SCFTs and susy RG flows

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Bay Area Particle Theory Seminar

Two topics: (1) based on work with Francesco Sannino and (2) with Clay Cordova and Thomas Dumitrescu

Spectacular Collaborators

Clay Córdova

Thomas Dumitrescu

1506.03807: 6d conformal anomaly a from 't Hooft anomalies. 6d a-thm. for N=(1,0) susy theories.

1602.01217: Classify susy-preserving deformations for d>2 SCFTs.

+ to appear & work in progress

“What is QFT?”

perturbation theory
around free field
Lagrangian theories

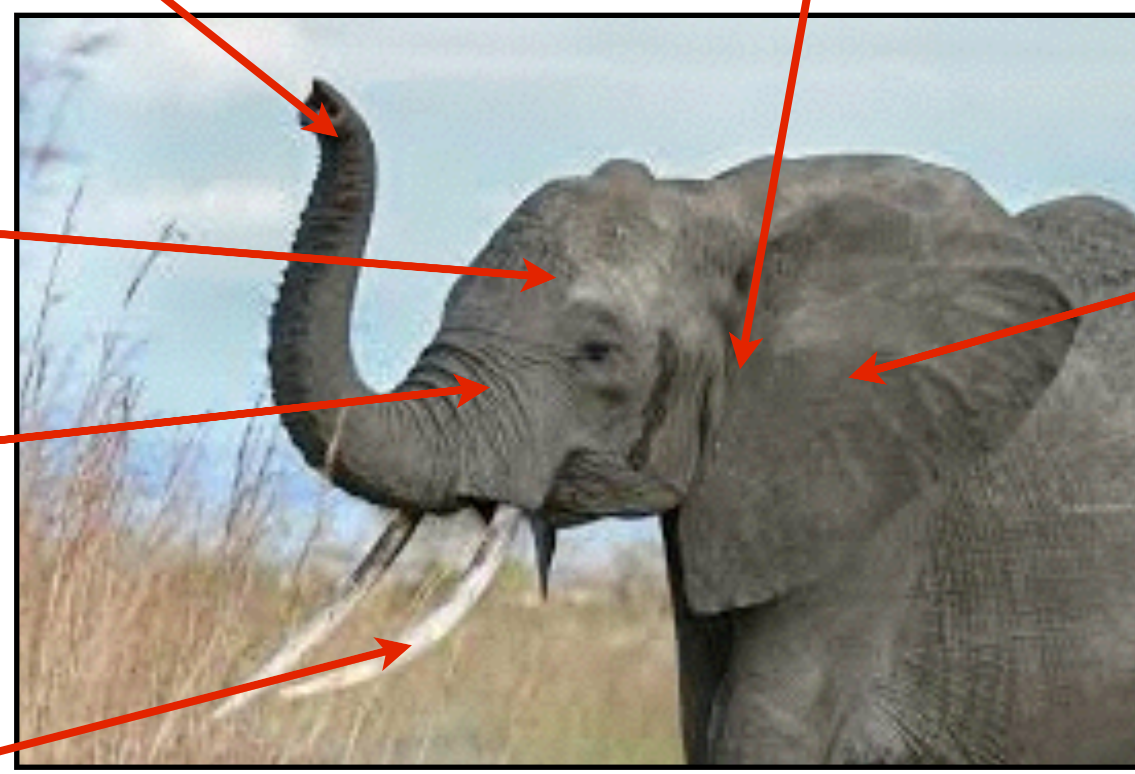
CFTs + perturbations

string, brane,
AdS, M, F-thy
realizations

susy

non-Lagrangian

higher dim’l thy
compactified



unexplored...
something
crucial for
the future?

RG flows

"# d.o.f."

UV CFT (+relevant)

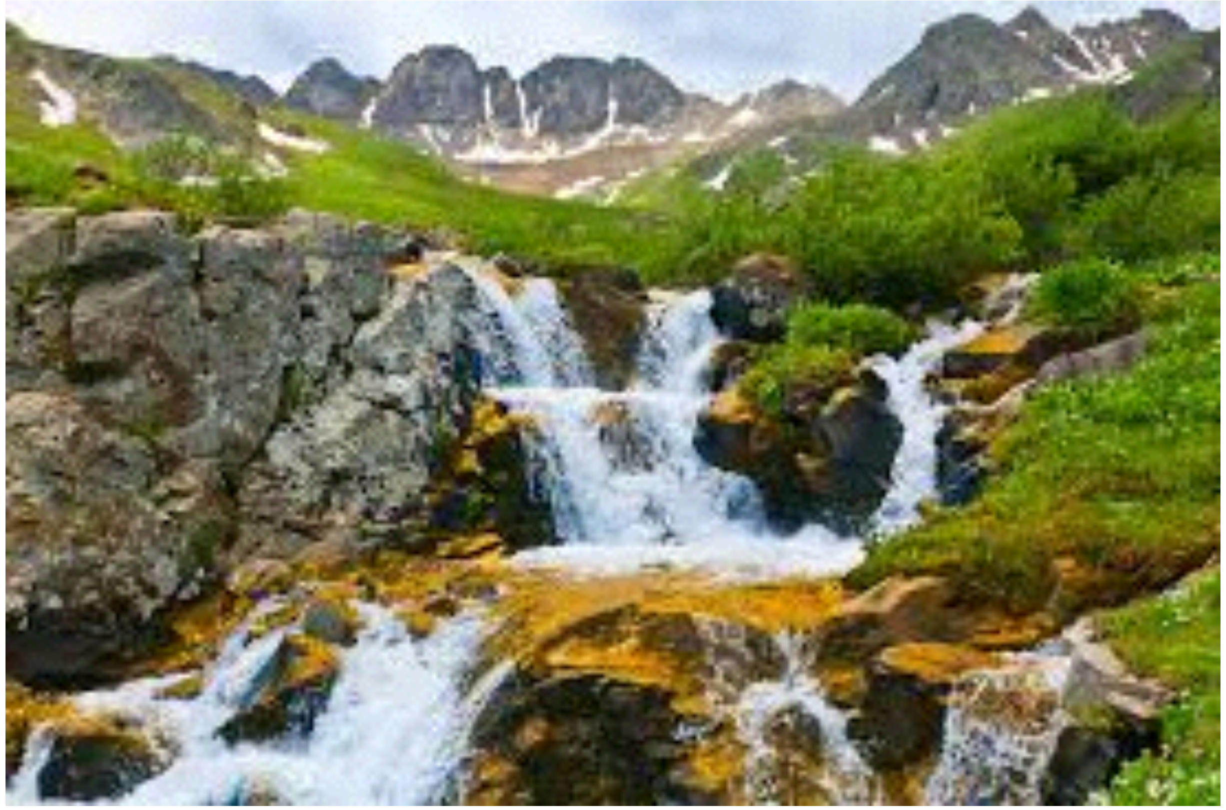
"chutes"

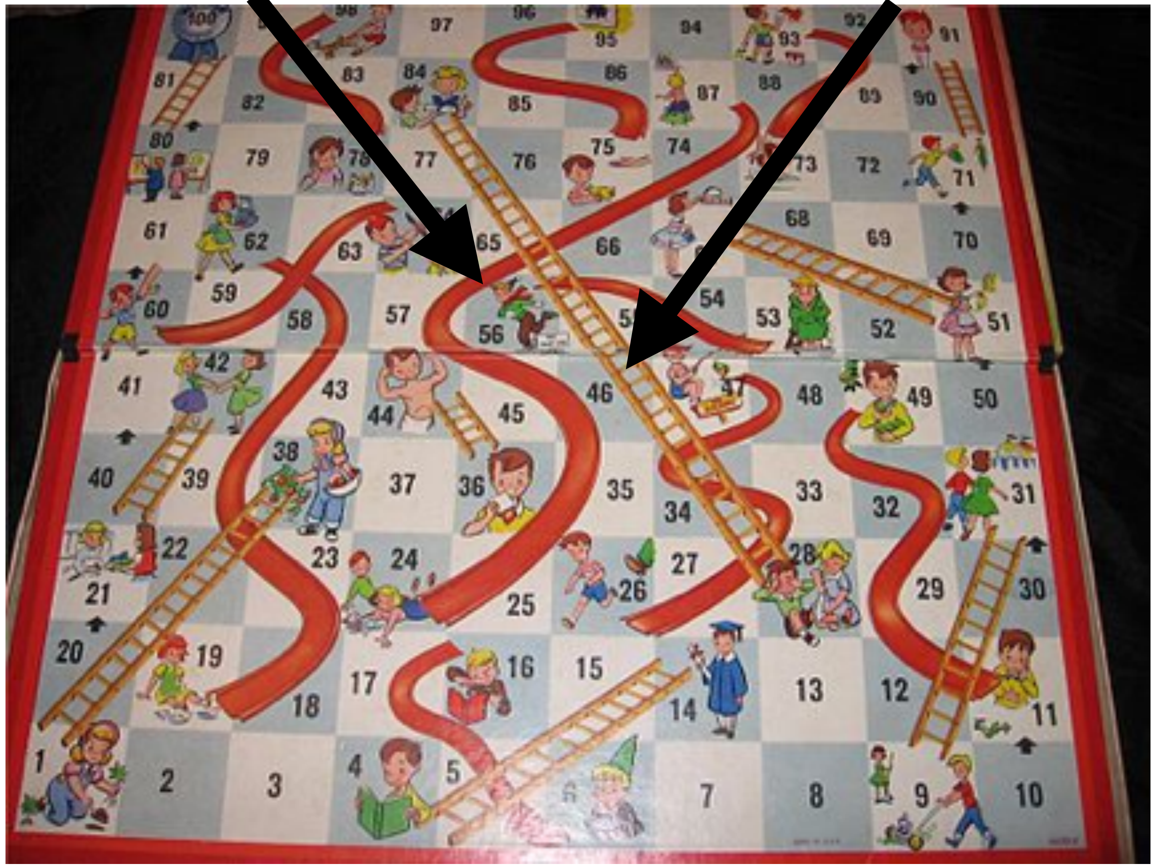
course graining

IR CFT (+irrelevant)

"ladders"

E.g. Higgs mass E.g. dim 6 BSM ops





RG flows

"# d.o.f."

UV CFT (+relevant)

RG course graining

IR CFT (+irrelevant)

The "**deformations**" examples:

- $$\text{``}\delta\mathcal{L}\text{''} = \sum_i g_i \mathcal{O}_i$$ (OK even if SCFT is non-Lagrangian)

- Move on the moduli space of (susy) vacua.

- Gauge a (e.g. UV or IR free) global symmetry.

- Will here focus on RG flows that preserve supersymmetry.

RG flow constraints

- d=even: 't Hooft anomaly matching for all global symmetries (including NGBs + WZW terms for spont. broken ones + Green-Schwarz contributions for reducible ones). Weaker d=odd analogs, e.g. parity anomaly matching in 3d.

- Reducing # of d.o.f. intuition. For d=2,4 (& d=6?) : a-theorem

$$a_{\rm UV} \geq a_{\rm IR} \qquad a \geq 0$$

For any unitary theory

d=even: $$\langle T^\mu_\mu \rangle \sim a E_d + \sum_i c_i I_i$$

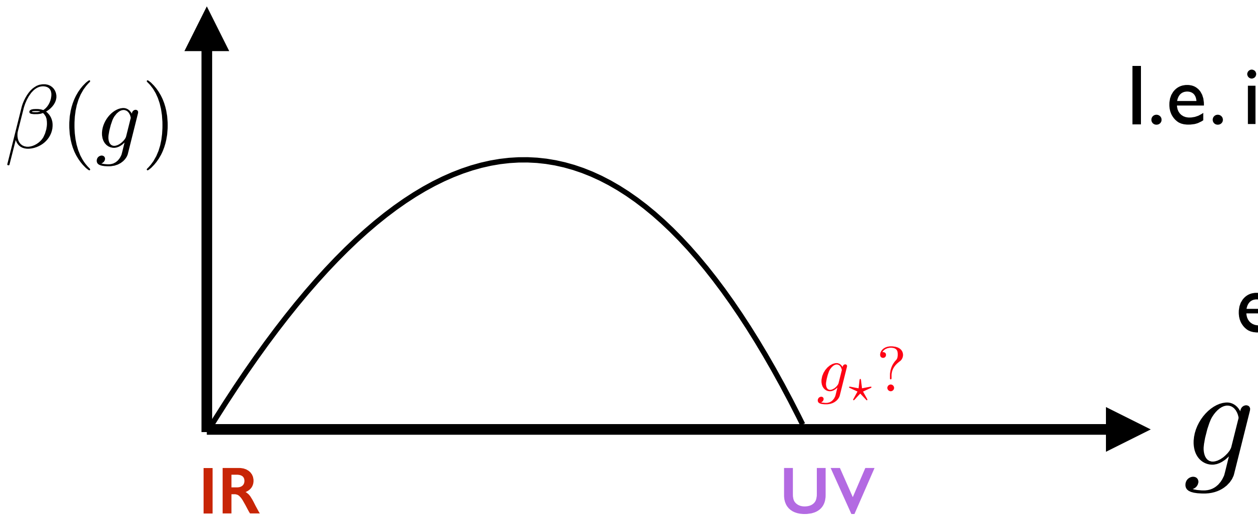
(d=odd: conjectured analogs, from sphere partition function / entanglement entropy.)

- Additional power from supersymmetry.

UV asymptotic safety?

- Suppose theory has too much matter, so not asymptotically free in UV.
- IR theory in free electric phase $g_{IR} \to 0.$
- UV safe, interacting CFT, completion?

$g_{UV} \to g_\star$ with $\beta(g_\star) = 0?$

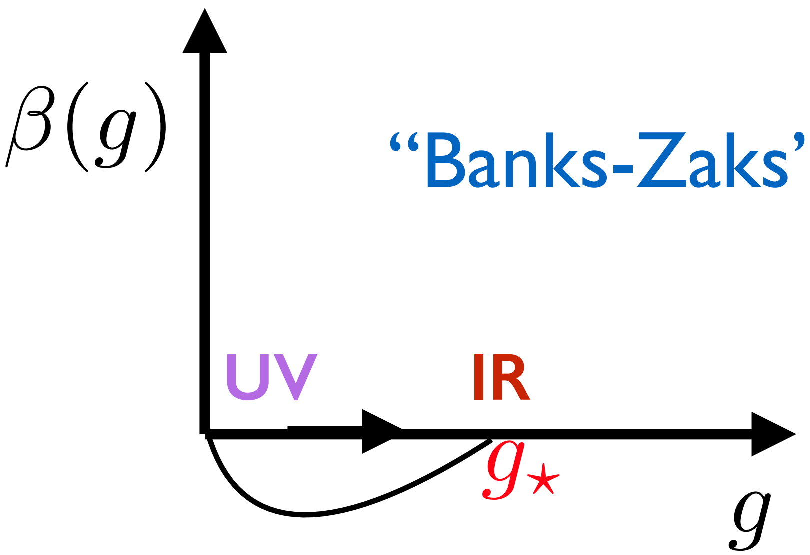


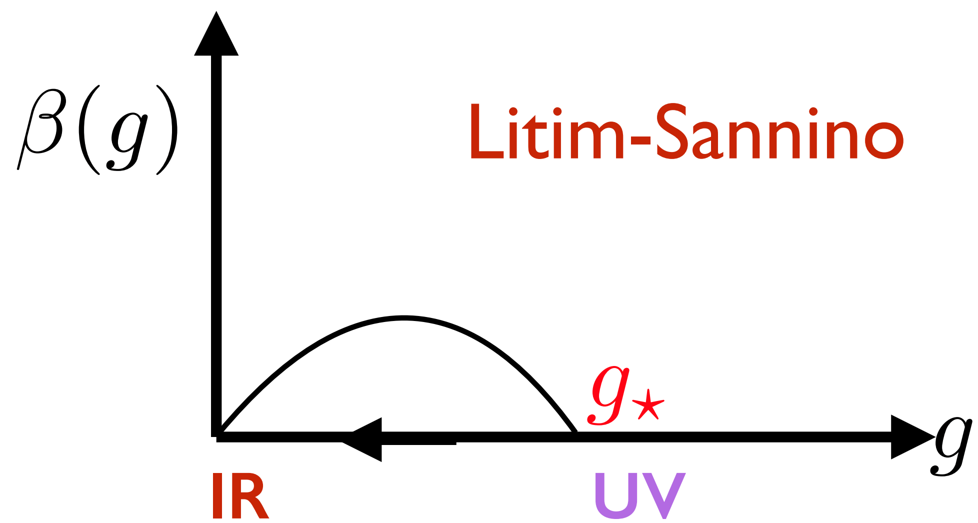
I.e. in same theory as opposed to some dual?

e.g. $\lambda\phi^4?$ no (lattice)

UV asymptotic safety

Surprise: examples found in non-susy QCD, D.F. Litim and F. Sannino '14.

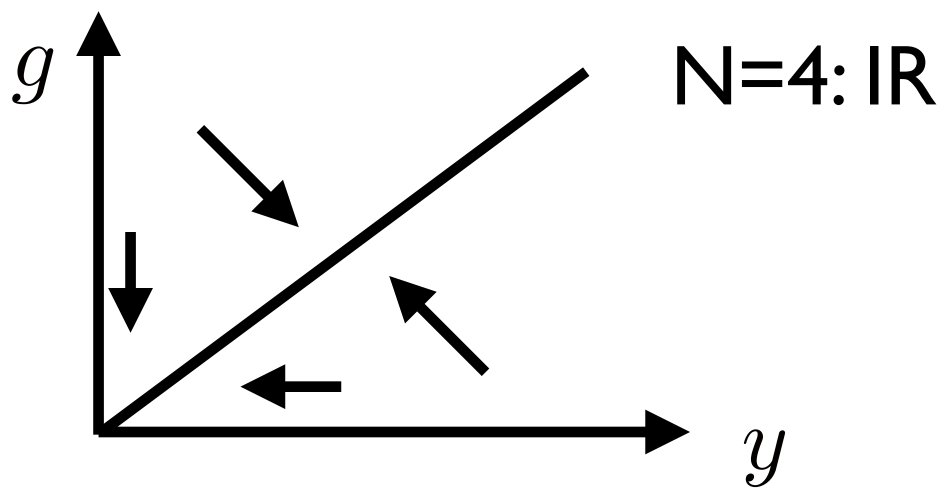




Theory: SU(N_c) QCD with N_f Dirac fermion flavors +Yukawa coupling to $N_f{}^2$ gauge singlet scalars, with also quartic self-interactions. $N_f > (11/2)N_c$ so not asymp. free. Multiple couplings, each IR free. Interacting CFT via cancellation of 1-loop and higher-loop beta fn contributions. Can be made perturbative, taking N_f just above $(11/2)N_c$.

Susy examples ?

Warmup: recall example of N=1 susy gauge theory with 3 adjoint matter chiral superfields, with superpotential $W = y\epsilon^{ijk}\mathrm{Tr}(\Phi_i\Phi_j\Phi_k)$



Individually IR free couplings combine to give IR-attractive, interacting SCFT.

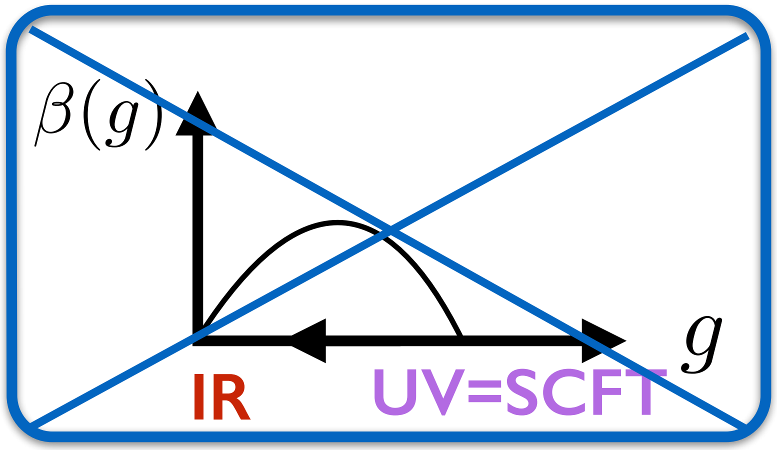
UV starting point of these RG flows? Needs a UV completion. Not UV safety.

“susy UV asyp. safety?”

K.I. and F. Sannino ’15.

“No.” at least not nearby, in perturbation theory, in broad classes of non-asymptotically free theories.

Via imposing



for susy theories

- (s) CFT unitarity constraints.
- a-theorem: $a_{UV} > a_{IR}$.

Also J. Wells & S. Martin ‘00. We were originally unaware of this excellent, early paper..

- apply a-maximization (KI, Wecht ’03), if needed.

4d, N=1 SQFTs

$$\mathcal{J}_\mu = J^R_\mu + \ldots + (\bar\theta\theta)^\nu T_{\mu\nu} + \ldots$$ Energy-momentum tensor supermultiplet, contains **U(1)$_R$ current**

$$\bar D^{\dot\alpha} \mathcal{J}_{\alpha\dot\alpha} = D_\alpha X,$$ Energy-momentum conservation. X=chiral superfield,

$$T^\mu_\mu + i\partial^\mu j^R_\mu = X|_{\theta^2}$$ supermultiplet of anomalies (useful!).

$$\Delta(Q_i) \equiv 1 + \frac{1}{2}\gamma_i(g) = \frac{3}{2}R(Q_i)$$ **Dimension of chiral fields ~ their running R-charges.**

Exact beta functions ~ linear combinations of R-charges of fields:

$$\beta_g^{NSVZ} \sim -g^3 \mathrm{Tr} R G^2$$ **U(1)$_R$ ABJ anomaly.**

$$W = h\mathcal{O} \qquad \beta_h = \frac{3}{2} h(R(\mathcal{O}) - 2)$$ **W's R-violation.**

4d N=1 SCFTs

$$T^\mu_\mu + i\partial_\mu J^\mu_R = X|_{\theta^2}$$ Curved background

$$X \supset \widehat{\beta}_{NSVZ}(R_i)W^2 + c\mathcal{W}^2 - a\Xi$$

Euler

Weyl^2

$$a = \cancel{\frac{3}{32}}(3\mathrm{Tr}R^3 - \mathrm{Tr}R)$$

$$c = \frac{1}{32}(9\mathrm{Tr}R^3 - 5\mathrm{Tr}R)$$

Anselmi, Freedman, Grisaru, Johansen.

Now vary $R \to R + \epsilon F, \quad X \to X + \epsilon \bar{D}^2 J_F$, find

$$\bar{D}^2 J_F = k_{FFF} W_F^2 + k_F \mathcal{W}^2 \qquad a \to a$$

The correct R-symmetry locally maximizes a(R). KI, B. Wecht

Can use the power of 't Hooft anomaly matching.

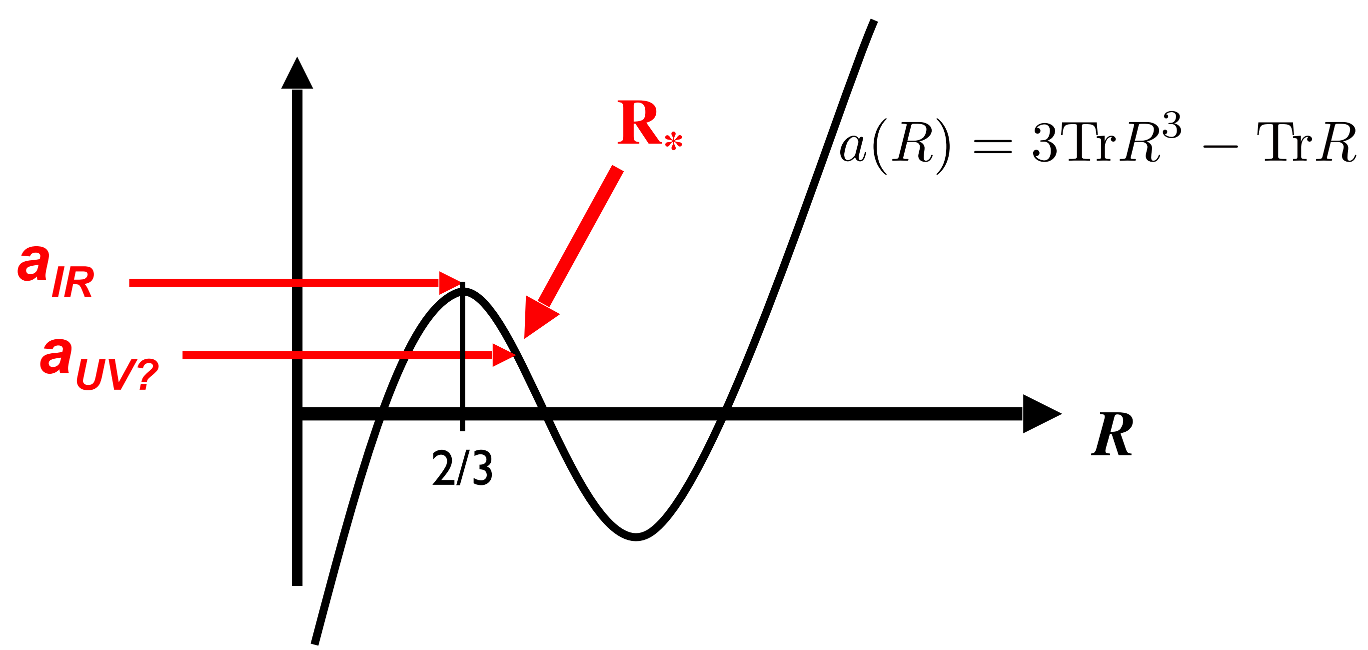
SQCD examples

SU(N_c) SQCD with N_f Dirac flavors, non AF: $N_f > 3N_c$

$$R_\star \equiv R(Q) = R(\widetilde{Q}) = \frac{N_f - N_c}{N_f}$$

Superconf'l U(1)$_R$ determined by anomaly free + symm

Would violate the a-theorem: (Instead UV-completes to asymp. free Seiberg dual.)



Also, no asymp safe UV SCFT is possible for $W = SQ\widetilde{Q}$

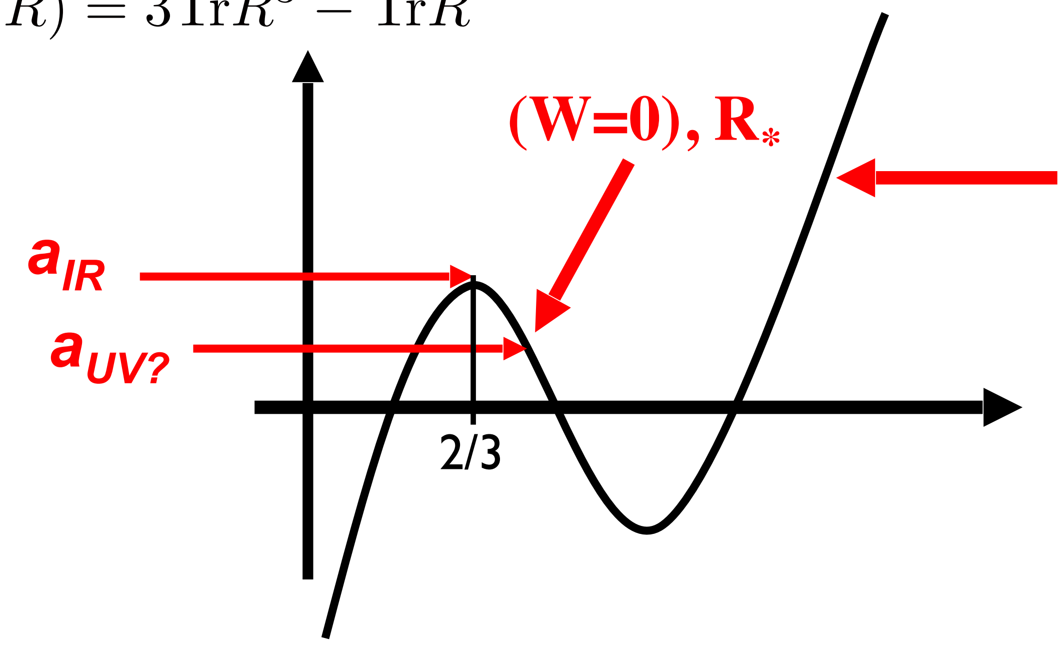
$$\Delta(S) = \frac{3}{2}R(S) = 3\frac{N_c}{N_f}$$

Would violate unitarity $\Delta(S) \geq 1$ for $N_f > 3N_c$

General N=1 cases

No N=1 susy theories with non-asymptotically free matter content and W=0 can have a UV-safe SCFT. By a-maximization all such cases would violate the a-theorem:

$a(R) = 3\mathrm{Tr}R^3 - \mathrm{Tr}R$



Can satisfy a-thm only if some fields have large R charge, far from perturbative limit. Some possible examples via W terms - see Martin & Wells. Various other constraints to check. All satisfied? Do these exist? TBD.

Change gears

Discuss work with C. Cordova and T. Dumitrescu

Study, and largely classify, possible susy-preserving deformations of SCFTs in various spacetime dims.

Especially consider 6d susy and SCFT constraints. 6 = the maximal spacetime dim for SCFTs. A growing list of interacting, 6d SCFTs. Yield many new QFTs in lower d, via compactification.

6d a-theorem?

For spontaneous conf'l symm breaking: dilaton has derivative interactions to give Δa anom matching Schwimmer, Theisen; Komargodski, Schwimmer

6d case: $$\mathcal{L}_{\rm dilaton} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6}$$ (schematic)

Maxfield, Sethi; Elvang, Freedman, Hung, Kiermaier, Myers, Theisen.

Can show that b>0 (b=0 iff free) but b's physical interpretation was unclear; no conclusive restriction on sign of Δa.

Clue: observed that, for case of (2,0) on Coulomb branch,

$$\Delta a \sim b^2 \quad >0.$$

Cordova, Dumitrescu, KI: this is a general req't of N=(1,0) susy, and b is related to an 't Hooft anomaly matching term.

Longstanding hunch

e.g. Harvey Minasian, Moore '98

Susy multiplet of anomalies: should be able to relate a-anomaly to R-symmetry 't Hooft-type anomalies in 6d, as in 2d and 4d.

$T^{\mu\nu} \leftrightarrow J_R^{\mu,a}$ Stress-tensor supermultiplet

$g_{\mu\nu} \leftrightarrow A_{R,\mu}^a$ Sources = bkgrd SUGRA supermultiplet

$T^{\mu\nu}$, $T^{\rho\sigma}$, $T^{\kappa\lambda}$, $T^{\zeta\psi}$: $\boxed{\text{a?}}$

susy?

$J^{\mu,a}, T^{\mu\nu}$, $J^{\rho,a}, T^{\rho\sigma}$, $J^{\kappa,a}, T^{\kappa\lambda}$, $J^{\zeta,a}, T^{\zeta\psi}$: $\boxed{I_8}$

4-point fn with too many indices. Hard to get a, and hard to compute.

Easier to isolate anomaly term, and enjoys anomaly matching

6d (1,0) 't Hooft anomalies

$$\mathcal{I}_8^{\rm origin} = \frac{1}{4!}\left(\alpha c_2^2(R) + \beta c_2(R)p_1(T) + \gamma p_1^2(T) + \delta p_2(T)\right)$$

$$c_2(R) \equiv \frac{1}{8\pi^2}{\rm tr}(F_{SU(2)_R}\wedge F_{SU(2)_R})$$

$$p_1(T) \equiv \frac{1}{8\pi^2}{\rm tr}(R\wedge R)$$

Background gauge fields and metric
(~ background SUGRA)

Computed for (2,0) SCFTs + many (1,0) SCFTs

Harvey, Minasian, Moore; KI; Ohmori, Shimizu, Tachikawa; Ohmori, Shimizu, Tachikawa, Yonekura; Del Zotto, Heckman, Tomasiello, Vafa; Heckman, Morrison, Rudelius, Vafa.

E.g. for theory of N small E8 instantons:

Ohmori, Shimizu, Tachikawa

$$\mathcal{E}_N:\ (\alpha,\beta,\gamma,\delta) = (N(N^2+6N+3), -\frac{N}{2}(6N+5), \frac{7}{8}N, -\frac{N}{2})$$

(Leading N^3 coeff. can be anticipated from Z_2 orbifold of A_{N-1} (2,0) case.)

(1,0) on tensor branch

$$\mathcal{I}_8^{\text{origin}} = \frac{1}{4!}\left(\alpha c_2^2(R) + \beta c_2(R) p_1(T) + \gamma p_1^2(T) + \delta p_2(T)\right)$$

't Hooft anomaly matching requires

$\Delta \mathcal{I}_8 \equiv \mathcal{I}_8^{\text{origin}} - \mathcal{I}_8^{\text{tensor branch}} \sim X_4 \wedge X_4$ must be a perfect square, match I_8 via X_4 sourcing B:

$\mathcal{L}_{GSWS} = -iB \wedge X_4$ KI ; Ohmori, Shimizu, Tachikawa, Yonekura

$X_4 \equiv 16\pi^2 (x c_2(R) + y p_1(T))$ for some real coefficients x, y

Our classification of defs. gives: $\mathcal{L}_{\text{tensor}} = Q^8(\mathcal{O}) \supset \mathcal{L}_{\text{dilaton}} + \mathcal{L}_{GSWS}$

Then $b = \frac{1}{2}(y - x)$ Adapting a SUGRA analysis of Bergshoeff, Salam, Sezgin '86 (!).

Upshot: $$\boxed{a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta}$$

Change gears (1602.01217)

Classify susy-preserving deformations of SCFTs

- $``\delta\mathcal{L}" = Q^{N_Q}\mathcal{O}_{\rm long}$ "D-term" e.g. Kaher potential in 4d N=1.

SCFT unitarity, bound grows with dim d: $\Delta(\delta\mathcal{L}) > \frac{1}{2}N_Q + \Delta_{min}(\mathcal{O}_{\rm long})$

Irrelevant. E.g. for 6d N=(1,0) such operators have $\Delta > \frac{1}{2}8 + 6 = 10.$

- $``\delta\mathcal{L}" = Q^{n_{\rm top}}\mathcal{O}_{\rm short}$ $\quad \Delta(\delta\mathcal{L}) = \frac{1}{2}n_{\rm top} + \Delta(\mathcal{O}_{\rm short})$ Constrained by SCFT unitarity.

  e.g. F-terms, W in 4d N=1.

Short reps classified, in terms of the superconf'l primary operator at the bottom of the multiplet. Theory independent, just using SCFT rep constraints. We study the Q descendants, looking for Lorentz scalar "top" ops. Some oddball susy-preserving ops do exist, including in middle of multiplet(!) We had to be careful - it's risky to claim a complete classification (embarrassing if something is overlooked)! Much more subtle and sporadic zoo than we originally expected (especially in 3d).

Some of our results:

- 6d (2,0): all 16 susy preserving deformations are irrel.
  least irrelevant operator has dim = 12.

- 6d (1,0): all 8 susy preserving deformations are irrel.
  least irrelevant operator has dim = 10. Also J. Luis, **S. Lust**.

- 5d: all susy preserving deformations are irrel., except for real mass terms associated with global symmetries.

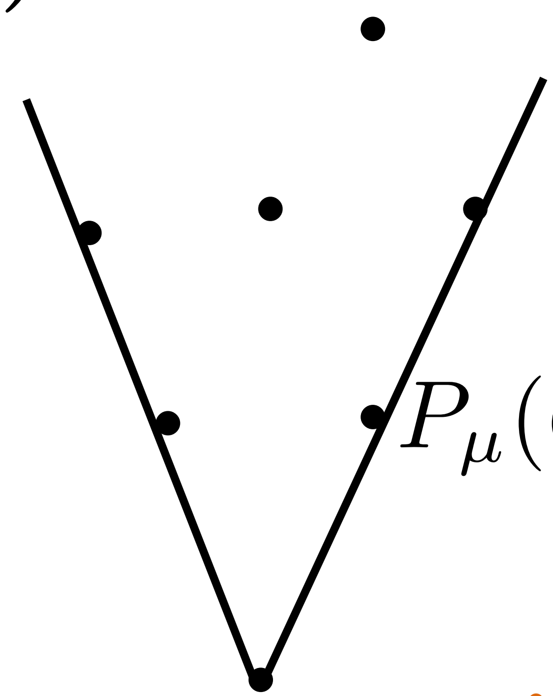
- 4d, N=3: no relevant or marginal deformations. Also O. Aharony and M. Evtikhiev.

- 3d, N>3: all have universal, relevant, mass deformations from stress-tensor; the only relevant deformations, and no marginal. For N=4, also flavor current masses, no others.

CFTs, first w/o susy

$SO(d,2)$ Operators form representations

$P_\mu \uparrow$ $K_\mu \downarrow$



$P_\mu(\mathcal{O}_\mathcal{R})$ descendants= total derivatives, such deformations are **trivial**.

$\mathcal{O}_\mathcal{R}$ **primary** $K_\mu(\mathcal{O}_\mathcal{R}) = 0$

Unitarity: primary + all descendants must have + norm, e.g.

$$\big|P_\mu|\mathcal{O}\rangle\big|^2 \sim \langle\mathcal{O}|[K_\mu, P_\mu]|\mathcal{O}\rangle \geq 0$$

$$[P_\mu, K_\nu] \sim \eta_{\mu\nu}D + M_{\mu\nu}$$

Zero norm, null states = set to zero. Nulls = both primary and descendant.

SCFT super-algebras

complete classification

$d > 6$ no SCFTs can exist

$d = 6$ $OSp(6,2|\mathcal{N}) \supset SO(6,2) \times Sp(\mathcal{N})_R$ $(\mathcal{N}, 0)$

$8\mathcal{N}Qs$

$d = 5$ $F(4) \supset SO(5,2) \times Sp(1)_R$ 8Qs

$d = 4$ $Su(2,2|\mathcal{N} \neq 4) \supset SO(4,2) \times SU(\mathcal{N})_R \times U(1)_R$

$4\mathcal{N}Qs$

$d = 4$ $PSU(2,2|\mathcal{N} = 4) \supset SO(4,2) \times SU(4)_R$

$d = 3$ $OSp(4|\mathcal{N}) \supset SO(3,2) \times SO(\mathcal{N})_R$ $2\mathcal{N}Qs$

$d = 2$ $OSp(2|\mathcal{N}_L) \times OSp(2|\mathcal{N}_R)$ $\mathcal{N}_L Qs + \mathcal{N}_R \bar{Q}s$

SCFT operator reps

$P_\mu \uparrow \qquad K_\mu \downarrow$

Q $\uparrow$ S $\downarrow$

$\{Q, Q\} = 2P_\mu$

$\{S, S\} = 2K_\mu$

$Q^\ell(\mathcal{O}_\mathcal{R})$

$Q(\mathcal{O}_\mathcal{R})$ descendants

$\mathcal{O}_\mathcal{R}$ super-primary $S(\mathcal{O}_\mathcal{R}) = 0$

$$\text{“}\delta\mathcal{L}\text{”} = \sum_i g_i \mathcal{O}_i$$ primary, modulo descendants.

$\{Q, Q\} \sim P_\mu \sim 0$ Grassmann algebra.

Level $Q^{\wedge \ell}(\mathcal{O}_\mathcal{R}) \qquad \ell = 0 \ldots \ell_{max} \leq N_Q$

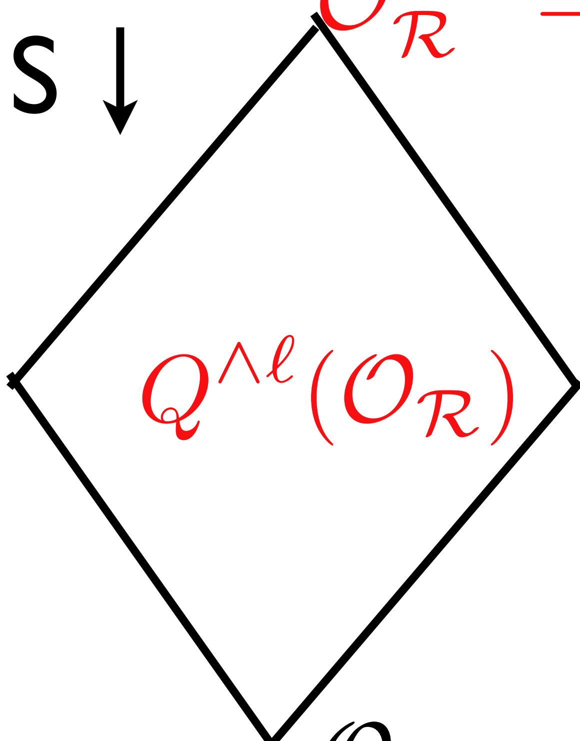
Typical, long multiplets

Q ↑   S ↓

$\mathcal{O}_{\mathcal{R}}^{\rm top} = Q^{\wedge N_Q}(\mathcal{O}_{\mathcal{R}})$

$\boxed{Q(\mathcal{O}_{\mathcal{R}}^{\rm top}) \sim 0}$ *

modulo descendants



$\binom{N_Q}{\ell} d_{\mathcal{O}_{\mathcal{R}}}$ conformal primary ops at level *l*, $2^{N_Q} d_{\mathcal{O}_{\mathcal{R}}}$ total

$\mathcal{O}_{\mathcal{R}}$ super-primary $S(\mathcal{O}_{\mathcal{R}}) = 0$

Can generate multiplet from bottom up, via Q,or from top down, via S. Reflection symmetry. Unique op at bottom, so unique op at the top. Operator at top = susy preserving deformation. No other susy preserving operators in long multiplets. Easy cases. D-terms. Unitarity bounds at bottom of give bounds at top.

Unitary bounds

All Q-descendants must have non-negative norm.
E.g. at Q-level one:

$$0 \leq \left| Q | \mathcal{O} \rangle \right|^2 \sim \langle \mathcal{O}^\dagger | S Q | \mathcal{O} \rangle \sim \langle \mathcal{O}^\dagger | \{S, Q\} | \mathcal{O} \rangle$$

$$\{S, Q\} \sim D - (M_{\mu\nu} + R)$$

$$\longrightarrow \quad \Delta \geq c(\mathrm{Lorentz}) + c(\mathrm{R-symmetry}) + \mathrm{shift}$$

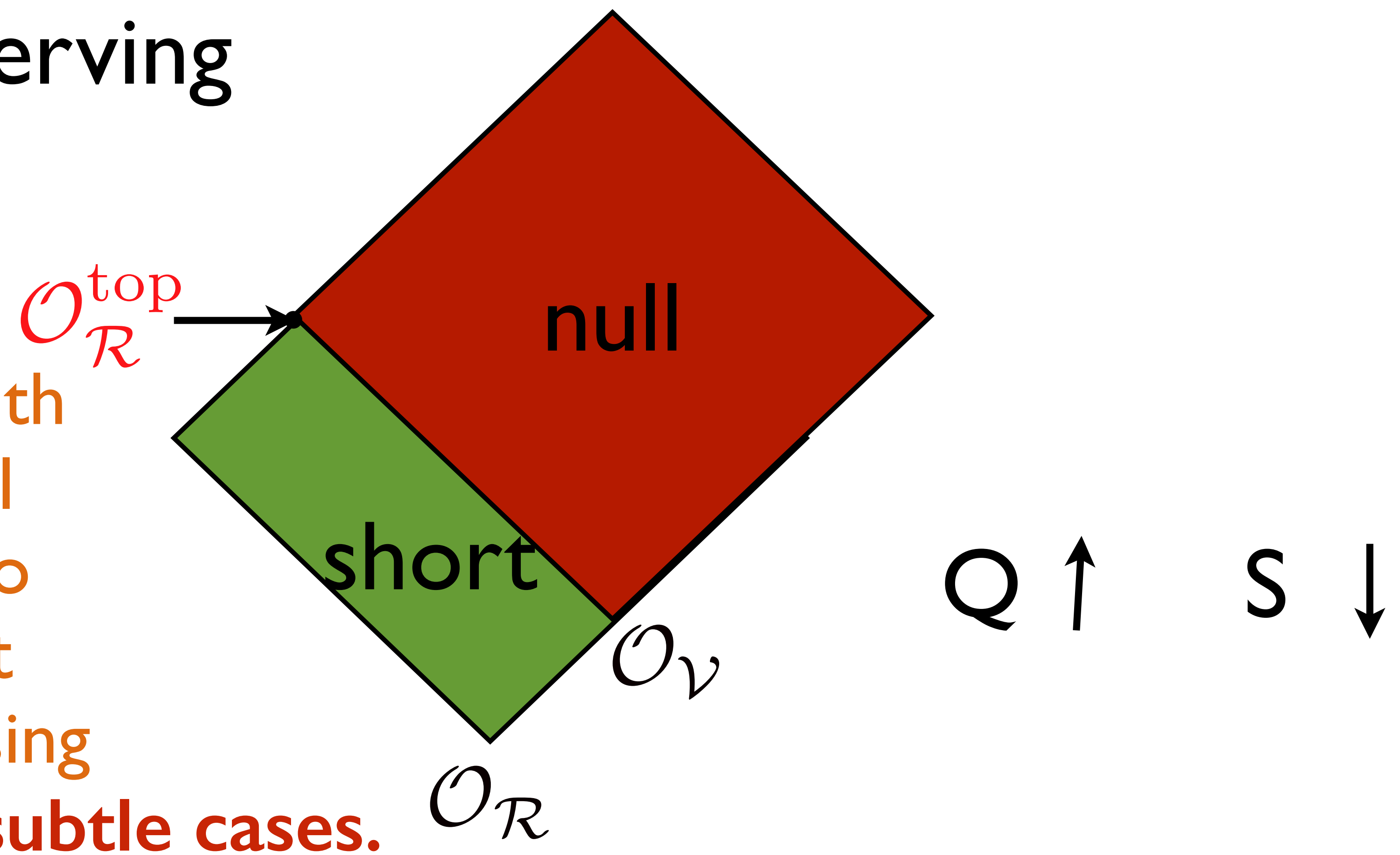
Saturated iff there is a **null state**: a Q-descendant that is also a superconformal primary:

$\mathcal{O}_{\mathcal{V}} = Q(\mathcal{O}')$ and $S(\mathcal{O}_{\mathcal{V}}) = 0$

Set $\mathcal{O}_{\mathcal{V}} = 0$ along with all its Q-descendants.

Long - null = short

Specific operator dimensions, in terms of Lorentz + R-symmetry + shifts, to get null states. Set null states to zero: a short multiplet. **Simplest cases** also have the reflection symmetry, unique operator at bottom and top = susy preserving deformation:



Act on bottom op. with all Q's, setting the null linear combinations to zero. But can also act with R-symmetry raising and lowering. **Some subtle cases.**

Multiple top op. cases

(Unique bottom operator, so no reflection symmetry.)

E.g. $T_{\mu\nu}$ multiplet of 4d N=4, top ops= $T_{\mu\nu},\ \mathcal{O}_\tau,\ \mathcal{O}_{\bar\tau}$

Conserved $J_\mu^{a,\mathrm{global}}$ of 5d N=1, top ops= $J_\mu^{a,\mathrm{global}},\ \mathcal{O}_{m^a}$

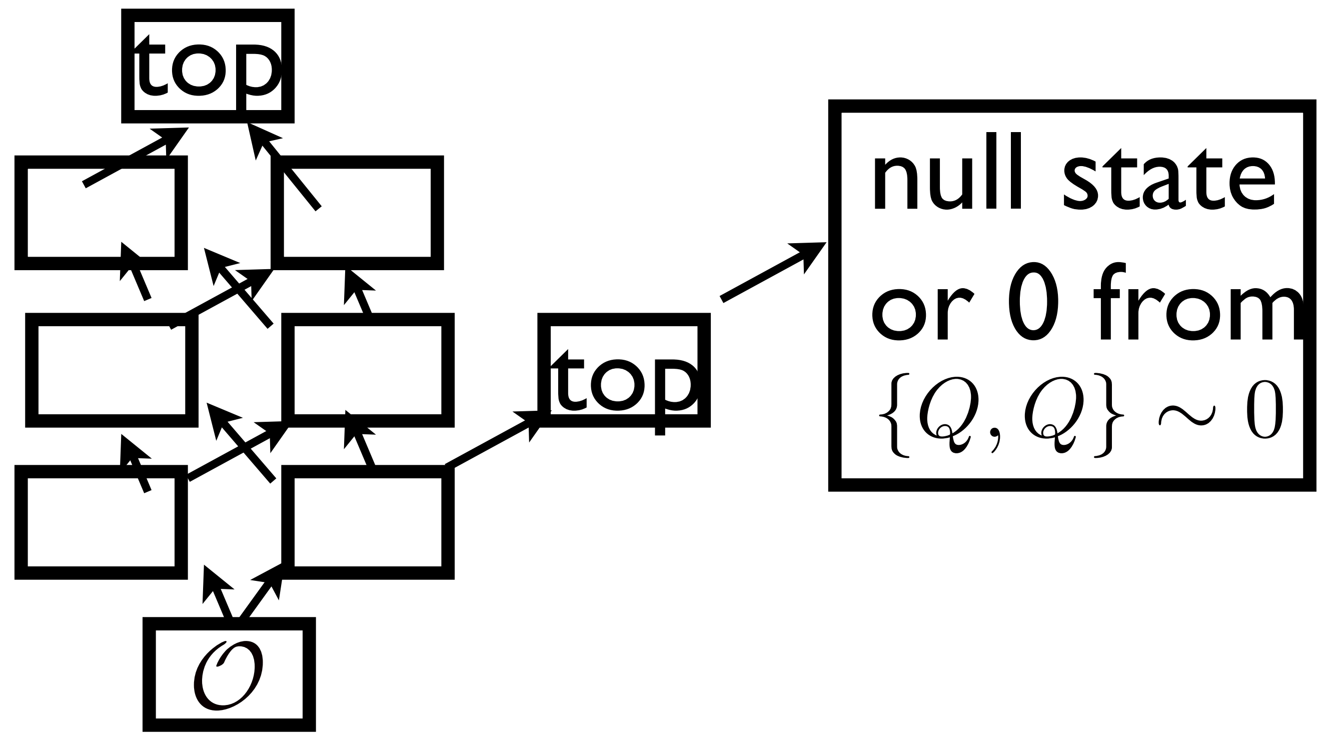
Many examples, especially with conserved currents; in such cases, setting $\{Q,Q\}\sim P_\mu\sim 0$ requires care, since current cons. laws are null, both primary and descendant. **But** also examples of multiple top operators without conserved currents, e.g. in 4d N=2,

$$\mathcal{O}^{\mathrm{bottom}} = A_2\bar{A}_2[0;0]^{R=1,r=0}_{\Delta=3} \qquad \mathcal{O}^{\mathrm{top}} = Q^3\bar{Q}^2\mathcal{O}^{\mathrm{bottom}}$$

No conserved currents
in this multiplet, yet 2 tops: and $\mathcal{O}^{\mathrm{top'}} = \bar{Q}^3 Q^2 \mathcal{O}^{\mathrm{bottom}}$

Mid-level susy tops(!)

Q $\uparrow$ S $\downarrow$



3d $\mathcal{N}\geq 4$ $T_{\mu\nu}$ multiplet: the stress-tensor is at top, at level 4.

Another top, at level 2, Lorentz scalar. Gives susy-preserving "universal mass term" relevant deformations. First found in 3d N=8 (KI '98, Bena & Warner '04; Lin & Maldacena '05). Seems special to 3d. Indeed, these examples give a deformed susy algebra with a "non-central extension" with R-symm gens R_{ij} playing role of central term (=3d loophole to Haag-Lopuszanski-Sohnius theorem).

Classify susy preserving deformations of SCFTs

Many multiplets have mid-level Lorentz scalars, in all dimensions. We do many cross checks that we're not overlooking any exotic susy deformations (e.g. verify that Q can map to an operator at the next level, check Bose-Fermi degeneracy, recombination rules, etc).

Detailed tables

Give all susy-preserving deformations, relevant, marginal, and all irrelevant deformations, for all N, d>2

E.g.

3d, N=8:

L=long, A,B,C,...=short.

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Primary $\mathcal{O}$	Deformation $\delta\mathscr{L}$	Comments
$B_1\left\{\begin{matrix}(0,0,2,0)\\ \Delta_{\mathcal{O}}=1\end{matrix}\right\}$	$Q^2\mathcal{O}\in\left\{\begin{matrix}(0,0,0,2)\\ \Delta=2\end{matrix}\right\}$	Stress Tensor (T)
$B_1\left\{\begin{matrix}(0,0,0,2)\\ \Delta_{\mathcal{O}}=1\end{matrix}\right\}$	$Q^2\mathcal{O}\in\left\{\begin{matrix}(0,0,2,0)\\ \Delta=2\end{matrix}\right\}$	Stress Tensor (T)
$B_1\left\{\begin{matrix}(0,0,R_3+4,0)\\ \Delta_{\mathcal{O}}=2+\frac{1}{2}R_3\end{matrix}\right\}$	$Q^8\mathcal{O}\in\left\{\begin{matrix}(0,0,R_3,0)\\ \Delta=6+\frac{1}{2}R_3\end{matrix}\right\}$	F-Term ($\widetilde{T}$)
$B_1\left\{\begin{matrix}(0,0,0,R_4+4)\\ \Delta_{\mathcal{O}}=2+\frac{1}{2}R_4\end{matrix}\right\}$	$Q^8\mathcal{O}\in\left\{\begin{matrix}(0,0,0,R_4)\\ \Delta=6+\frac{1}{2}R_4\end{matrix}\right\}$	F-Term ($\widetilde{T}$)
$B_1\left\{\begin{matrix}(0,0,R_3+2,R_4+2)\\ \Delta_{\mathcal{O}}=2+\frac{1}{2}(R_3+R_4)\end{matrix}\right\}$	$Q^{10}\mathcal{O}\in\left\{\begin{matrix}(0,0,R_3,R_4)\\ \Delta=7+\frac{1}{2}(R_3+R_4)\end{matrix}\right\}$	–
$B_1\left\{\begin{matrix}(0,R_2+2,R_3,R_4)\\ \Delta_{\mathcal{O}}=2+R_2+\frac{1}{2}(R_3+R_4)\end{matrix}\right\}$	$Q^{12}\mathcal{O}\in\left\{\begin{matrix}(0,R_2,R_3,R_4)\\ \Delta=8+R_2+\frac{1}{2}(R_2+R_3)\end{matrix}\right\}$	–
$B_1\left\{\begin{matrix}(R_1+2,R_2,R_3,R_4)\\ \Delta_{\mathcal{O}}=2+R_1+R_2+\frac{1}{2}(R_3+R_4)\end{matrix}\right\}$	$Q^{14}\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2,R_3,R_4)\\ \Delta=9+R_1+R_2+\frac{1}{2}(R_3+R_4)\end{matrix}\right\}$	–
$L\left\{\begin{matrix}(R_1,R_2,R_3,R_4)\\ \Delta_{\mathcal{O}}>1+R_1+R_2+\frac{1}{2}(R_3+R_4)\end{matrix}\right\}$	$Q^{16}\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2,R_3,R_4)\\ \Delta>9+R_1+R_2+\frac{1}{2}(R_3+R_4)\end{matrix}\right\}$	D-Term

Table 16: Deformations of three-dimensional $\mathcal{N}=8$ SCFTs. The R-charges of the deformation are denoted by the $\mathfrak{so}(8)_R$ Dynkin labels $R_1,R_2,R_3,R_4\in\mathbb{Z}_{\geq 0}$.

universal mass

all others irrelev.

4d, N=3 (all irrelevant)

Primary $\mathcal{O}$	**Deformation $\delta\mathscr{L}$**	**Comments**
$B_1\overline{B}_1\left\{\begin{matrix}(R_1+4,0\,;\,2R_1+8)\\ \Delta_{\mathcal{O}}=4+R_1\end{matrix}\right\}$	$Q^4\overline{Q}^2\mathcal{O}\in\left\{\begin{matrix}(R_1,0\,;\,2R_1+6)\\ \Delta=7+R_1\end{matrix}\right\}$	F-Term $(*)$
$B_1\overline{B}_1\left\{\begin{matrix}(0,R_2+4\,;\,-2R_2-8)\\ \Delta_{\mathcal{O}}=4+R_2\end{matrix}\right\}$	$Q^2\overline{Q}^4\mathcal{O}\in\left\{\begin{matrix}(0,R_2\,;\,-2R_2-6)\\ \Delta=7+R_2\end{matrix}\right\}$	F-Term $(*)$
$B_1\overline{B}_1\left\{\begin{matrix}\left(R_1+2,R_2+2\,;\,2(R_1-R_2)\right)\\ \Delta_{\mathcal{O}}=4+R_1+R_2\end{matrix}\right\}$	$Q^4\overline{Q}^4\mathcal{O}\in\left\{\begin{matrix}\left(R_1,R_2\,;\,2(R_1-R_2)\right)\\ \Delta=8+R_1+R_2\end{matrix}\right\}$	–
$L\overline{B}_1\left\{\begin{matrix}(0,0\,;\,r+6)\ ,\ r>0\\ \Delta_{\mathcal{O}}=1+\frac{1}{6}r\end{matrix}\right\}$	$Q^6\mathcal{O}\in\left\{\begin{matrix}(0,0\,;\,r)\ ,\ r>0\\ \Delta=4+\frac{1}{6}r>4\end{matrix}\right\}$	F-term $(\star)$
$B_1\overline{L}\left\{\begin{matrix}(0,0\,;\,r-6)\ ,\ r<0\\ \Delta_{\mathcal{O}}=1-\frac{1}{6}r\end{matrix}\right\}$	$\overline{Q}^6\mathcal{O}\in\left\{\begin{matrix}(0,0\,;\,r)\ ,\ r<0\\ \Delta=4-\frac{1}{6}r>4\end{matrix}\right\}$	F-Term $(\star)$
$L\overline{B}_1\left\{\begin{matrix}(R_1+2,0\,;\,r+4)\ ,\ r>2R_1+6\\ \Delta_{\mathcal{O}}=2+\frac{2}{3}R_1+\frac{1}{6}r\end{matrix}\right\}$	$Q^6\overline{Q}^2\mathcal{O}\in\left\{\begin{matrix}(R_1,0\,;\,r)\ ,\ r>2R_1+6\\ \Delta=6+\frac{2}{3}R_1+\frac{1}{6}r>7+R_1\end{matrix}\right\}$	$(\dagger)$
$B_1\overline{L}\left\{\begin{matrix}(0,R_2+2\,;\,r-4)\ ,\ r<-2R_2-6\\ \Delta_{\mathcal{O}}=2+\frac{2}{3}R_2-\frac{1}{6}r\end{matrix}\right\}$	$Q^2\overline{Q}^6\mathcal{O}\in\left\{\begin{matrix}(0,R_2\,;\,r)\ ,\ r<-2R_2-6\\ \Delta=6+\frac{2}{3}R_2-\frac{1}{6}r>7+R_2\end{matrix}\right\}$	$(\dagger)$
$L\overline{B}_1\left\{\begin{matrix}(R_1,R_2+2\,;\,r+2)\ ,\ r>2(R_1-R_2)\\ \Delta_{\mathcal{O}}=3+\frac{2}{3}(R_1+2R_2)+\frac{1}{6}r\end{matrix}\right\}$	$Q^6\overline{Q}^4\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2\,;\,r)\ ,\ r>2(R_1-R_2)\\ \Delta=8+\frac{2}{3}(R_1+2R_2)+\frac{1}{6}r>8+R_1+R_2\end{matrix}\right\}$	$(\ddagger)$
$B_1\overline{L}\left\{\begin{matrix}(R_1+2,R_2\,;\,r-2)\ ,\ r<2(R_1-R_2)\\ \Delta_{\mathcal{O}}=3+\frac{2}{3}(2R_1+R_2)-\frac{1}{6}r\end{matrix}\right\}$	$Q^4\overline{Q}^6\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2\,;\,r)\ ,\ r<2(R_1-R_2)\\ \Delta=8+\frac{2}{3}(2R_1+R_2)-\frac{1}{6}r>8+R_1+R_2\end{matrix}\right\}$	$(\ddagger)$
$L\overline{L}\left\{\begin{matrix}(R_1,R_2\,;\,r)\\ \Delta_{\mathcal{O}}>2+\max\left\{\begin{matrix}\frac{2}{3}(2R_1+R_2)-\frac{1}{6}r\\ \frac{2}{3}(R_1+2R_2)+\frac{1}{6}r\end{matrix}\right\}\end{matrix}\right\}$	$Q^6\overline{Q}^6\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2\,;\,r)\\ \Delta>8+\max\left\{\begin{matrix}\frac{2}{3}(2R_1+R_2)-\frac{1}{6}r\\ \frac{2}{3}(R_1+2R_2)+\frac{1}{6}r\end{matrix}\right\}\end{matrix}\right\}$	D-Term

Table 25: Deformations of four-dimensional $\mathcal{N}=3$ SCFTs. The $\mathfrak{su}(3)_R$ Dynkin labels $R_1, R_2 \in \mathbb{Z}_{\geq 0}$ and the $\mathfrak{u}(1)_R$ charge $r \in \mathbb{R}$ denote the R-symmetry representation of the deformation.

d=5, 6 = simpler

No exotic susy deformations (not yet a 100% proof).

5d, N=1:

$$Q^2 C_1[0,0]^{R=2} = [0,0]_4^{R=0}$$ mass terms via flavor symms

(E.g. gauge kinetic terms)

$$Q^4 C_1[0,0]^{R+4} = [0,0]^R_{8+\frac{3}{2}R}$$ irrel. F-terms

$$Q^8 L_1[0,0]^R = [0,0]^R_{\Delta>8+\frac{3}{2}R}$$ irrel. D-terms

6d, N=(1,0):

$$Q^4 D_1[0,0,0]^{R+4} = [0,0,0]^R_{\Delta=10+2R}$$ irrel. F-terms

$$Q^8 L[0,0,0]^R = [0,0,0]^R_{\Delta>10+2R}$$ irrel. D-terms

Conclude

- QFT is vast, still much to be found.
- susy QFTs and RG flows are rich, useful testing grounds for exploring QFT. Strongly constrained: unitarity, a-thm., etc. Can rule out some things. Exact results for others.
- Thank you !