion self energy are
power suppressed

$$\Gamma[E] \sim E^2 \qquad\qquad R \sim T^2$$

Generalizing to Metals: Framid gets gapped but constraint is unchanged

Defines a Fermi Liquid. Fermi liquid universality class is remarkably robust. How can we avoid Fermi liquid behavior!!

Hi Tc compounds: $R \sim T$

Modifying the constraints to allow for non-Fermi Liquid behavior

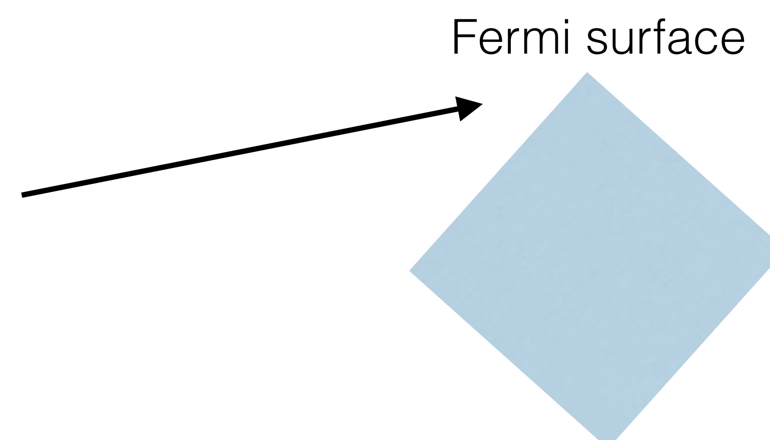
$$S_0 = \int d^d x dt \; \psi^\dagger \left[i(\partial_0 - \eta^i \partial_i) + \frac{1}{2} m \vec{\eta}^2 + \varepsilon(i\partial_i + m\eta_i) \right] \psi$$

Suppose: $\dfrac{\partial \epsilon}{\partial k} = 0$ ``Von-Hove Singularity"

$$\rho(E_f) \sim \infty$$

Classic example: 2-D Hubbard model at half-filling

$$\vec{k} = (0, \pi)$$



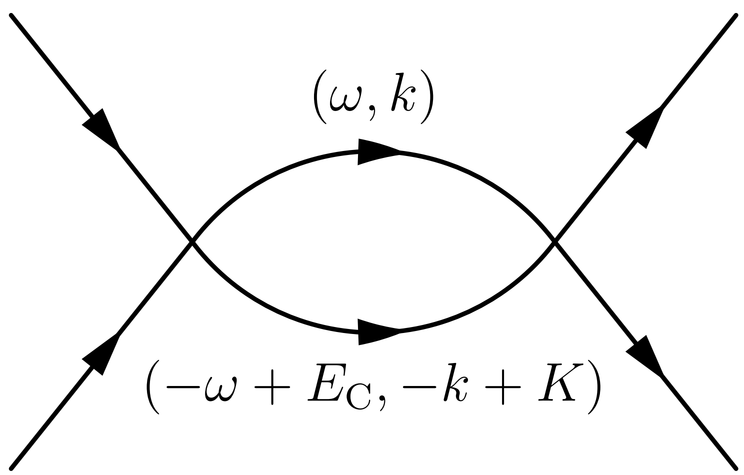
$$\epsilon(k) = k_x^2 - k_y^2 \equiv k_+ k_-$$

fluctuations near singularity

$$\epsilon(k) = \vec{v}_F \cdot \vec{k}$$

Away from singularity

One loop beta function



VH or NVH fluctuations

Impose energy cut-off $k_+ k_- < \Lambda$

Cut-off on energy does NOT lead to a finite integral,

$k_+ \to \infty, k_- \to 0$ Collinear region

needs a ``rapidity" regulator $k_\pm < \Upsilon$

$$\mathcal{A}_{VH}(E) = \frac{g^2}{8\pi^2}\left[-2\log\frac{2\Lambda}{E}\log\frac{2\Lambda}{\Upsilon^2} + \log^2\frac{E}{2\Lambda} + i\pi\log\frac{\Upsilon^2}{E}\right]$$

Adding NVH region leads to a regulator independent result with $\Upsilon \to v_F$

Phenomenological consequences

gap:

$$|\Delta| \simeq 2V_F^2 \exp\left(-\sqrt{\log^2 \frac{V_F^2}{\Lambda} + \frac{8\pi^2}{|g|}}\right)$$

quasi-particle width:

$$\Gamma(E) \sim E$$

(Gopalon et al)

$$R \sim T$$

Here we have only analyzed effects one singularity, richer phenomenology arise when considering the full Hubbard model.

Note: High Tc behavior stable under filling fraction changes 5-10 percent.

Fermi Liquids at Unitarity

Consider a systems whose coupling is tuned to unitarity limit (Feshbach resonance)

> Open Question: Above Tc does this system behave like a Fermi liquid?

Fermi sea spontaneously breaks conformal symmetry

Where is the NR dilaton?

Seems that such a Goldstone is not realizable

(Oz et. al.)

Under Dilatations

$$\sigma \to \sigma + \delta$$

Boosts

$$\sigma(x,t) \to \sigma(x+vt,t)$$

$$L = F(e^{2\sigma}\partial_t, e^{\sigma}\partial_i)e^{(2+d)\sigma}$$

Seems like any attempt to write down an invariant kinetic term fails. However, this is no longer true once we include framon

$$\partial_t \to \partial_t + \vec{\eta}\cdot\vec{\partial}_x$$

Can realize all the symmetries non-linearly.
No reason why dilaton can't be there a priori.

Fate of Goldstones $\lambda, \vec{\eta}, \sigma$

Inverse Higgs Relations

$$[H, C] \sim D \qquad [P, C] \sim K$$

can trade special conformal for dilaton, and dilaon for longitudinal part of framid, but still have transverse framid degrees of freedom, which (in 2D) lead to a set of constraints. In 3D we need not choose to impose the constraints and leave the (non-derivatively coupled Goldstones in the theory, leading to non-Fermi liquid behavior.)

Suppose we treat the Framid as a Lagrange Multiplier ? In 2D no choice (power counting).

The additional constraint beyond the Landau conditions due to dilatations

(Only BCS and FS allowed by boost)

$$\mathcal{O}_\phi = \int \psi^\dagger_{\vec{\mathbf{p}}} \left(2\varepsilon(\vec{\mathbf{p}}) - \frac{\vec{\mathbf{p}}^2}{m}\right) \psi_{\vec{\mathbf{p}}} + \int \left((2-d) g(\vec{\mathbf{p}}_i, \mu) - \beta(g)\right) \psi^\dagger_{\vec{\mathbf{p}}_4} \psi_{\vec{\mathbf{p}}_3} \psi^\dagger_{\vec{\mathbf{p}}_2} \psi_{\vec{\mathbf{p}}_1}$$

2-D: $\beta = 0$ $\qquad \epsilon = p^2$

3-D: $\beta = -g\mu$ $\qquad \epsilon = p^2$

Same as linearly realized theory! Not consistent with the assumptions