Solutions of the Strong CP Problem: A Scorecard

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Usually speak of three solutions of the strong CP problem

1. $m_u = 0$
2. Spontaneous CP violation with nearly vanishing θ (“Nelson-Barr" or NB) (and related: spontaneous breaking of P -Barr, Mohapatra, Senjanovic)
3. The axion, or the Peccei-Quinn symmetry

There are others (e.g. Hiller and Schmaltz, Anson Hook) which can be shoehorned into this classification scheme).

Among naturalness problems, the strong CP problem is special in that it is of almost no consequence. We don't have to invoke anthropic selection to realize that if the cosmological constant was a few orders of magnitude larger than observed, the universe would be dramatically different. The same is true for the value of the weak scale and of the light quark and lepton masses. But if θ were, say, 10^{-3}, nuclear physics would hardly be different than we observe, since effects of θ are shielded by small quark masses.

So while theorists may be endlessly clever in providing solutions to the problem, we might choose to be guided by a principle that the smallness of θ should be *incidental to* other aspects of physical theory, or, at the least, a plausible accident of features of an underlying theory.

Our goal today is to ask to look at how each of these solutions might fare under this principle.

Outline

- θ renormalization in the Standard Model
- $m_u = 0$.
  1. Theoretical justifications
  2. Lattice status and a proposed calibration of lattice measurements of m_q
  3. Generalizations
- The axion: PQ *Quality*
- Spontaneous CP Violation (Nelson-Barr)
  1. Two issues: higher dimension operators (bound the CP-violating scale); loop corrections (matching)
  2. Tuning of parameters
  3. Role of axions (and reprise for $m_u = 0$)
  4. Nelson-Barr in a landscape
- Spontaneous P Violation
- Conclusions

What is the Strong CP Problem

Two sources of CP violation in QCD, related by anomaly:

$$\frac{\theta}{16\pi^2} F\tilde{F}; \ \arg\, \det m_q$$

$\bar{\theta} = \theta - \arg \det m_q < 10^{-10}$.

As convenient, can use anomaly redefine fields so θ or $\arg \det m_q$ is zero.

Suppose there is a scale, Λ_{SM}, below which the only degrees of freedom are those of the SM. Define fields so $\arg \det m_q = 0$. Then ask about radiative corrections to this quantity.

Loop Corrections at Low Energies in the Standard Model

Loop corrections to θ in the Standard Model are highly suppressed. Focussing on divergent corrections, one requires Higgs loops. These involve the Hermitian matrices

$$A = y_d^\dagger y_d; \quad B = y_u^\dagger y_u \tag{1}$$

Contributions to θ are proportional to traces of the form

$$Tr(ABA^2B\dots) \tag{2}$$

one additional matrix factor for each loop.

It is easy to check that the first complex combination involves six matrices, e.g.

$$\mathrm{Tr}(ABA^2B^2) \tag{3}$$

but this and its complex conjugate both appear with the same coefficient. It is necessary to add a $U(1)$ gauge loop (which distinguishes u and d) to have the possibility of a complex traces. [Ellis, Gaillard]

So if θ is small at Λ_{SM}, further corrections are extremely tiny (finite corrections are also very small).

Why might this be?

$m_u = 0$

If $m_u = 0$, one can rotate away θ. More precisely, one requires

$$\frac{m_u}{m_d} < 10^{-10} \tag{4}$$

at the scale Λ_{SM}. There are two issues with this proposal:

1. Why might m_u be so small?
2. We can measure m_u (with the help of the lattice). Is this consistent with lattice results?

Accounting for small m_u

Banks, Nir, Seiberg put forward models which, in accounting for quark flavor, gave rise to small or zero m_u.

A simple possibility is suggested by string theory, which often exhibits anomalous discrete symmetries; more precisely, the chiral content of the theory is anomalous, with the anomaly being cancelled by the non-linear transformation of an axion-like field. In the supersymmetric case, this means that one has a modulus field, coupling to the $\bar{u}$ quark as

$$e^{-\Phi} Q H_U \bar{u}. \tag{5}$$

One requires that the exponential be very small, but this is plausible. One can speculate as to whether or not a suitable discrete symmetry structure is typical of underlying theories.

How might $m_u = 0$ be consistent with known facts of hadron physics

Instantons suggestive (Georgi-McArthur). With three light quarks, generate an effective u quark mass (two point function) proportional to $m_d m_s$. Simple dimensional analysis suggests the effect goes as

$$\frac{m_d m_s}{\Lambda} \tag{6}$$

with Λ a suitable QCD scale. This could easily be of order the few MeV expected from current algebra. Kaplan and Manohar expressed this as an ambiguity in current algebra, i.e. they isolated a term and second order in quark masses which could mimic the effects of a u quark mass.

Summary of lattice results for light quark masses

Current results from lattice simulations (summarized by the FLAG working group) are inconsistent with $m_u = 0$.

$$m_u = 2.16\ (9)(7)\mathrm{MeV} \quad m_d = 4.68\ (14)(7)\mathrm{MeV} \tag{7}$$

$$m_s = 93.5(2.5)\mathrm{MeV}$$

Numbers are in $\overline{MS}$ scheme at 2 GeV.

So m_u is many standard deviations from zero. Probably end of story, but some proposals for dedicated tests (Kitano), calibrations (Dine, Draper, Festuccia).

The Axion

(Pseudo)-scalar field, a, with approximate (*Peccei-Quinn*) symmetry $a \to a + \alpha$ and coupling

$$\frac{a}{32\pi^2 f_a} F\tilde{F} \tag{8}$$

Absorb θ into a. a part of low energy theory; low energy theory breaks PQ symmetry, favors $a = 0$.

The Challenge for the PQ Solution: Axion *Quality*

Global symmetries should arise only as accidents of gauge symmetry and the structure of low dimension terms in an effective action. It has been recognized almost from the beginning that this is a challenge for the axion solution of the strong CP problem.

With δV the contribution to $V(a)$ from scales above the QCD scale, we can define an axion quality factor, Q_a, as

$$Q_a = \frac{f_a \frac{\partial \delta V(a)}{\partial a}}{m_\pi^2 f_\pi^2} \tag{9}$$

Solving the strong CP problem requires

$$Q_a < 10^{-10} \tag{10}$$

In a conventional effective field theory analysis (i.e. finite number of degrees of freedom above f_a), this is quite a challenge. If

$$\langle \Phi \rangle = f_a e^{ia/f_a} \tag{11}$$

symmetry violating operators like

$$\frac{\Phi^{n+4}}{M_p^n} \tag{12}$$

make too large a contribution to Q_a even for $f_a = 10^{11}$ GeV unless $n > 7$. We might try to achieve this with a discrete Z_N, but this requires $N = 11$ at least, which certainly violates our minimalist principle.

Witten pointed out early on that string theory provides a possible resolution to this conundrum.

This is most easily understood in the framework of supersymmetry. Typically string models possess moduli, Φ, whose imaginary component obeys a discrete shift symmetry:

$$\Phi = x + ia; \quad a \to a + 2\pi \tag{13}$$

This insures, for example, that any superpotential is a function of $e^{-\Phi}$ at large x. Here x might be $\frac{8\pi^2}{g^2}$ for some gauge coupling g.

So the issue becomes: why or whether the theory sits in an asymptotic region of the moduli space where e^{-X} is very small. One can put forward various scenarios (and this is consistent at least with the fact that the observed gauge couplings are small), but reliable computations are not possible at present. Correlated with possibility of large field inflation (Laurel Stephenson-Haskins, M.D.)

String axions point to large f_a. Requires a separate talk (tied up with so-called *moduli problem*.

The Nelson-Barr mechanism

Unlike axion, $m_u = 0$ solutions, no obvious low energy consequences.

Attempts to achieve a setup where θ at the scale Λ_{SM} is extremely small.

Invokes spontaneous CP violation to argue “bare θ" is zero. Constructs a mass matrix such that spontaneous CP breaking gives a large CKM angle (as observed, $\delta = 1.2$) with $\arg \det m_q = 0$.

Bare θ is tree level θ (presumes some perturbative approximation). Must insure that $\theta(\Lambda_{SM})$ is small.

Such a structure is perhaps made plausible by string theory, where CP is a (gauge) symmetry, necessarily spontaneously broken. At string scale, $\theta = 0$ a well-defined notion. Some features of the required mass matrices appear, e.g., in Calabi-Yau compactifications of the heterotic string.

Simple realization of the NB structure

Complex scalars η_i with complex (CP-violating) vev's.
Additional vectorlike quark with charge $1/3$.

$$\mathcal{L} = \mu \bar{q} q + \lambda_{if} \eta_i \bar{d}_f q + y_{fg} Q_f \bar{d}_g \phi \tag{14}$$

where ϕ is Higgs; y, λ, μ real.

$$M = \begin{pmatrix} \mu & B \\ 0 & m_d \end{pmatrix} \tag{15}$$

$B_f = \lambda_{if} \eta_i$ is complex. M has real determinant.

The structure is reminiscent of an E_6 gauge theory, which has the requisite vector-like quarks and singlets.

Requirements for a successful NB Solution

1. Symmetries: It is important that η_i not couple to $\bar{q}q$, for example. So, e.g., η's complex, subject to a Z_N symmetry.
2. Coincidences of scale: if only one field η, CKM angle vanishes (can make d quark mass matrix real by an overall phase redefinition). Need at least two, and their vev's (times suitable couplings) have to be quite close:

$$\delta_{CKM} \propto \frac{B_{small}}{B_{large}} \qquad (16)$$

3. Similarly, μ (which might represent vev of another field) can not be much larger than η_i, and if much smaller the Yukawa's and B's have to have special features.

Constraints on the Overall Scale

Before considering radiative effects, possible higher dimension operators in $\mathcal{L}$ constrain the scales η_i, μ. E.g.

$$\frac{\eta_i^* \eta_j}{M_p} \bar{q} q \tag{17}$$

requires $\frac{|\eta|}{M_p} < 10^{-10}$.

Barr-Nelson With/Without Supersymmetry

Without supersymmetry, highly tuned. Two light scalars and μ (or three light scalars), with masses 10 orders of magnitude below M_p. Far worse than θ.

Even ignoring that, require close coincidence of scales.

Supersymmetry helps. Allows light scalars. Coincidences still required (and more chiral multiplets to achieve desired symmetry breakings – typically at least seven). Some of the high dimension operators better controlled (e.g. if μ, η_i much larger than susy breaking scale, don't have analogs of the $\eta_i^* \eta_j \bar{q} q$ operator).

Axions in the NB Scenario

What does it mean that the “bare" θ is naturally zero in a model which is CP-conserving at some underlying level? String theory provides a realization. Here one might mean that the vev’s of the moduli are CP conserving, i.e. that the various axions have vanishing vev. These axions might be presumed to be heavier than the conventional QCD axion (otherwise they would provide a PQ resolution of strong CP). Such masses could arise from strong string effects, or other strong gauge groups.

So NB might be considered a particular limit of the PQ picture. Here it is not necessary that the quality be particular good, provided that $\arg\det m_q \approx 0$ and the axion coupling to the fields which break CP is weak enough.

How plausible is $\theta_{bare} = 0$

Thinking of "θ_{bare}" as the expectation value of some axion-like field, one can ask: how likely is it that this quantity vanishes. One model: flux landscapes. Here, "KKLT" as a model. Superpotential

$$W = e^{-\Phi/b} + W_0. \quad (18)$$

Supersymmetric stationary points have

$$\phi \approx b \log(W_0). \quad (19)$$

$\theta_{bare} = 0$ requires that W_0 is real.

In a landscape, this is likely to be extremely rare. W_0 a sum of many determines determined by fluxes. Roughly speaking requires that all CP-odd fluxes (presumably 1/2) should vanish. Exponential suppression.

Loop Corrections in Nelson-Barr: Non-Supersymmetric case

In the non-supersymmetric case, in the simplest model, potential corrections arise at one loop order. Consider, in particular, couplings of the form

$$\lambda_{ij}\eta_i\eta_j|H|^2$$

give rise to one loop contributions.

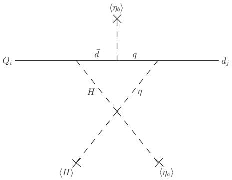

Figure 1: Example threshold correction to Arg det m_d.

If the new couplings are of order one these are six or seven orders of magnitude too large.

In the past these have sometimes been dismissed on the grounds that these couplings contribute to the Higgs mass, but this is just part of the usual fine tuning problem.

Supersymmetry breaking and Nelson-Barr

Many possible phases once allow soft breaking **Note: these effects don't decouple for large susy-breaking scale**. E.g. is susy breaking described by Goldstino superfield, X, superpotential couplings

$$\frac{\mathcal{O}_d}{M_p^{d-2}} X \tag{20}$$

where $\langle \mathcal{O} \rangle$ is complex can lead to large phases in soft breakings. Similarly phases in W. Phases in gaugino masses feed directly into θ.

Loop Corrections in Supersymmetric Nelson-Barr

If tree level phases in soft terms suppressed, loops still pose a problem (Kagan, Leigh, M.D.). Loop corrections to gaugino mass from loops with q, $\bar{q}$, fields. Require, e.g., A terms small or proportional to Yukawas. Gauge mediation (with real F) most plausible solution (A terms small). (Luty, Schmaltz in a slightly different context)

A variant: Spontaneous P Violation

P can forbid θ. So perhaps CP phases order one when allowed, but enforce P.

Long history. Often motivated by unification in $O(10)$. Examples include model of Barr and Senjanovic; recent work of Hook.

Take Hook's model as an example.

Gauge group: $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$

Hook's Model: Particle content:

Ordinary quarks and leptons

$$Q = (3,2,1)_{1/3} \; \bar{u} = (\bar{3},1,1,)_{-4/3} \; \bar{d} = (\bar{3},1,1)_{2/3};$$

$$L = (1,2,1)_{-1} \; \bar{e} = (1,1,1)_{2}.$$

Mirror quarks and leptons:

$$\bar{Q}' = (\bar{3},1,2)_{-1/3} \; u' = (3,1,1,)_{4/3} \; d' = (3,1,1)_{-2/3};$$

$$L' = (1,1,2)_{1} \; e' = (1,1,1)_{-2}.$$

Parity takes $SU(2)_L \leftrightarrow SU(2)_R$ $Q \leftrightarrow \bar{Q}'^*$, $\bar{u} \leftrightarrow u'^*$ etc.

Problems typically more severe than NB:

1. Higher dimension operators place upper bounds on scales. Fine tuning severe without supersymmetry.
2. Radiative corrections problematic.
3. With supersymmetry, phases in $H_U H_D$ and $H'_U H'_D$ independent, problematic (in addition to problems encountered in NB case). Gauge mediation does not automatically fix this difficulty.

Conclusions

Each proposed solution to the strong CP problem raises troubling questions. We have argued, indeed, that θ is of so little importance that any solution should be an outcome of some other constraint on the physical theory. Solutions which require many additional degrees of freedom, intricate symmetries, or significant fine tuning have little plausibility.

1. A very light *u* quark might be a consequence of horizontal symmetries, or might arise as a result of the anomalous discrete symmetries which seem ubiquitous in string theory. However, there are now lattice computations which appear to definitively rule out the possibility.
2. The axion raises the issues of the quality of the PQ symmetry. String theory suggests a plausible answer, but our understanding is limited.
3. Nelson-Barr: The basic premise, that if the underlying theory is CP conserving, the “bare" θ vanishes, is open to question; it requires an understanding of how certain moduli are stabilized, and in a landscape would seem unlikely. Allowing this, the mechanism requires a low scale for CP violation. Without supersymmetry this is highly tuned. With supersymmetry, still coincidences. Loop corrections very problematic without supersymmetry. With supersymmetry, severe difficulties except, perhaps, with gauge mediation.

Other variants exist (Hiller-Schmaltz, Hook). Similar issues.

I will leave it to you to make a final scoresheet, and a viewpoint on which solution of the strong CP problem is the most likely.